# Exercise 1.3

**Q.1. Factorize the following.**

i) $a^2+4b^2$

Sol: $a^2+4b^2=a^2-(-4b^2)$

$=a^2-(i2b)^2$

$\therefore\ \{(i2b)^2=i^2 2^2 b^2=(-1)4b^2=-4b^2\}$

$=(a-i2b)(a+i2b)$

ii) $9a^2+16b^2$

Sol: $9a^2+16b^2=(3a)^2-(-16b^2)$

$=(3a)^2-(i4b)^2$

$\therefore\ \{(i4b)^2=i^2 4^2 b^2=(-1)16b^2=-16b^2\}$

$=(3a-i4b)(3a+i4b)$

iii) $3x^2+3y^2$

Sol: $3x^2+3y^2=3(x^2+y^2)$

$=3\{(x)^2-(-y^2)\}$

$=3\{(x)^2-(iy)^2\}$

$\therefore\ \{(iy)^2=i^2y^2=(-1)y^2=-y^2\}$

$=3(x-iy)(x+iy)$

iv) $144x^2+225y^2$

Sol: $144x^2+225y^2=9(16x^2+25y^2)$

$=9\{16x^2-(-25y^2)\}$

$=9\{(4x)^2-(i5y)^2$

$\therefore\ \{(i5y)^2=i^2 5^2 y^2=(-1)25y^2=-25y^2\}$

$=9(4x-i5y)(4x+i5y)$

v) $z^2-2iz-1$

Sol: $z^2-2iz-1=z^2-2iz+(-1)$

$=z^2-2iz+i^2\ =(z-i)^2=(z-i)(z-i)$

vi) $z^2+6z+13$

Sol: $z^2+6z+13=z^2+6z+9+4$

$=(z+3)^2+4$

$=(z+3)^2-(-4)$

$=(z+3)^2-(2i)^2$

$\therefore\ \{(2i)^2=2^2i^2=4(-1)=-4\}$

$=(z+3-2i)(z+3+2i)$

vii) $z^2+4z+5$

Sol: $z^2+4z+5=z^2+4z+4+1$

$=(z+2)^2+1=(z+2)^2-(-1)$

$=(z+2)^2-i^2=(z+2-i)(z+2+i)$

viii) $2z^2-22z+65$

Sol: $2z^2-22z+65=2\left(z^2-11z+\frac{65}{2}\right)$

$=2\left(z^2-11z+\frac{121}{4}+\frac{65}{2}-\frac{121}{4}\right)$

$=2\left(\left(z-\frac{11}{2}\right)^2+\frac{65}{2}-\frac{121}{4}\right)$

$=2\left(\left(z-\frac{11}{2}\right)^2+\frac{130-121}{4}\right)=2\left(\left(z-\frac{11}{2}\right)^2+\frac{9}{4}\right)$

$=2\left(\left(z-\frac{11}{2}\right)^2-\left(\frac{3}{2}i\right)^2\right)$

$\therefore\ \left\{\left(\frac{3}{2}i\right)^2=\left(\frac{3}{2}\right)^2 i^2=\frac{9}{4}(-1)=-\frac{9}{4}\right\}$

$=2\left(z-\frac{11}{2}-\frac{3}{2}i\right)\left(\left(z-\frac{11}{2}+\frac{3}{2}i\right)\right)$

**Q.2. Factorize the following polynomials into linear factors.**

i) $z^3+8$

Sol: $z^3+8=z^3+2^3$

$=(z+2)(z^2-2z+4)=(z+2)(z^2-2z+1+3)$

$=(z+2)((z-1)^2-(-3))$

$=(z+2)(z-1)^2-(\sqrt{3}i)^2)$

$\therefore\ \{(\sqrt{3}i)^2=(\sqrt{3})^2 i^2=3(-1)=-3\}$

$=(z+2)(z-1-\sqrt{3}i)(z-1+\sqrt{3}i)$

ii) $z^3+27$

Sol: $z^3+27=z^3+3^3=(z+3)(z^2-3z+9)$

$=(z+3)\left(z^2-3z+\frac{9}{4}-\frac{9}{4}+9\right)$

$=(z+3)\left(\left(z-\frac{3}{2}\right)^2+9-\frac{9}{4}\right)$

$$=(z+3)\left(\left(z-\frac{3}{2}\right)^2+\frac{36-9}{4}\right)$$

$$=(z+3)\left(\left(z-\frac{3}{2}\right)^2+\frac{27}{4}\right)$$

$$=(z+3)\left(\left(z-\frac{3}{2}\right)^2-\left(-\frac{27}{4}\right)\right)$$

$$=(z+3)\left(z-\frac{3}{2}\right)^2-\left(\frac{3\sqrt{3}}{2}i\right)^2$$

$$\therefore \left\{\left(\frac{3\sqrt{3}}{2}i\right)^2=\left(\frac{3\sqrt{3}}{2}\right)^2 i^2=\frac{27}{4}(-1)=-\frac{27}{4}\right\}$$

$$=(z+3)\left(z-\frac{3}{2}-\frac{3\sqrt{3}}{2}i\right)\left(z-\frac{3}{2}+\frac{3\sqrt{3}}{2}i\right)$$

**iii)** $z^3-2z^2+16z-32$

**Sol:** $z^3-2z^2+16z-32$

$=z^2(z-2)+16(z-2)$

$=(z-2)(z^2+16)=(z-2)(z^2-(-16))$

$=(z+3)((z)^2-(4i)^2)$

$\therefore \{(4i)^2=(4)^2 i^2=16(-1)=-16\}$

$=(z+3)(z-4i)(z+4i)$

**iv)** $z^4+21z^2-100$

**Sol:** $z^4+21z^2-100=z^4-4z^2+25z^2-100$

$=z^2(z^2-4)+25(z^2-4)$

$=(z^2-4)(z^2+25)=(z^2-2^2)(z^2-(-25))$

$=(z-2)(z+2)(z^2-(5i)^2)$

$\therefore \{(5i)^2=(5)^2 i^2=25(-1)=-25\}$

$=(z-2)(z+2)(z-5i)(z+5i)$

**v)** $z^4-16$

**Sol:** $z^4-16=(z^2)^2-(4)^2$

$=(z^2-4)(z^2+4)=(z^2-2^2)(z^2-(-4))$

$=(z-2)(z+2)(z^2-(2i)^2)$

$\therefore \{(2i)^2=(2)^2 i^2=4(-1)=-4\}$

$=(z-2)(z+2)(z-2i)(z+2i)$

**vi)** $z^4+3z^2-4$

**Sol:** $z^4+3z^2-4=z^4-z^2+4z^2-4$

$=z^2(z^2-1)+4(z^2-1)=(z^2-1)(z^2+4)$

$=(z^2-1^2)(z^2-(-4))$

$=(z-1)(z+1)(z^2-(2i)^2)$

$\therefore \{(2i)^2=(2)^2 i^2=4(-1)=-4\}$

$=(z-1)(z+1)(z-2i)(z+2i)$

**vii)** $z^4+5z^2+6$

**Sol:** $z^4+5z^2+6=z^4+2z^2+3z^2+6$

$=z^2(z^2+2)+3(z^2+2)=(z^2+2)(z^2+3)$

$=(z^2-(-2))(z^2-(-3))$

$=\left(z^2-(\sqrt{2}i)^2\right)\left(z^2-(\sqrt{3}i)^2\right)$

$=(z-\sqrt{2}i)(z+\sqrt{2}i)(z-\sqrt{3}i)(z+\sqrt{3}i)$

**viii)** $z^4-32z^2-3969$

**Sol:** $z^4-32z^2-3969$

$=z^4-81z^2+49z^2-3969$

$=z^4-81z^2+49z^2-3969$

$=z^2(z^2-81)+49(z^2-81)$

$=(z^2-81)(z^2+49)$

$=(z-9)(z+9)[z^2+(7i)^2]$

$=(z-9)(z+9)(z+7i)(z-7i)$

**Q.3. Find roots of $z^4+7z^2-144=0$ and hence expresses it as a product of linear factors.**

**Sol:** $z^4+7z^2-144=0$

$z^4-9z^2+16z^2-144=0$

$z^2(z^2-9)+16(z^2-9)=0$

$(z^2-9)(z^2+16)=0$

$z^2-9=0, z^2+16=0$

$z^2=9,\ z^2=-16$

$z=\pm 3,\ z=\pm 4i$

$(z-3)(z+3)(z-4i)(z+4i)=0$

$\Rightarrow z-3=0$ or $z+3=0$ or $z-4i=0$ or $z+4i=0$

$z=3$ or $z=-3$ or $z=4i$ or $z=-4i$

Hence roots are $3,-3,4i,-4i$ and linear product is

$(z-3)(z+3)(z-4i)(z+4i)$

**Q.4.** **Solve the following complex quadratic equations by completing square method.**

**i)** $2z^2-3z+4=0$

**Sol:** $2z^2-3z+4=0$

$\frac{2z^2}{2}-\frac{3}{2}z+\frac{4}{2}=0$ (Divide by 2)

$z^2-\frac{3}{2}z+2=0$

$z^2-\frac{3}{2}z=-2$

Add $\left(\frac{3}{4}\right)^2$ on both sides

$z^2-\frac{3}{2}z+\left(\frac{3}{4}\right)^2=-2+\left(\frac{3}{4}\right)^2$

$\left(z-\frac{3}{4}\right)^2=-2+\frac{9}{16}=\frac{-32+9}{16}$

$\left(z-\frac{3}{4}\right)^2=\frac{-23}{16}$

$\sqrt{\left(z-\frac{3}{4}\right)^2}=\sqrt{\frac{-23}{16}}$

$z-\frac{3}{4}=\pm\frac{i\sqrt{23}}{4}$

$z-\frac{3}{4}=\frac{i\sqrt{23}}{4}$ , $z-\frac{3}{4}=-\frac{i\sqrt{23}}{4}$

$z=\frac{3}{4}+\frac{i\sqrt{23}}{4}$ , $z=\frac{3}{4}-\frac{i\sqrt{23}}{4}$

$z=\frac{3+i\sqrt{23}}{4}$ , $z=\frac{3-i\sqrt{23}}{4}$

**ii)** $z^2-6z+30=0$

**Sol** $z^2-6z+30=0$

$z^2-6z=-30$

Add $(3)^2$ on both sides

$z^2-6z+(3)^2=-30+(3)^2$

$(z-3)^2=-30+9$

$(z-3)^2=-21$

$\sqrt{(z-3)^2}=\sqrt{-21}$

$z-3=\pm i\sqrt{21}$

$z-3=i\sqrt{21}$ , $z-3=-i\sqrt{21}$

$z=3+i\sqrt{21}$ , $z=3-i\sqrt{21}$

**iii)** $3z^2-18z+50=0$

**Sol:** $3z^2-18z+50=0$

$\frac{3z^2}{3}-\frac{18}{3}z+\frac{50}{3}=0$

$z^2-6z+\frac{50}{3}=0$

$z^2-6z=-\frac{50}{3}$

Add $(3)^2$ on both sides

$z^2-6z+(3)^2=-\frac{50}{3}+(3)^2$

$(z-3)^2=-\frac{50}{3}+9$

$(z-3)^2=\frac{-50+27}{3}$

$(z-3)^2=\frac{-23}{3}$

$\sqrt{(z-3)^2}=\sqrt{\frac{-23}{3}}$

$z-3=\pm i\sqrt{\frac{23}{3}}$

$z-3=i\sqrt{\frac{23}{3}}$ , $z-3=-i\sqrt{\frac{23}{3}}$

$z=3+i\sqrt{\frac{23}{3}}$ , $z=3-i\sqrt{\frac{23}{3}}$

**iv)** $z^2+4z+13=0$

**Sol:** $z^2+4z+13=0$

$z^2+4z=-13$

Add $(2)^2$ on both sides

$z^2+4z+(2)^2=-13+(2)^2$

$(z+2)^2=-13+4$

$(z+2)^2=-9$

$\sqrt{(z+2)^2}=\sqrt{-9}$

$z+2=\pm 3i$

$z+2=3i$ , $z+2=-3i$

$z=-2+3i, z=-2-3i$

**v)** $\mathbf{2z^2+6z+9=0}$

**Sol:** $2z^2+6z+9=0$

$$\frac{2z^2}{2}+\frac{6}{2}z+\frac{9}{2}=0$$

Divide by 2

$$z^2+3z+\frac{9}{2}=0$$

$$z^2+3z=-\frac{9}{2}$$

Add $\left(\frac{3}{2}\right)^2$ on both sides

$$z^2+3z+\left(\frac{3}{2}\right)^2=-\frac{9}{2}+\left(\frac{3}{2}\right)^2$$

$$\left(z+\frac{3}{2}\right)^2=-\frac{9}{2}+\frac{9}{4}$$

$$\left(z+\frac{3}{2}\right)^2=\frac{-18+9}{4}$$

$$\left(z+\frac{3}{2}\right)^2=\frac{-9}{4}$$

$$\sqrt{\left(z+\frac{3}{2}\right)^2}=\sqrt{\frac{-9}{4}}$$

$$z+\frac{3}{2}=\pm i\frac{3}{2}$$

$$z+\frac{3}{2}=i\frac{3}{2}\ ,\ z+\frac{3}{2}=-i\frac{3}{2}$$

$$z=-\frac{3}{2}+\frac{3}{2}i\ ,\ z=-\frac{3}{2}-\frac{3}{2}i$$

**vi)** $\mathbf{3z^2-5z+7=0}$

**Sol:** $3z^2-5z+7=0$

$$\frac{3z^2}{3}-\frac{5}{3}z+\frac{7}{3}=0$$

$$z^2-\frac{5}{3}z+\frac{7}{3}=0$$

$$z^2-\frac{5}{3}z=-\frac{7}{3}$$

Add $\left(\frac{5}{6}\right)^2$ on both sides

$$z^2-\frac{5}{3}z+\left(\frac{5}{6}\right)^2=-\frac{7}{3}+\left(\frac{5}{6}\right)^2$$

$$\left(z-\frac{5}{6}\right)^2=-\frac{7}{3}+\frac{25}{36}$$

$$\left(z-\frac{5}{6}\right)^2=\frac{-84+25}{36}$$

$$\left(z-\frac{5}{6}\right)^2=\frac{-59}{36}$$

$$\sqrt{\left(z-\frac{5}{6}\right)^2}=\sqrt{\frac{-59}{36}}$$

$$z-\frac{5}{6}=\pm i\frac{\sqrt{59}}{6}$$

$$z-\frac{5}{6}=i\frac{\sqrt{59}}{6}\ ,\ z-\frac{5}{6}=-i\frac{\sqrt{59}}{6}$$

$$z=\frac{5}{6}+i\frac{\sqrt{59}}{6}\ ,\ z=\frac{5}{6}-i\frac{\sqrt{59}}{6}$$

$$z=\frac{5+i\sqrt{59}}{6}\ ,\ z=\frac{5-i\sqrt{59}}{6}$$

**Q.5. Solve the following equations.**

**i)** $\mathbf{2z^4-32=0}$

**Sol:** $2z^4-32=0$

$$2\left(z^4-16\right)=0$$

$$z^4-16=0$$

$$\left(z^2\right)^2-(4)^2=0$$

$$\left(z^2-4\right)\left(z^2+4\right)=0$$

$$z^2-4=0, z^2+4=0$$

$$z^2=4\ ,\ z^2=-4$$

$$z=\pm2\ ,\ z=\pm2i$$

**ii)** $\mathbf{3z^5-243z=0}$

**Sol:** $3z^5-243z=0$

$$3z\left(z^4-81\right)=0$$

$$z=0\ ,\ z^4-81=0$$

$$\left(z^2\right)^2-(9)^2=0$$

$$\left(z^2-9\right)\left(z^2+9\right)=0$$

$$z^2-9=0, z^2+9=0$$

$$z^2=9\ ,\ z^2=-9$$

$$z=\pm3\ ,\ z=\pm3i$$

$$z=0\ ,\ z=\pm3\ ,\ z=\pm3i$$

**iii)** $5z^5-5z=0$

**Sol:** $5z^5-5z=0$

$5z(z^4-1)=0$

$5z=0\,,\ z^4-1=0$

$z=0\ ,\ \left((z^2)^2-(1)^2\right)=0$

$(z^2-1)(z^2+1)=0$

$z^2-1=0\ ,\ z^2+1=0$

$z^2=1\ ;\ z^2=-1$

$z^2=\pm1\,,\ z^2=\pm i$

$z=0,\ z^2=\pm1\,,\ z^2=\pm i$

**iv)** $z^3-5z^2+z-5=0$

**Sol:** $z^3-5z^2+z-5=0$

$z^2(z-5)+1(z-5)=0$

$(z-5)(z^2+1)=0$

$z-5=0$ and $z^2+1=0$

$z=5$ and $z^2=-1\Rightarrow z^2=\pm i$

$z=5$ and $z=\pm i$

Hence $z=5\ ,z=i\,,z=-i$

**v)** $4z^4-25z^2-21=0$

**Sol:** $4z^4-25z^2-21=0$

$4(z^2)^2-25z^2-21=0$

$$z^2=\frac{-(-25)\pm\sqrt{(-25)^2-4(4)(-21)}}{2(4)}$$

$$z^2=\frac{25\pm\sqrt{625+336}}{8}$$

$$=\frac{25\pm\sqrt{961}}{8}=\frac{25\pm31}{8}$$

$z^2=\frac{25+31}{8}$ *and* $z^2=\frac{25-31}{8}$

$z^2=\frac{56}{8}$ *and* $z^2=\frac{-6}{8}$

$z^2=7$ *and* $z^2=\frac{-3}{4}$

$z=\pm\sqrt{7}$ *and* $z=\pm\frac{\sqrt{3}\,i}{2}$

Hence $z=\sqrt{7}\,,z=-\sqrt{7}\ ,z=\frac{\sqrt{3}}{2}i\,,z=-\frac{\sqrt{3}}{2}i$

**vi)** $z^3+z^2+z+1=0$

**Sol:** $z^3+z^2+z+1=0$

$z^2(z+1)+1(z+1)=0$

$(z+1)(z^2+1)=0$

$z+1=0$ and $(z^2+1)=0$

$z=-1$ and $z^2+1=0\Rightarrow z^2=-1$

$z=\pm i$

Hence $z=-1\,,i\,,-i$

**Q.6. Find a polynomial $P(z)$ of degree 3 with zeros $3,-2i,2i$ and satisfying $P(1)=20$.**

**Sol:** Let $P(z)$ be polynomial with zeros $3,2i,-2i$.

According to Fundamental Theorem of Algebra.

$\therefore P(z)=A(z-3)(z-2i)(z+2i)$

$P(z)=A(z-3)\left((z)^2-(2i)^2\right)$

$P(z)=A(z-3)\left(z^2-(-4)\right)$

$P(z)=A(z-3)(z^2+4)$

$P(z)=A(z^3+4z-3z^2-12)$

$P(z)=A(z^3-3z^2+4z-12)$ (i)

Here $P(1)=20$

$A\left((1)^3-3(1)^2+4(1)-12\right)=20$

$A(1-3+4-12)=20$

$A(-10)=20$

$A=\frac{20}{-10}=-2$

(i) becomes

$P(z)=(-2)(z^3-3z^2+4z-12)$

$P(z)=-2z^3+6z^2-8z+24$

**Q.7. Find a polynomial $P(z)$ of degree 4 with zeros $2i,-2i,1,-1$ and satisfying $P(2)=240$.**

**Sol:** Let $P(z)$ be polynomial with zeros $2i,-2i,1,-1$.

According to Fundamental Theorem of Algebra

$\therefore P(z)=A(z-1)(z+1)(z-2i)(z+2i)$

$P(z)=A\left((z)^2-(1)^2\right)\left((z)^2-(2i)^2\right)$

$P(z)=A(z^2-1)\left(z^2-(-4)^2\right)$

$$P(z)=A(z^2-1)(z^2+4)$$

$$P(z)=A(z^4+4z^2-z^2-4)$$

$$P(z)=A(z^4+3z^2-4) \qquad \text{(i)}$$

Here $P(2)=240$

$$A\left((2)^4+3(2)^2-4\right)=240$$

$$A(16+12-4)=240$$

$$A(24)=240 \Rightarrow A=\frac{240}{24}=10$$

(i) becomes

$$P(z)=(10)(z^4+3z^2-4)$$

$$P(z)=10z^4+30z^2-40$$

**Q.8. Find a polynomial $P(z)$ of degree 4 with zeros $4,-4,1+i$, $1-i$ and satisfying $P(2)=72$.**

**Sol** Let $P(z)$ be polynomial with zeros $4,-4,1+i,1-i$

According to Fundamental Theorem of Algebra

$$\therefore P(z)=A(z-4)(z+4)(z-(1+i))(z-(1-i))$$

$$P(z)=A(z- \qquad 1+i)$$

$$P(z)=A\left((z)^2-(4)^2\right)($$

$$P(z)=A(z^2-16)(z^2-2z+1-(-1))$$

$$P(z)=A(z^2-16)(z^2-2z+1+1)$$

$$P(z)=A(z^2-16)(z^2-2z+2)$$

$$P(z)=A(z^4-2z^3+2z^2-16z^2+32z-32)$$

$$P(z)=A(z^4-2z^3-14z^2+32z-32) \qquad \text{(i)}$$

Here $P(2)=72$

$$P(z)=A\left((2)^4-2(2)^3-14(2)^2+32(2)-32\right)=72$$

$$A(-24)=72$$

$$\Rightarrow A=\frac{72}{-24}=-3$$

(i) becomes

$$P(z)=(-3)(z^4-2z^3-14z^2+32z-32)$$

$$P(z)=-3z^4+6z^3+42z^2-96z+96$$