Exercise 1.2

1. Find the values of x and y in each of the following.

i) $x+iy+2-3i=i(5-i)(3+4i)$

Sol: $x+2+iy-3i=i(15+20i-3i-4i^2)$

$(x+2)+i(y-3)=i(15+17i-4(-1))$

$(x+2)+i(y-3)=i(15+17i+4)$

$(x+2)+i(y-3)=i(19+17i)$

$(x+2)+i(y-3)=19i+17i^2$

$(x+2)+i(y-3)=19i+17(-1)$

$(x+2)+i(y-3)=-17+19i$

Comparing real and imaginary parts

$x+2=-17$, $y-3=19$

$x=-17-2$, $y=19+3$

$x=-19$, $y=22$

ii) $(x+iy)(1-i)=(2-3i)(-5+5i)\left(-i\frac{3}{5}\right)$

Sol: $(x+iy)\cancel{(1-i)}=(2-3i)(-\cancel{5})\cancel{(1-i)}\frac{-3i}{\cancel{5}}$

$x+iy=3i(2-3i)$

$x+iy=6i-9i^2$

$x+iy=6i-9(-1)$

$x+iy=9+6i$

$x=9$, $y=6$

iii) $\frac{x}{2+i}+\frac{y}{3-i}=4+5i$

Sol: $\frac{x}{2+i}.\frac{2-i}{2-i}+\frac{y}{3-i}.\frac{3+i}{3+i}=4+5i$

$\frac{(2-i)x}{(2)^2-(i)^2}+\frac{(3+i)y}{(3)^2-(i)^2}=4+5i$

$\frac{2x-ix}{4-(-1)}+\frac{3y+iy}{9-(-1)}=4+5i$

$\frac{2x-ix}{5}+\frac{3y+iy}{10}=4+5i$

$\frac{10(2x-ix)+5(3y+iy)}{50}=4+5i$

$20x-10ix+15y+5iy=50(4+5i)$

$20x+15y-10ix+5iy=200+250i$

$20x+15y+i(-10x+5y)=200+250i$

Comparing real and imaginary parts

$20x+15y=200$ (i),

$-10x+5y=250$ (ii)

Multiply (ii) by 2 and add to (i)

$\cancel{20x}+15y=200$

$\underline{-\cancel{20x}+10y=500}$

$25y=700$ $\Rightarrow y=28$

(ii) $\Rightarrow -10x+5(28)=250$

$-10x+140=250$

$-10x=250-10$

$\Rightarrow -10x=110$

$\Rightarrow x=-11$

2. If $z_1=-13+24i$, $z_2=x+yi$ find the real values of x and y such that $z_1-z_2=-27+15i$

Sol: $z_1-z_2=-27+15i$

$(-13+24i)-(x+yi)=-27+15i$

$-13+24i-x-yi=-27+15i$

$-13-x+24i-yi=-27+15i$

$(-13-x)+i(24-y)=-27+15i$

$-13-x=-27$, $24-y=15$

$-13+27=x$, $24-15=y$

$x=14$, $y=9$

3. Find the real values of x and y if:

i) $(x+iy)^2=25+60i$

Sol: $x^2+(iy)^2+2ixy=25+60i$

$x^2-y^2+2ixy=25+60i$

$x^2-y^2=25$, $2xy=60$

$x^2-y^2=25$ (i) , $xy=30$

$\Rightarrow y=\frac{30}{x}$ (ii)

Put (ii) in (i)

$x^2-\left(\frac{30}{x}\right)^2=25 \Rightarrow x^2-\frac{900}{x^2}=25$

$x^2.x^2-\frac{900}{x^2}.x^2=25x^2$ (Multiply by x^2)

$x^4-900=25x^2$

$x^4-25x^2-900=0$

$$x^4-45x^2+20x^2-900=0$$
$$x^2\left(x^2-45\right)+20\left(x^2-45\right)=0$$
$$\left(x^2-45\right)\left(x^2+20\right)=0$$
$$x^2-45=0\quad,\ x^2+20=0$$
$$x^2=45\quad,\ x^2=-20$$
$$x=\pm\sqrt{45}\quad,\ x=\pm\sqrt{-20}$$
$x=\pm3\sqrt{5}\quad,x=\pm2i\sqrt{5}$ (ignore complex number)

Put $x=\pm3\sqrt{5}$ in (ii)
$$y=\frac{30}{\pm3\sqrt{5}}=\pm\frac{10}{\sqrt{5}}=\pm2\sqrt{5}$$

ii) $\boldsymbol{(x+iy)^2=64+48i}$

Sol: $x^2+(iy)^2+2ixy=64+48i$
$$x^2-y^2+2ixy=64+48i$$
$$x^2-y^2=64\quad,2xy=48$$
$$x^2-y^2=64\quad(i)\quad,\quad xy=24$$
$$\Rightarrow y=\frac{24}{x}\qquad(ii)$$
Put in (i)
$$x^2-\left(\frac{24}{x}\right)^2=64$$
$$x^2-\frac{576}{x^2}=64$$
$x^2.x^2-\frac{576}{x^2}.x^2=64x^2$ (Multiply by x^2)
$$x^4-576=64x^2$$
$$x^4-64x^2-576=0$$
$$x^4-72x^2+8x^2-576=0$$
$$x^2\left(x^2-72\right)+8\left(x^2-72\right)=0$$
$$\left(x^2-72\right)\left(x^2+8\right)=0$$
$$x^2-72=0\ ,\ x^2+8=0$$
$$x^2=72\quad,\ x^2=-8$$
$$x=\pm\sqrt{72}\quad,\ x=\pm\sqrt{-8}$$
$x=\pm6\sqrt{2}\quad,\ x=\pm2i\sqrt{2}$ (ignore complex number)

Put $x=\pm6\sqrt{2}$ in (ii)
(Taking only real values as $x,y\in R$)
$$y=\frac{24}{\pm6\sqrt{2}}=\pm\frac{4}{\sqrt{2}}=\pm2\sqrt{2}$$

iii) $\boldsymbol{(x+iy)^2=\frac{2i-3}{3+i}}$

Sol: $(x+iy)^2=\frac{2i-3}{3+i}$
$$=\frac{2i-3}{3+i}\times\frac{3-i}{3-i}=\frac{6i-2i^2-9+3i}{(3)^2-(i)^2}$$
$$=\frac{6i+2-9+3i}{9-(-1)}$$
$$(x+iy)^2=\frac{-7+9i}{10}$$
$$x+iy=\pm\sqrt{\frac{-7}{10}+\frac{9}{10}i}\text{-----}(1)$$
Let $\quad z=\frac{-7}{10}+\frac{9}{10}i=-0.7+0.9i$
$$|z|=\sqrt{\left(\frac{-7}{10}\right)^2+\left(\frac{9}{10}\right)^2}$$
$$=\sqrt{\frac{49}{100}+\frac{81}{100}}=\sqrt{\frac{130}{100}}=\sqrt{\frac{13}{10}}\approx1.14$$
Now using formula
$$\sqrt{z}=\left(\pm\sqrt{\frac{|z|+x}{2}}+i\frac{y}{|y|}\sqrt{\frac{|z|-x}{2}}\right)$$
$$\sqrt{\frac{-7}{10}+\frac{9}{10}i}=\pm\left(\sqrt{\frac{1.14-0.7}{2}}+i\frac{0.9}{|0.9|}\sqrt{\frac{1.14-(-0.7)}{2}}\right)$$
$$=\pm(0.46+0.95i)$$
Hence $x=\pm0.46\quad,\quad y=\pm0.95$

4. **If $\boldsymbol{z_1=2+3i}$ and $\boldsymbol{z_2=1-\alpha i}$,**
Find real values of α such that $Im(z_1z_2)=7$

Sol: $z_1.z_2=(2+3i)(1-\alpha i)$
$$z_1z_2=2-2\alpha i+3i-3\alpha i^2$$
$$z_1z_2=2+i(-2\alpha+3)-3\alpha(-1)$$
$$z_1z_2=2+i(-2\alpha+3)+3\alpha$$
$$z_1z_2=2+3\alpha+i(-2\alpha+3)$$
$$Im(z_1z_2)=-2\alpha+3$$
Given $Im(z_1z_2)=7$
$$-2\alpha+3=7$$
$$3-7=2\alpha$$
$$\Rightarrow 2\alpha=-4$$
$$\Rightarrow \alpha=-2$$

5. If $z_1=x+yi$, $z_2=a+bi$, find x, y, a and b Such that $z_1+z_2=10+4i$ and $z_1-z_2=6+2i$

Sol: $z_1+z_2=10+4i \quad (i)$

$z_1-z_2=6+2i \quad (ii)$

$z_1+\not{z_2}=10+4i$

$z_1-\not{z_2}=6+2i$

$2z_1=16+6i$

$z_1=8+3i$

$x+yi=8+3i$

$x=8,\ y=3$

$\not{z_1}+z_2=10+4i$

$-\not{z_1}\mp z_2=-6\pm 2i$

$2z_2=4+2i$

$z_2=2+i$

$a+bi=2+i$

Comparing real and imaginary parts

$a=2 \quad b=1$

6. Show that for $\forall\ z_1,z_2\in C$, $\overline{z_1z_2}=\overline{z_1}\,\overline{z_2}$

Sol: $z_1=a+bi \qquad \overline{z_1}=a-bi$

$z_2=c+di \qquad \overline{z_2}=c-di$

$z_1z_2=(a+ib)(c+id)$

$=ac+iad+ibc+bdi^2$

$=ac+i(ad+bc)+bd(-1)$

$=ac+i(ad+bc)-bd$

$z_1z_2=ac-bd+i(ad+bc)$

L.H.S $=\overline{z_1z_2}=ac-bd-i(ad+bc)$ (i)

R.H.S. $=\overline{z_1}\,\overline{z_2}$

$=(a-ib)(c-id)$

$=ac-iad-ibc+bdi^2$

$=ac-i(ad+bc)+bd(-1)$

$=ac-i(ad+bc)-bd$

$=ac-bd+i(ad+bc)$ (ii)

From (i) & (ii)

L.H.S= R.H.S.

7. Find the square root of the following complex numbers.

i. $-7-24i$

Sol: Let $z=-7-24i$ here $x=-7,\ y=-24$

$|z|=\sqrt{(-7)^2+(-24)^2}=\sqrt{49+576}=\sqrt{625}=25$

$\sqrt{z}=\pm\left(\sqrt{\frac{|z|+x}{2}}+\frac{iy}{|y|}\sqrt{\frac{|z|-x}{2}}\right)$

$\sqrt{z}=\pm\left(\sqrt{\frac{25+(-7)}{2}}+\frac{i(-\not{24})}{\not{24}}\sqrt{\frac{25-(-7)}{2}}\right)$

$\sqrt{z}=\pm\left(\sqrt{\frac{25-7}{2}}-i\sqrt{\frac{25+7}{2}}\right)=\pm\left(\sqrt{9}-i\sqrt{16}\right)$

$=\pm(3-i4)$

Hence $\sqrt{z}=3-4i$ or $-3+4i$

ii. $8-6i$

Sol: Let $z=8-6i$ here $x=8,\ y=-6$

$|z|=\sqrt{(8)^2+(-6)^2}=\sqrt{64+36}=\sqrt{100}=10$

Now, $\sqrt{z}=\pm\left(\sqrt{\frac{|z|+x}{2}}+\frac{iy}{|y|}\sqrt{\frac{|z|-x}{2}}\right)$

$\sqrt{z}=\pm\left(\sqrt{\frac{10+8}{2}}+\frac{i(-\not{6})}{\not{6}}\sqrt{\frac{10-8}{2}}\right)$

$\sqrt{z}=\pm\left(\sqrt{9}-i\sqrt{1}\right)=\pm(3-i)$

Hence $\sqrt{z}=3-i,-3+i$

iii. $-15-36i$

Sol Let $z=-15-36i$ here $x=-15,\ y=-36$

$|z|=\sqrt{(-15)^2+(-36)^2}=\sqrt{225+1296}=\sqrt{1521}=39$

Now, $\sqrt{z}=\pm\left(\sqrt{\frac{|z|+x}{2}}+\frac{iy}{|y|}\sqrt{\frac{|z|-x}{2}}\right)$

$\sqrt{z}=\pm\left(\sqrt{\frac{39+(-15)}{2}}+\frac{i(-\not{36})}{\not{36}}\sqrt{\frac{39-(-15)}{2}}\right)$

$\sqrt{z}=\pm\left(\sqrt{\frac{39-15}{2}}-i\sqrt{\frac{39+15}{2}}\right)=\pm\left(\sqrt{\frac{24}{2}}-i\sqrt{\frac{54}{2}}\right)$

$=\pm\sqrt{12}-i\sqrt{27}=\pm\left(2\sqrt{3}-i3\sqrt{3}\right)$

Hence $\sqrt{z}=2\sqrt{3}-i3\sqrt{3}$ or $-2\sqrt{3}+i3\sqrt{3}$

iv. $119+120i$

Sol Let $z=119+120i$ here $x=119$, $y=120$

$$|z|=\sqrt{(119)^2+(120)^2}=\sqrt{1416+14400}=\sqrt{25561}=169$$

Now, $$\sqrt{z}=\pm\left(\sqrt{\frac{|z|+x}{2}}+\frac{iy}{|y|}\sqrt{\frac{|z|-x}{2}}\right)$$

$$\sqrt{z}=\pm\left(\sqrt{\frac{169+119}{2}}+\frac{i(-120)}{120}\sqrt{\frac{169-119}{2}}\right)$$

$$\sqrt{z}=\pm\left(\sqrt{\frac{288}{2}}+i\sqrt{\frac{50}{2}}\right)=\left(\sqrt{\frac{114}{2}}+i\sqrt{25}\right)$$

$$=\pm(12+5i)$$

Hence $\sqrt{z}=12+5i$ or $-12+5i$

8. Find the square root of $13-20\sqrt{3}\,i$ and represent it on an Argand Diagram.

Sol Let $z=13-20\sqrt{3}\,i$ here $x=13$, $y=-20\sqrt{3}$

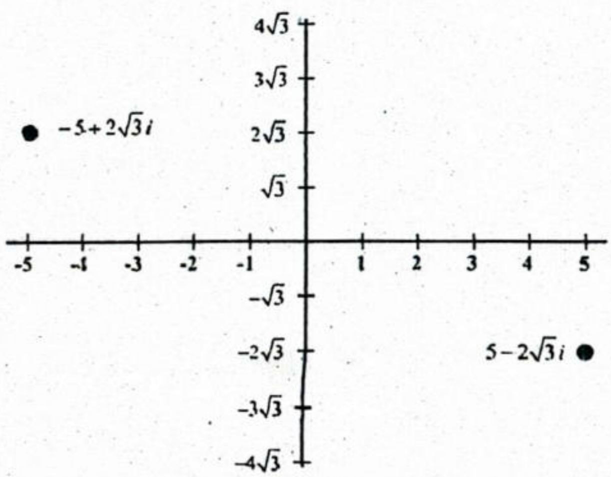
$$|z|=\sqrt{(13)^2+(-20\sqrt{3})^2}=\sqrt{169+1200}=\sqrt{1369}=37$$

Now, $$\sqrt{z}=\pm\left(\sqrt{\frac{|z|+x}{2}}+\frac{iy}{|y|}\sqrt{\frac{|z|-x}{2}}\right)$$

$$\sqrt{z}=\pm\left(\sqrt{\frac{37+13}{2}}+\frac{i(-20\sqrt{3})}{20\sqrt{3}}\sqrt{\frac{37-13}{2}}\right)$$

$$=\pm\left(\sqrt{25}-i\sqrt{12}\right)=\pm\left(5-2\sqrt{3}\,i\right)$$

Hence $\sqrt{z}=5-2\sqrt{3}\,i$ or $\sqrt{z}=-5+2\sqrt{3}\,i$



9. Find the real values of x and y if $(-7+i)(x+iy)+(-1-5i)=i(11-i)$:

Sol: $(-7+i)(x+iy)+(-1-5i)=i(11-i)$

$(-7+i)(x+iy)=(11i-i^2)-(-1-5i)$

$(-7+i)(x+iy)=(11i-(-1))+1+5i$

$(-7+i)(x+iy)=11i+1+1+5i$

$(-7+i)(x+iy)=2+16i$

$$x+iy=\frac{2+16i}{-7+i}$$

$$x+iy=\frac{2+16i}{-7+i}\cdot\frac{-7-i}{-7-i}$$

$$=\frac{(2+16i)(-7-i)}{(-7)^2-i^2}=\frac{-14-2i-112i-16i^2}{49-(-1)}$$

$$=\frac{-14-114i-16(-1)}{50}=\frac{-14-114i+16}{50}$$

$$=\frac{2-114i}{50}=\frac{2}{50}-\frac{114i}{50}$$

$$x+iy=\frac{1}{25}-\frac{57}{25}i$$

$$x=\frac{1}{25},\ y=-\frac{57}{25}$$

10. Find the real value of x and y if $(5-2i)(x+iy)+3=i(11-i)-4i$:

Sol $(5-2i)(x+iy)+3=i(11-i)-4i$

$(5-2i)(x+iy)=(11i-i^2)-4i-3$

$(5-2i)(x+iy)=(11i-(-1))-4i-3$

$(5-2i)(x+iy)=11i+1-4i-3$

$(5-2i)(x+iy)=-2+7i$

$$x+iy=\frac{-2+7i}{5-2i}$$

$$x+iy=\frac{-2+7i}{5-2i}\cdot\frac{5+2i}{5+2i}$$

$$=\frac{(-2+7i)(5+2i)}{(5)^2-(2i)^2}=\frac{-10-4i+35i+14i^2}{25-(-4)}$$

$$=\frac{-10+31i+14(-1)}{29}=\frac{-10+31i-14}{29}$$

$$=\frac{-24+31i}{29}=\frac{-24}{29}+\frac{31i}{29}$$

$$x+iy=\frac{-24}{29}+\frac{31}{29}i$$

$$x=\frac{-24}{29},\ y=\frac{31}{29}$$

11. **Find the real value of u and v. If $\frac{u-2}{2+i}+\frac{v-3}{2-i}=4i$**

Sol: $\frac{u-2}{2+i}+\frac{v-3}{2-i}=4i$

$\frac{(2-i)(u-2)+(2+i)(v-3)}{(2)^2-(i)^2}=4i$

$\frac{2(u-2)-i(u-2)+2(v-3)+i(v-3)}{4-(-1)^2}=4i$

$\frac{2(u-2)+2(v-3)+i(v-3)-i(u-2)}{5}=4i$

$2u-4+2v-6+i(v-3-(u-2))=5(4i)$

$2u+2v-10+i(v-3-u+2)=20i$

$2u+2v-10+i(-u+v-1)=0+20i$

Comparing real and imaginary parts

$2u+2v-10=0$ and $-u+v-1=20$

$u+v-5=0 \quad (i)$

$-u+v-21=0 \quad (ii)$

Adding (i) and (ii)

$\cancel{u}+v-5=0$

$-\cancel{u}+v-21=0$

$2v-26=0$

$v=\frac{26}{2}=13$

$u+13-5=0$

$u+8=0$

$\Rightarrow u=-8$

12. **If $z_1=4+5i$, $z_2=\alpha-2i$, find the real value of α such that $\text{Re}(z_1z_2)=20$**

Sol: $z_1.z_2=(4+5i)(\alpha-2i)$

$z_1z_2=4\alpha-8i+5\alpha i-10i^2$

$z_1z_2=4\alpha+i(-8+5\alpha)-10(-1)$

$z_1z_2=4\alpha+i(-8+5\alpha)+10$

$z_1z_2=4\alpha+10+i(-8+5\alpha)$

$Re(z_1z_2)=4\alpha+10$

Given $Re(z_1z_2)=20$

$4\alpha+10=20$

$4\alpha=20-10$

$\Rightarrow 4\alpha=10$

$\Rightarrow \alpha=\frac{10}{4}=\frac{5}{2}$