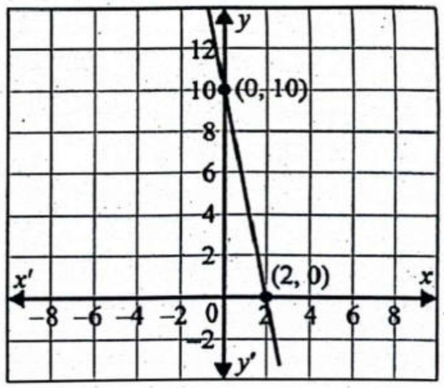
Exercise 2.2

Q1. **Find the point of intersection of the coordinate axes and the following linear functions graphically:**

(i) $y=-5x+10$



Sol: we need to find:

Y-intercept (Intersection with the *y*-axis):

Put $x=0$

$y=-5(0)+10=10\Rightarrow$ The line intersects the *y*-axis at the point $(0,10)$.

X-intercept (Intersection with the *x*-axis):

Put $y=0$

$0=-5x+10$

$\Rightarrow 5x=10\Rightarrow x=2$ $\Rightarrow$ The line intersects the *x*-axis at the point $(2,0)$.

Graphical Summary:

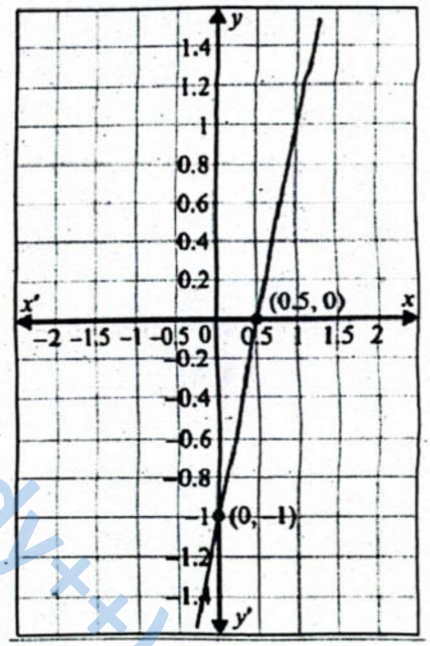
- *x*-intercept: $(2,0)$
- *y*-intercept: $(0,10)$

This graph shows the line $y=-5x+10$ intersecting:

- the *y*-axis at $(0,10)$
- the *x*-axis at $(2,0)$

These are the required points of intersection with the coordinate axes.

(ii) $y=2x-1$



Sol: *y*-intercept (where $x=0$):

$y=2(0)-1=-1$

$\Rightarrow$ *y*-intercept is $(0,-1)$

x-intercept (where $x=0$):

$0=2x-1$ $\Rightarrow$ $2x=1$

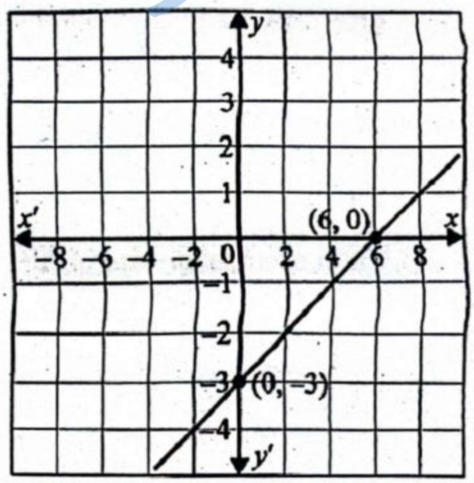
$\Rightarrow$ $x=\frac{1}{2}$ $\Rightarrow$ *x*-intercept is $\left(\frac{1}{2},0\right)$

Now let's draw the graph of the line using these points.

Here is the graph of the line $y=2x-1$. It intersects:

- the *y*-axis at $(0,-1)$
- the *x*-axis at $(0.5,0)$

(iii) $y=\frac{1}{2}x-3$



Sol: *y*-intercept (where $x=0$):

$y=\frac{0}{2}-3=-3$ $\Rightarrow$ *y*-intercept is $(0,-3)$

x-intercept (where $y=0$):

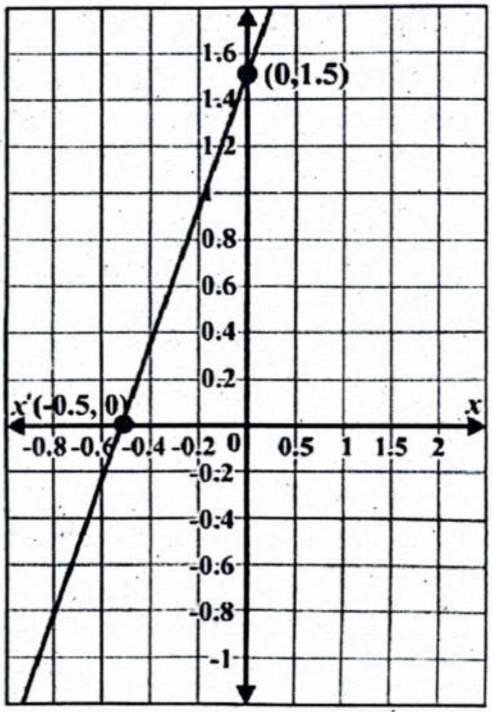
$0=\frac{x}{2}-3$ $\Rightarrow$ $\frac{x}{2}=3$ $\Rightarrow$ $x=6$ $\Rightarrow$ *x*-intercept is $(6,0)$

Now let's draw the graph using these points.

Here is the graph of the line $y=\frac{x}{2}-3$. It intersects:

- *x*-intercept at $(6,0)$ (red point)
- *y*-intercept at $(0,-3)$ (green point)

(iv) $y=3x+\frac{3}{2}$



Sol: y-intercept (set $x=0$):

$y=3(0)+\frac{3}{2}=\frac{3}{2} \Rightarrow y\text{-intercept is}\left(0,\frac{3}{2}\right)=(0,1.5)$

X-intercept (set $y=0$):

$0=3x+\frac{3}{2} \quad \Rightarrow \quad 3x=\frac{-3}{2} \quad \Rightarrow \quad x=\frac{-1}{2}$

$\Rightarrow x\text{-intercept is}\left(\frac{-1}{2},0\right)=(-0.5,0)$

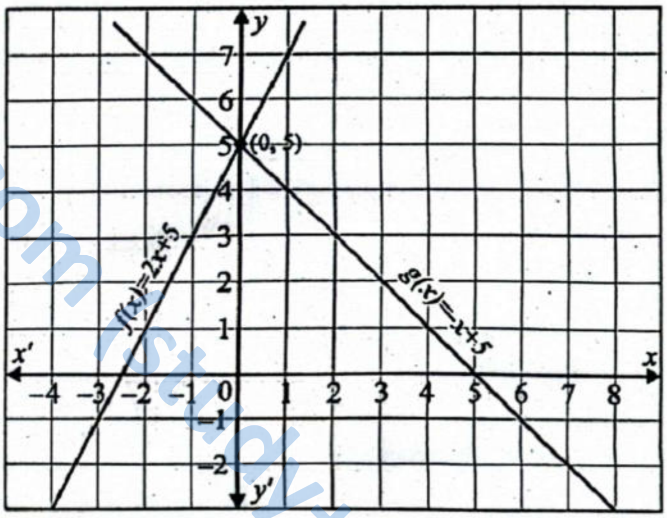
Now let's draw the graph of this line.

Here is the graph of the line $y=3x+\frac{3}{2}$. It intersects:

- x-intercept at $\left(\frac{-1}{2},0\right)$
- y-intercept at $\left(0,\frac{3}{2}\right)$

Q2: Find the pint(s) of intersection of the following functions graphically:

(i) $f(x)=2x+5, g(x)=-x+5$



Sol: For $f(x)=2x+5$

For x-intercept put $f(x)=0$

$\Rightarrow 2x+5=0 \Rightarrow x=\frac{-5}{2};\left(\frac{-5}{2},0\right)$

For y-intercept put $x=0$

$\Rightarrow f(x)=y=2(0)+5=5;(0,5)$

For $g(x)=-x+5$

For x-intercept put $g(x)=0$

$\Rightarrow 0=-x+5 \Rightarrow x=5;(5,0)$

For y-intercept put $x=0$

$\Rightarrow g(x)=y=-0+5=5(0,5)$

Point of Intersection (Algebraically)

Set $f(x)=g(x)$

$2x+5=-x+5 \quad \Rightarrow \quad 2x+x=5-5 \quad \Rightarrow \quad 3x=0 \Rightarrow \quad x=0$

Now substitute x=0 *into either* function:

$f(0)=2(0)+5=5 \qquad \Rightarrow$ Point of intersection is $(0,5)$

Now let's plot both functions and show where they intersect.

Graph shows that two lines $f(x)=2x+5$ and $g(x)=-x+5$ intersect at the point $(0,5)$.

(ii) $f(x)=3x-2, g(x)=10-x$

Sol: Point of intersection $f(x)=3x-2$ and $g(x)=10-x$ graphically:

To plot the graphs find the points of intersection with coordinate axes of the functions:

For $f(x)=3x-2$:

x-intercept (where $f(x)=0$):

$3x-2=0 \Rightarrow x=\frac{2}{3}$, the point: $\left(\frac{2}{3},0\right)$ lies on x-axes.

y-intercept (where $x=0$):

$f(0)=3(0)-2=-2$, the point: $(0,-2)$ lies on y-axes.

For $g(x)=-x+10$:

x-intercept (where $g(x)=0$):

$-x+10=0 \Rightarrow x=10$, the point: $(10,0)$ lies on x-axes.

y-intercept (where $x=0$):

$g(0)=-0+10=10$, the point: $(0,10)$ lies on y-axes.

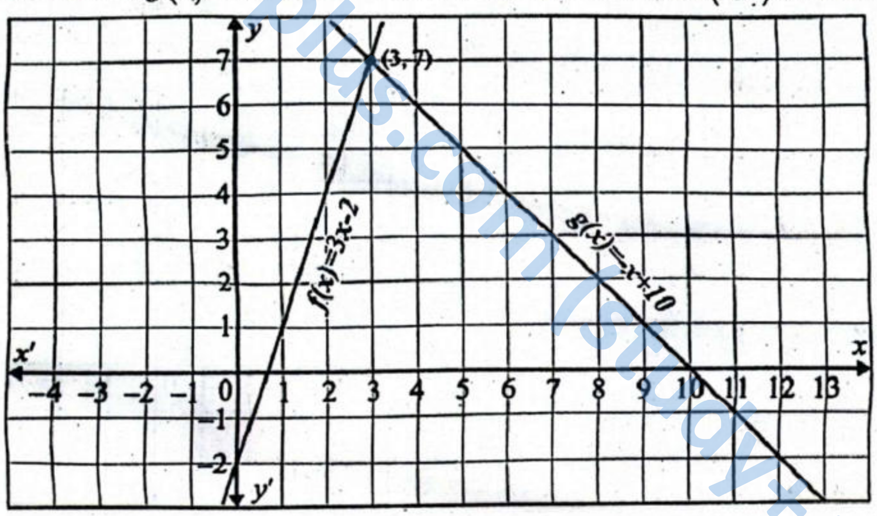
To find point of intersection of $f(x)=3x-2$ and $g(x)=-x+10$

set $f(x)=g(x)$:

$3x-2=-x+10 \Rightarrow 3x+x=10+2 \Rightarrow 4x=12 \Rightarrow x=3$

Substitute $x=3$ into either function:

$f(3)=3(3)-2=9-2=7$ or $g(3)=10-3=7 \Rightarrow$ Point of intersection is $(3,7)$ it is clear in graph.



Graph shows that two lines $f(x)=3x-2$ and $g(x)=-x+10$ intersect at the point $(3,7)$.

(iii) $f(x)=2x-4, g(x)=3x-1$

Sol: Points of intersection with coordinate axes:

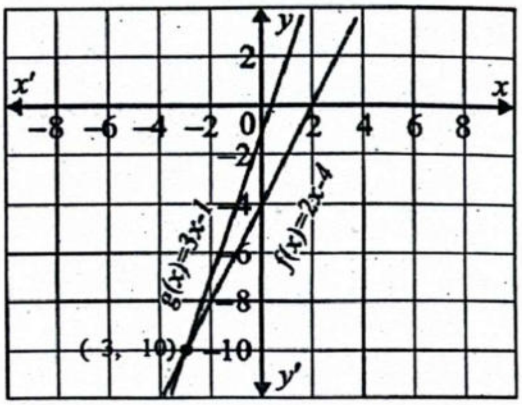
For $f(x)=2x-4$:

x-intercept (Put $f(x)=0$):

$2x-4=0 \Rightarrow x=2 \Rightarrow$ Point on x-axes is $(2,0)$

y-intercept (Put $x=0$):

$f(0)=2(0)-4=-4 \Rightarrow$ Point on y-axes is $(0,-4)$



For $g(x)=3x-1$:

x-intercept (Put $g(x)=0$):

$3x-1=0 \quad \Rightarrow \quad x=\frac{1}{3} \quad \Rightarrow$ Point on x-axes is $\left(\frac{1}{3},0\right)$

y-intercept (Put $x=0$):

$g(0)=3(0)-1=-1 \quad \Rightarrow$ Point on y-axes is $(0,-1)$

Point of intersection of $f(x)=2x-4$ and $g(x)=3x-1$:

Set $f(x)=g(x)$:

$2x-4=3x-1 \;\Rightarrow\; -4+1=3x-2x \;\Rightarrow\; -3=x$

Now putting $x=-3$ into any of the function:

$f(-3)=2(-3)-4=-6-4=-10 \Rightarrow$ Point of intersection is $(-3,-10)$

Graph shows that two lines $f(x)=2x-4$ and $g(x)=3x-1$ intersect at the point $(-3,-10)$.

(iv) $\boldsymbol{f(x)=-3x-4, g(x)=\frac{1}{2}x+3}$

Sol: Points of intersection with coordinate axes:

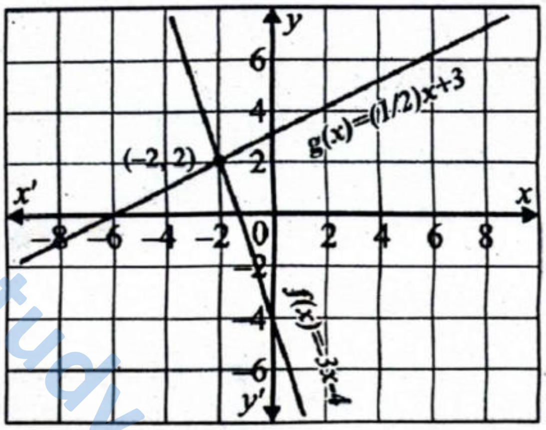
For $f(x)=-3x-4$:

x-intercept (Put $f(x)=0$):

$-3x-4=0 \Rightarrow -3x=4 \Rightarrow x=-4/3 \Rightarrow$ Point on x-axes is $\left(-\frac{4}{3},0\right)$

y-intercept (Put $x=0$):

$f(0)=-3(0)-4=-4 \;\Rightarrow$ Point on y-axes is $(0,-4)$



For $g(x)=\frac{x}{2}+3$:

x-intercept (Put $g(x)=0$):

$\frac{x}{2}+3=0 \Rightarrow \frac{x}{2}=-3 \Rightarrow x=-6 \Rightarrow$ Point on x-axes is $(-6,0)$

y-intercept (Put $x=0$):

$g(0)=(0)/2+3=3 \quad \Rightarrow$ Point on y-axes is $(0,3)$

Point of intersection of $f(x)=-3x-4$ and $g(x)=\frac{x}{2}+3$:

Set them $f(x)=g(x)$:

$-3x-4=\frac{x}{2}+3$

$\Rightarrow -6x-8=x+6 \Rightarrow -6x-x=6+8 \Rightarrow -7x=14 \Rightarrow x=-2$

Now substitute $x=-2$ into any of the function:

$f(-2)=-3(-2)-4=6-4=2 \quad \Rightarrow$ Point of intersection is $(-2,2)$

Graph shows that two lines $f(x)=-3x-4$ and $g(x)=\frac{x}{2}+3$ intersect at the point $(-2,2)$.

(v) $\boldsymbol{f(x)=x-1, g(x)=x^2-4x+3}$

Sol: Points of intersection with coordinate axes

For $f(x)=x-1$:

x-intercept (Put $f(x)=0$):

$x-1=0 \quad \Rightarrow \quad x=1 \quad \Rightarrow$ the point on x-axes: $(1,0)$

y-intercept (Put $x=0$):

$f(0)=0-1=-1 \quad \Rightarrow$ the point on y-axes: $(0,-1)$

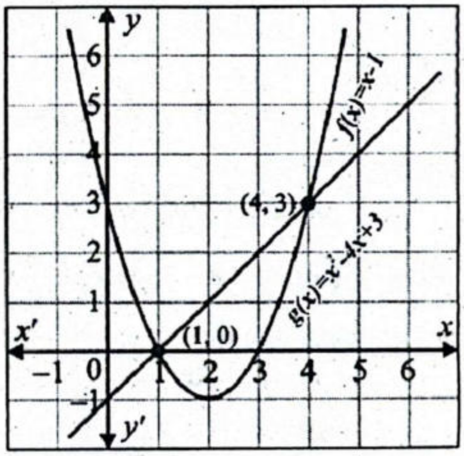
For $g(x)=x^2-4x+3$:

x-intercepts (Put $g(x)=0$):

$x^2-4x+3=0 \quad \Rightarrow \quad (x-1)(x-3)=0 \quad \Rightarrow \quad x=1, x=3 \quad \Rightarrow$ The points are: $(1,0)$ and $(3,0)$

y-intercept (Put $x=0$):

$g(0)=(0)^2-4(0)+3=3 \Rightarrow$ the point: $(3,0)$



Points of intersection of $f(x)=x-1$ and $g(x)=x^2-4x+3$

Set the functions equal:

$x-1=x^2-4x+3 \Rightarrow 0=x^2-5x+4 \Rightarrow x^2-x-4x+4=0$

$\Rightarrow x(x-1)-4(x-1)=0 \Rightarrow (x-4)(x-1)=0 \Rightarrow x=1, x=4$

Now find corresponding y values:

For $x=1$:

$f(1)=1-1=0 \quad \Rightarrow$ the point is $(1,0)$

For $x=4$:

$f(4)=4-1=3 \Rightarrow (4,3)$

Points of intersection: $(1,0)$ and $(4,3)$

Graph shows that two lines $f(x)=x-1$ and $g(x)=x^2-4x+3$ intersect at the points $(1,0)$ and $(4,3)$.

(vi) $\boldsymbol{f(x)=3x+4, g(x)=x^2+2x-8}$

Sol: Points of Intersection with Coordinate Axes

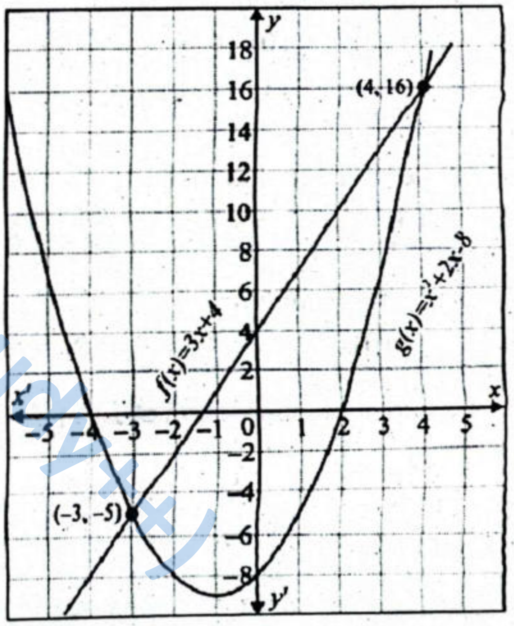
For $f(x)=3x+4$:

x-intercept (Put $f(x)=0$):

$3x+4=0 \quad \Rightarrow \quad x=-4/3 \quad \Rightarrow$ The point on x-axes: $\left(-\frac{4}{3},0\right)$

y-intercept (Put $x=0$):

$f(0)=3(0)+4=4 \quad \Rightarrow \quad$ The point on y-axes: $(0,4)$



For $g(x)=x^2+2x-8$:

x-intercepts (Put $g(x)=0$):

$x^2+2x-8=0 \Rightarrow (x+4)(x-2)=0 \Rightarrow x=-4, x=2$

The points on x-axes are: $(-4,0)$ and $(2,0)$

y-intercept (Put $x=0$):

$g(0)=(0)^2+2(0)-8=-8 \Rightarrow \quad$ The point on y-axes: $(0,-8)$

Points of Intersection of $f(x)=3x+4$ and $g(x)=x^2+2x-8$

Take $f(x) = g(x)$:

$3x+4=x^2+2x-8 \quad \Rightarrow \quad 0=x^2-x-12 \quad \Rightarrow \quad (x-4)(x+3)=0 \quad \Rightarrow \quad x=4, x=-3$

Now find y values for both:

For $x=4$:

$f(4)=3(4)+4=12+4=16 \quad \Rightarrow$ the point is $(4,16)$:

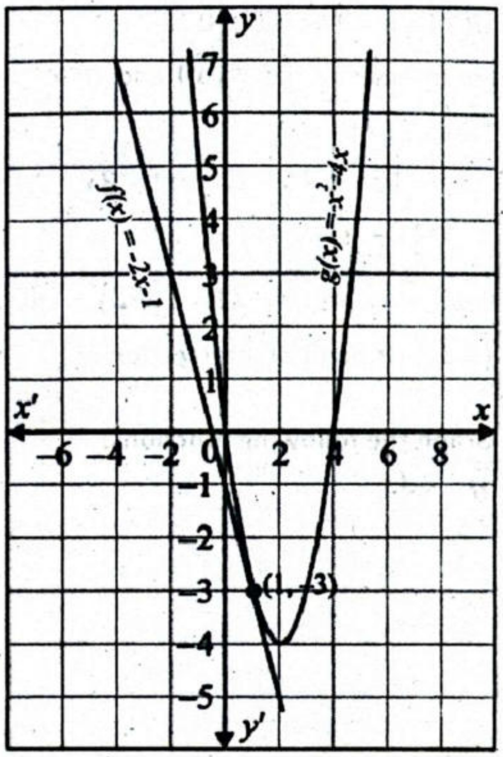
For x=−3:

$f(-3)=3(-3)+4=-9+4=-5$ $\Rightarrow$ the other point is $(-3,-5)$:

From graph the Intersection points of $f(x)$ and $g(x)$ are $(4,16)$ and $(-3,-5)$:

(vii) $\boldsymbol{f(x)=-2x-1, g(x)=x^2-4x}$

Sol: For $f(x)=-2x-1$



x-intercept (Put $f(x)=0$):

$-2x-1=0 \Rightarrow -2x=1 \Rightarrow x=\dfrac{-1}{2}$; $\left(-\dfrac{1}{2},0\right)$

y-intercept (Put $x=0$):

$f(x)=y=-2(0)-1 \Rightarrow y=-1$; $(0,-1)$

For $g(x)=x^2-4x$:

x-intercepts (Put $g(x)=0$):

$x^2-4x=0 \Rightarrow x(x-4)=0 \Rightarrow x=0$ & $x=4$

We get $(0,0),(4,0)$

y-intercept (Put $x=0$):

$g(x)=y=(0)^2-4(0)=0 \Rightarrow (0,0)$

For Intersection of $f(x)$ and $g(x)$

Take $f(x)=g(x)$

$-2x-1=x^2-4x \Rightarrow x^2-4x+2x+1=0$

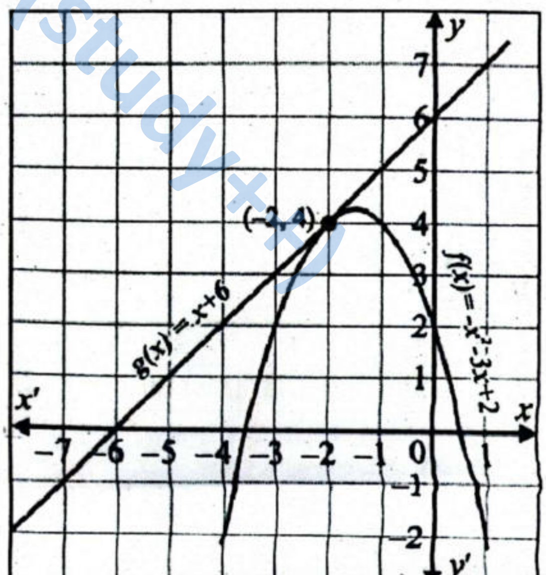
$x^2-2x+1=0 \Rightarrow (x-1)^2=0 \Rightarrow x-1=0 \Rightarrow x=1$

Put $x=1$ in $f(x)$ we get $f(1)=-2(1)-1=-3$

Hence point of intersection is $(1,-3)$

(viii) $\boldsymbol{f(x)=-x^2-3x+2, g(x)=x+6}$

Sol: For $f(x)=-x^2-3x+2$



x-intercept (Put $f(x)=0$):

$-x^2-3x+2=0 \Rightarrow x^2+3x-2=0$

$x^2+3x-2=0 \Rightarrow x=\dfrac{-3\pm\sqrt{(3)^2-4(1)(-2)}}{2(1)}$

$\Rightarrow x=\dfrac{-3\pm\sqrt{9+8}}{2}=\dfrac{-3\pm\sqrt{17}}{2}=\dfrac{-3\pm 4.123}{2}$

$x=\dfrac{-3+4.123}{2}$, $x=\dfrac{-3-4.123}{2}$

$x=\dfrac{1.123}{2}$, $x=\dfrac{-7.123}{2}$

$x=0.56$, $x=-3.56$

We get $(0.56,0)$ and $(-3.56,0)$

y-intercept (Put $x=0$):

$f(x)=y=-(0)^2-3(0)+2=2$; $(0,2)$

For $g(x)=x+6$:

x-intercepts (Put $g(x)=0$):

$x+6=0 \Rightarrow x=-6$; $(-6,0)$

y-intercept (Put $x=0$):

$g(x)=y=0+6=6$; $(0,6)$

For Intersection of $f(x)$ and $g(x)$

Take $f(x)=g(x)$

$-x^2-3x+2=x+6 \Rightarrow x^2+3x-2+x+6=0$

$x^2+4x+4=0 \Rightarrow (x+2)^2=x+2=0 \Rightarrow x=-2$

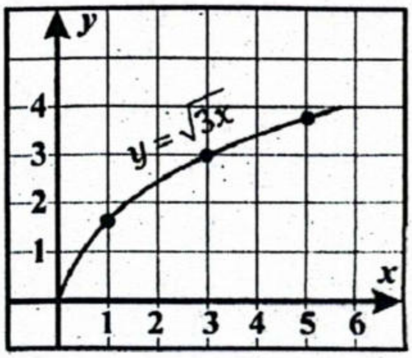
Put $x=-2 \Rightarrow f(-2)=-(-2)^2-3(-2)+2=-4+6+2=4$

$(-2,4)$ Is point of intersection

Q3. Graph the following functions:

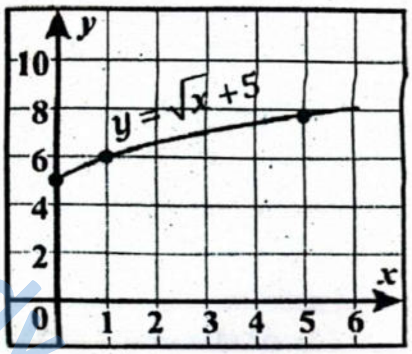
(i) $y=\sqrt{3x}$

x	0	1	3	5
y	0	1.73	3	3.87



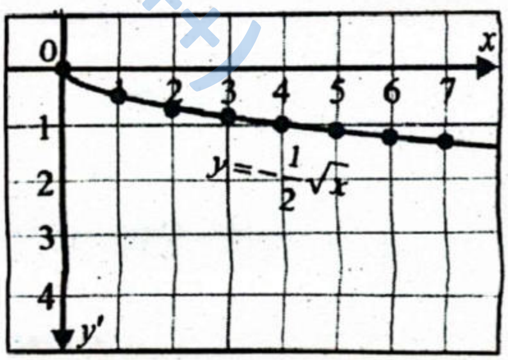
(ii) $y=\sqrt{x}+5$

x	0	1	3	5
y	5	6	6.73	7.23



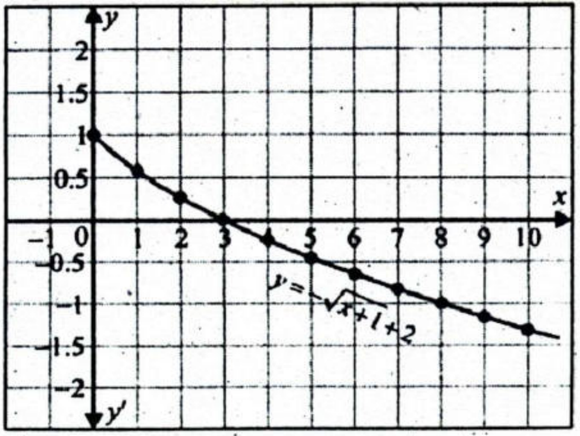
(iii) $y=-\frac{1}{2}\sqrt{x}$

x	0	1	3	5
y	0	–0.5	–0.86	–1.12



(iv) $y=-\sqrt{x+1}+2$

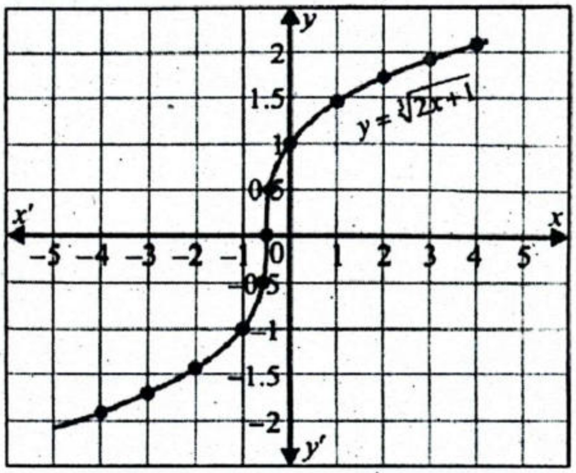
x	0	1	3	5
y	1	0.59	0	−0.45

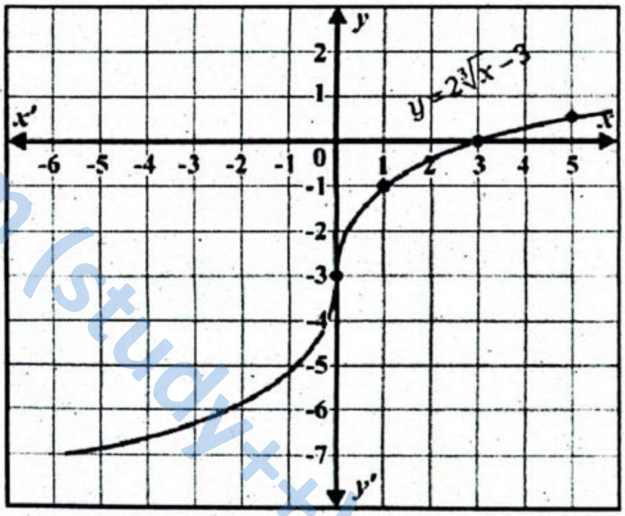


(v) $y=\sqrt[3]{2x+1}$

x	0	1	3	5
y	1	1.44	1.91	2.22

Graph of $y=\sqrt[3]{2x+1}$

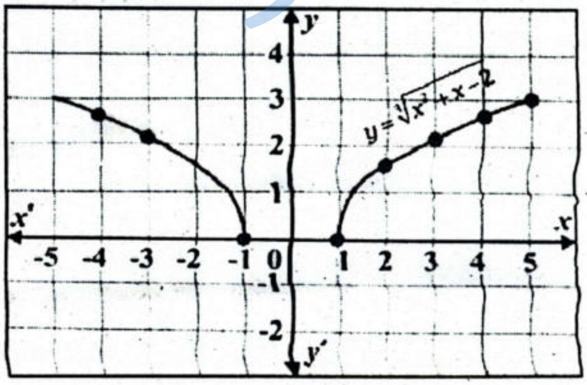




(vi) $y=2\sqrt[3]{x}-3$

X	0	1	3	5
Y	−3	−1	−0.16	0.42

Graph of $y=2\sqrt[3]{x}-3$



(vii) $y=\sqrt[3]{x^2+x-2}$

x	-1	1	2	3	4
y	0	0	1.58	2.13	2.59

Graph of $y=\sqrt[3]{x^2+x-2}$

Q4. **A building's height over time is modeled by $H(t)=100+20t$ which is in mete[illegible] in months.**
The height of growing tree nearby is given by $T(t)=50+10t+t^2$.

(i) **At what time will the building and tree have the same height?**

(ii) **What will that height?**

Sketch the graphs of both functions and determine the time when the tree will overtake the height of the building.

Sol: We are given two functions for height over time t (in months):

Building: $H(t)=100+20t$

Tree: $T(t)=50+10t+t^2$

We are to find:

(i) **When the building and tree have the same height:**

Set the two height functions equal:

$100+20t=50+10t+t^2$

Rearranging:

$100+20t=t^2+10t+50 \quad \Rightarrow \quad 0=t^2-10t-50$

This is a quadratic equation:

$t^2-10t-50=0$

Use the quadratic formula:

$$t=\frac{-(-10)\pm\sqrt{(-10)^2-4(1)(-50)}}{2(1)}$$

$$\Rightarrow t=\frac{10\pm\sqrt{300}}{2}=\frac{10\pm\sqrt{300}}{2}=5\pm5\sqrt{3}$$

So, the two times are:

$t=5+5\sqrt{3}$ and $t=5-5\sqrt{3}$. Since t is time (months), and must be positive, discard the negative one:

$t=5+5\sqrt{3}\approx5+5(1.732)=5+8.66=13.66$ months ≈ 14 months (approximately)

(ii) **What is that height?**

Substitute $t=5+5\sqrt{3}$ into either function (let's use building height):

$H(t)=100+20t=100+20(5+5\sqrt{3})=100+100+100\sqrt{3}=200+100\sqrt{3}$

$\Rightarrow \quad H(t)\approx200+100(1.732)=200+173.2=373.2$ meters (approx.)

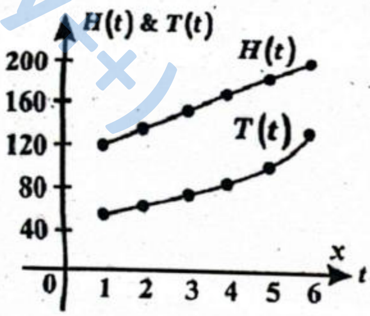
Graph of the functions: Building height: $H(t)=100+20t$ and Tree height: $T(t)=50+10t+t^2$.

Building Height $H(t)=100+20t$

t	1	2	3	4	5
$H(t)$	120	140	160	180	200

For Tree $T(t)=50+10t+t^2$

t	1	2	3	4	5
$T(t)$	61	74	89	106	125



Q5: **A radioactive substance has half-life (*h*) of 2 years. If the initial quantity Q_0 is 200 grams and the exponential decay function is $Q(t) = Q_0\left(\frac{1}{2}\right)^{\frac{t}{h}}$, then find the remaining quantity after 6 years graphically.**

Sol: We are given:

- Half-life: $h = 2$ years
- Initial quantity: $Q_0 = 200$ grams
- Time: t=6 years
- Decay model:

$$Q(t) = Q_0 \cdot \left(\frac{1}{2}\right)^{\frac{t}{h}}$$

Step-by-step calculation:

$$Q(6) = 200 \cdot \left(\frac{1}{2}\right)^{\frac{6}{2}} = 200 \cdot \left(\frac{1}{2}\right)^{3} = 200 \cdot \frac{1}{8} = 25 \text{ Grams.}$$

Graph of $Q = 200 \cdot \left(\frac{1}{2}\right)^{\frac{t}{2}}$

t=x	0	2	4	6	7
Q=y	200	100	50	25	17.68