Exercise 3.2

Q1. Solve the following equations.

i. $\dfrac{1}{3x} + \dfrac{4x}{6} = 1$, $x \neq 0$

Sol: $\dfrac{1}{3x} + \dfrac{4x}{6} = 1 \Rightarrow \dfrac{2+4x^2}{6x} = 1$

$2 + 4x^2 = 6x \Rightarrow 4x^2 - 6x + 2 = 0$

$4x^2 - 4x - 2x + 2 = 0 \Rightarrow 4x(x-1) - 2(x-1) = 0$

$(4x-2)(x-1) = 0 \Rightarrow x-1 = 0$ or $4x-2 = 0$

$x = 1$ or $4x = 2 \Rightarrow x = \dfrac{\cancel{2}}{\cancel{4}} = \dfrac{1}{2}$

$S.S = \left\{ 1, \dfrac{1}{2} \right\}$

ii. $\dfrac{x}{x+1} + \dfrac{x+1}{x} = \dfrac{5}{2}$; $x \neq -1, 0$

Sol: $\dfrac{x^2 + (x+1)^2}{x(x+1)} = \dfrac{5}{2}$

$\dfrac{x^2 + x^2 + 2x + 1}{x^2 + x} = \dfrac{5}{2}$

$\dfrac{2x^2 + 2x + 1}{x^2 + x} = \dfrac{5}{2}$ (By cross multiplication)

$\Rightarrow 2(2x^2 + 2x + 1) = 5(x^2 + x)$

$4x^2 + 4x + 2 = 5x^2 + 5x$

$5x^2 + 5x - 4x^2 - 4x - 2 = 0$

$x^2 + x - 2 = 0$

$\Rightarrow x^2 + 2x - x - 2 = 0$

$x(x+2) - 1(x+2) = 0$

$\Rightarrow (x+2)(x-1) = 0$

$x + 2 = 0$ or $x - 1 = 0$

$x = -2$ or $x = 1$

$S.S = \{1, -2\}$

iii. $\dfrac{1}{x+1} + \dfrac{2}{x+2} = \dfrac{7}{x+5}$; $x \neq -1, -2, -5$

Sol. $\dfrac{1(x+2) + 2(x+1)}{(x+1)(x+2)} = \dfrac{7}{x+5}$

$\dfrac{x+2+2x+2}{x^2+3x+2} = \dfrac{7}{x+5}$

$\dfrac{3x+4}{x^2+3x+2} = \dfrac{7}{x+5}$

$\Rightarrow (3x+4)(x+5) = 7(x^2+3x+2))$

$3x^2 + 15x + 4x + 20 = 7x^2 + 21x + 14$

$7x^2 + 21x + 14 - 3x^2 - 19x - 20 = 0$

$4x^2 + 2x - 6 = 0$

$4x^2 + 6x - 4x - 6 = 0$

$2x(2x+3) - 2(2x+3) = 0$

$(2x+3)(2x-2) = 0$

$2x + 3 = 0$ or $2x - 2 = 0$

$x = \dfrac{-3}{2}$ or $x = 1$

$S.S = \left\{ \dfrac{-3}{2}, 1 \right\}$

iv. $\dfrac{a}{ax-1} + \dfrac{b}{bx-1} = a+b$; $x \neq \dfrac{1}{a}, \dfrac{1}{b}$

Sol. or $\dfrac{a}{(ax-1)} - b + \dfrac{b}{bx-1} - a = 0$

$\dfrac{a - b(ax-1)}{(ax-1)} + \dfrac{b - a(bx-1)}{bx-1} = 0$

$\dfrac{a - abx + b}{(ax-1)} + \dfrac{b - abx + a}{bx-1} = 0$

$(a - abx + b)\left[\dfrac{1}{ax-1} + \dfrac{1}{bx-1} \right] = 0$

$(a - abx + b)\left(\dfrac{bx-1+ax-1}{(ax-1)(bx-1)} \right) = 0$

$(a - abx + b)(ax + bx - 2) = 0$

$a - abx + b = 0$ or $ax + bx - 2 = 0$

$abx = a + b$ or $x(a+b) = 2$

$x = \dfrac{a+b}{ab}$ or $x = \dfrac{2}{a+b}$

$S.S = \left\{ \dfrac{a+b}{ab}, \dfrac{2}{a+b} \right\}$

v. $\dfrac{x+1}{x-1} + \dfrac{x-1}{x+1} = 2$, $x \neq 1$, $x \neq -1$

Sol: $\dfrac{x+1}{x-1} + \dfrac{x-1}{x+1} - 2 \Rightarrow \dfrac{(x+1)^2 + (x-1)^2}{(x-1)(x+1)} = 2$

$\dfrac{x^2 + 2x + 1 + x^2 - 2x + 1}{x^2 - 1} = 2 \Rightarrow \cancel{x^2} + 1 - \cancel{x^2} + 1 = 0$

$\Rightarrow 2 = 0$ not possible so, $S.S = \{\ \}$

vi. $3x^2 + 15x - 2\sqrt{x^2 + 5x + 1} = 2$

Sol: $3(x^2 + 5x) - 2\sqrt{x^2 + 5x + 1} = 2$

Put $\sqrt{x^2 + 5x + 1} = y \Rightarrow x^2 + 5x + 1 = y^2$

$\Rightarrow x^2 + 5x = y^2 - 1$

We get $3(y^2 - 1) - 2y = 2 \Rightarrow 3y^2 - 3 - 2y - 2 = 0$

$3y^2 - 2y - 5 = 0 \Rightarrow 3y^2 - 5y + 3y - 5 = 0$

$y(3y - 5) + 1(3y - 5) = 0 \Rightarrow (3y - 5)(y + 1) = 0$

$3y - 5 = 0$ or $y + 1 \Rightarrow y = \dfrac{5}{3}$ or $y = -1$

When $y = \dfrac{5}{3}$ then $\sqrt{x^2 + 5x + 1} = \dfrac{5}{3}$

$\Rightarrow x^2 + 5x + 1 = \dfrac{25}{9} \Rightarrow 9x^2 + 45x + 9 = 25$

$9x^2 + 45x - 16 = 0$

$x = \dfrac{-45 \pm \sqrt{(45)^2 - 4(9)(-16)}}{2(9)}$

$x = \dfrac{-45 \pm \sqrt{2025 + 576}}{18} = \dfrac{-45 \pm \sqrt{2601}}{18} = \dfrac{-45 \pm 51}{18}$

$x = \dfrac{-45 + 51}{18}$ and $x = \dfrac{-45 - 51}{18}$

$x = \dfrac{6}{18} = \dfrac{1}{3}$ and $x = \dfrac{-96}{18} = \dfrac{-16}{3}$

When $y = -1$ then $\sqrt{x^2 + 5x + 1} = -1$

$\Rightarrow x^2 + 5x + 1 = 1 \Rightarrow x^2 + 5x + 1 - 1 = 0$

$x(x + 5) = 0 \Rightarrow x = 0$ or $x + 5 = 0$

$x = 0$ or $x = -5$

Checking

$3x^2 + 15x - 2\sqrt{x^2 + 5x + 1} = 2$

For $x = 0$

$3(0)^2 + 15(0) - 2\sqrt{(0)^2 + 5(0) + 1} = 2$

$-2 = 2$ False

For $x = -5$

$3(-5)^2 + 15(-5) - 2\sqrt{(-5)^2 + 5(-5) + 1} = 2$

$75 - 75 - 2\sqrt{25 - 25 + 1} = 2 \Rightarrow -2 = 2$ False

For $x = \dfrac{1}{3}$

$3\left(\dfrac{1}{3}\right)^2 + 15\left(\dfrac{1}{3}\right) - 2\sqrt{\left(\dfrac{1}{3}\right)^2 + 5\left(\dfrac{1}{3}\right) + 1} = 2$

$3\left(\dfrac{1}{9}\right) + 15\left(\dfrac{1}{3}\right) - 2\sqrt{\left(\dfrac{1}{9}\right) + \dfrac{5}{3} + 1} = 2$

$\dfrac{1}{3} + 5 - 2\sqrt{\dfrac{1 + 15 + 9}{9}} = 2$

$\dfrac{1 + 15}{3} - 2\sqrt{\dfrac{25}{9}} = 2 \Rightarrow \dfrac{16}{3} - 2\left(\dfrac{5}{3}\right) = 2$

$\dfrac{16 - 10}{3} = 2 \Rightarrow \dfrac{6}{3} = 2 \Rightarrow 2 = 2$ True

For $x = -\dfrac{16}{3}$

$3\left(\dfrac{-16}{3}\right)^2 + 15\left(\dfrac{-16}{3}\right) - 2\sqrt{\left(\dfrac{-16}{3}\right)^2 + 5\left(\dfrac{-16}{3}\right) + 1} = 2$

$3\left(\dfrac{256}{9^3}\right) + 15\left(\dfrac{-16}{3}\right) - 2\sqrt{\dfrac{256}{9} - \dfrac{80}{3} + 1} = 2$

$\dfrac{256 - 240}{3} - 2\sqrt{\dfrac{256 - 240}{9} + 1} = 2$

$\dfrac{16}{3} - 2\sqrt{\dfrac{256 - 240 + 9}{9}} = 2 \Rightarrow \dfrac{16}{3} - 2\sqrt{\dfrac{25}{9}} = 2$

$\dfrac{16}{3} - 2\left(\dfrac{5}{3}\right) = 2 \Rightarrow \dfrac{16}{3} - \dfrac{10}{3} = 2$

$\Rightarrow \dfrac{6}{3} = 2 \Rightarrow 2 = 2$

S.S $\left\{\dfrac{1}{3}, \dfrac{-16}{3}\right\}$ and Extraneous roots 0 and -5

vii. $\sqrt{2x + 8} + \sqrt{x + 5} = 7$

Sol. $\sqrt{2x + 8} + \sqrt{x + 5} = 7$ _____ I

Squaring both sides

$2x + 8 + x + 5 + 2\sqrt{2x + 8}\sqrt{x + 5} = 49$

$3x + 13 + 2\sqrt{(2x + 8)(x + 5)} = 49$

$2\sqrt{2x^2 + 10x + 8x + 40} = 49 - 13 - 3x$

$2\sqrt{2x^2 + 18x + 40} = 36 - 3x$

Again Squaring

$4(2x^2 + 18x + 40) = 1296 + 9x^2 - 216x$

$8x^2 + 72x + 160 = 9x^2 - 216x + 1296$

$9x^2 - 216x + 1296 - 8x^2 - 72x - 160 = 0$

$x^2 - 288x + 1136 = 0$

$x^2 - 4x - 284x + 1136 = 0$

$x(x - 4) - 284(x - 4) = 0$

$(x-4)(x-284)=0$

$x-4=0 \quad or \quad x-284=0$

$x=4 \quad or \quad x=284$

CHECKING for x = 4, I become

$\sqrt{2(4)+8}+\sqrt{4+5}=7$

$\sqrt{16}+\sqrt{9}=7 \quad \Rightarrow \quad 4+3=7$

$\Rightarrow \quad 7=7 \ TRUE$

For $\quad$ x=284 $\quad$ I become

$\sqrt{2(284)+8}+\sqrt{284+5}=7$

$\sqrt{256+8}+\sqrt{289}=7$

$\sqrt{276}+\sqrt{289}=7$

$\sqrt{276}+\sqrt{289}=7$

$24+17=7 \quad$ **FALSE**

$S.S=\{4\} \ and \ \text{Extraneous Root} = 284$

viii.

Sol. $\sqrt{3x+4}=2+\sqrt{2x-4}$

$\sqrt{3x+4}=2+\sqrt{2x-4}$

Squaring both sides

$3x+4=4+2x-4+2(2)\sqrt{2x-4}$

$3x+4-2x=4\sqrt{2x-4}$

$x+4=4\sqrt{2x-4}$

Again Squaring

$x^2+8x+16=16(2x-4)$

$x^2+8x+16=32x-64$

$x^2+8x+16-32x+64=0$

$x^2-24x+80=0$

$x^2-4x-20x+80=0$

$x(x-4)-20(x-4)=0$

$(x-4)(x-20)=0$

$x-4=0 \quad or \quad x-20=0$

$x=4 \quad or \quad x=20$

Checking for x=4 I become

$\sqrt{3(4)+4}=2+\sqrt{2(4)-4}$

$\sqrt{16}=2+\sqrt{4}$

$4=2+2=4 \quad \Rightarrow \quad 4=4 \ True$

For $\quad$ x=20 $\quad$ I become

$\sqrt{3(20)+4}=2+\sqrt{2(20)-4}$

$\sqrt{60+4}=2+\sqrt{40-4}$

$\sqrt{64}=2+\sqrt{36} \quad \Rightarrow \quad 8=2+6$

$8=8 \quad$ **TRUE**

$S.S=\{4,20\}$

ix. $\sqrt{x+7}+\sqrt{x+2}=\sqrt{6x+13}$

Sol: $\sqrt{x+7}+\sqrt{x+2}=\sqrt{6x+13}$ _____ I

Squaring both sides

$x+7+x+2+2\sqrt{x+7}\sqrt{x+2}=6x+13$

$2x+9+2\sqrt{(x+7)(x+2)}=6x+13$

$2\sqrt{x^2+7x+2x+14}=6x+13-2x-9$

$\not{2}\sqrt{x^2+9x+14}=4x+4=\not{2}(2x+2)$

$\sqrt{x^2+9x+14}=2x+2$

Again Squaring

$x^2+9x+14=4x^2+4+8x$

$4x^2+8x+4-x^2-9x-14=0$

$3x^2-x-10=0$

$3x^2-6x+5x-10=0$

$3x(x-2)+5(x-2)=0$

$(x-2)(3x+5)=0$

$x-2=0 \quad or \quad 3x+5=0$

$x=2 \quad or \quad x=-5/3$

CHECKING for x= 2 I become

$\sqrt{2+7}+\sqrt{2+2}=\sqrt{6(2)+13}$

$\sqrt{9}+\sqrt{4}=\sqrt{25} \quad \Rightarrow \quad 3+2=5$

$5 \ = \ 5 \quad$ **TRUE**

For $\quad$ x =-5/3 $\quad$ I become

$\sqrt{\dfrac{-5}{3}+7}+\sqrt{\dfrac{-5}{3}+2}=\sqrt{6\left(\dfrac{-5}{3}\right)+13}$

$\sqrt{\dfrac{-5+21}{3}}+\sqrt{\dfrac{-5+6}{3}}=\sqrt{\dfrac{-30}{3}+13}$

$\sqrt{\dfrac{16}{3}}+\sqrt{\dfrac{1}{3}}=\sqrt{\dfrac{-30+39}{3}}$

$\dfrac{4}{\sqrt{3}}+\dfrac{1}{\sqrt{3}}=\sqrt{\dfrac{9}{3}} \quad \Rightarrow \quad \dfrac{5}{\sqrt{3}}=\sqrt{3} \quad$ **FALSE**

$S.S=\{2\} \quad and \ \text{Extraneous Root} =\dfrac{-5}{3}$

x.

Sol: $\sqrt{x+5}-\sqrt{x-3}=2$

$\sqrt{x+5}-\sqrt{x-3}=2$

Squaring both sides

$\left(\sqrt{x+5}-\sqrt{x-3}\right)^2=(2)^2$

$\left(\sqrt{x+5}\right)^2+\left(\sqrt{x-3}\right)^2-2\sqrt{x+5}\sqrt{x-3}=4$

$x+5+x-3-2\sqrt{(x+5)(x-3)}=4$

$$2x + 2 - 2\sqrt{x^2 - 3x + 5x - 15} = 4$$

$$\cancel{2}\left(x+1-\sqrt{x^2+2x-15}\right) = 2 \times \cancel{2}$$

$$x + 1 - 2 = \sqrt{x^2 + 2x - 15}$$

$$x - 1 = \sqrt{x^2 + 2x - 15}$$

Squaring again

$$\left(x-1\right)^2 = \left(\sqrt{x^2+2x-15}\right)^2$$

$$x^2 - 2x + 1 = x^2 + 2x - 15$$

$$\cancel{x^2} + 2x - 15 - \cancel{x^2} + 2x - 1 = 0 \Rightarrow 4x - 16 = 0$$

$$4x = 16 \Rightarrow x = 4$$

Checking $\sqrt{x+5} - \sqrt{x-3} = 2$

Put

$$x = 4 \Rightarrow \sqrt{4+5} - \sqrt{4-3} = 2 \Rightarrow \sqrt{9} - \sqrt{1} = 2$$

$$3 - 1 = 2 \Rightarrow 2 = 2 \quad true$$

$$S.S = \{4\}$$

Q2. A former bought some sheep for Rs.9000. If he had paid Rs.100 less for each, he would have got 3 sheep more for the same money. How many sheep did he buy, when the rate in each case is uniform?

Sol. Let number of sheep $= x$

Rate of x sheep $= 9000$

Rate of one sheep $= \dfrac{9000}{x}$

If three sheep are more,

then rate of one sheep $= \dfrac{9000}{x+3}$

According to the given condition equation is

$$\frac{9000}{x} - 100 = \frac{9000}{x+3} \Rightarrow \frac{9000 - 100x}{x} = \frac{9000}{x+3}$$

$$\Rightarrow (x+3)(9000 - 100x) = 9000x$$

$$\Rightarrow 9000x - 100x^2 + 27000 - 300x = 9000x$$

$$\Rightarrow \cancel{9000x} - \cancel{9000x} + 100x^2 - 27000 + 300x = 0$$

$$\Rightarrow 100x^2 + 300x - 27000 = 0 \quad (\div) \ by \ 100$$

$$x^2 + 3x - 270 = 0$$

$$\Rightarrow x^2 + 18x - 15x - 270 = 0$$

$$\Rightarrow x(x+18) - 15(x+18) = 0$$

$$\Rightarrow (x+18)(x-15) = 0$$

$$\Rightarrow x = -18 \ or \ x = 15$$

$-18 \ Not \ possible \ so \ x = 15 = $ no of sheep's

Q3. A man sold his stock of eggs for Rs. 2400. If he had 2 dozen more, he would have got the same money by selling the whole for Rs. 0.50 per dozen cheaper. How many dozen eggs did he sell?

Sol: Suppose man sold x dozen eggs.

Rate of x dozen $= 2400$

Rate of one dozen $= \dfrac{2400}{x}$

Rate of one dozen if sold 2 dozen more for some many $= \dfrac{2400}{x+2}$

According to the given condition

$$\frac{2400}{x} - 0.5 = \frac{2400}{x+2} \Rightarrow \frac{2400}{x} - \frac{1}{2} = \frac{2400}{x+2}$$

$$\Rightarrow \frac{4800 - x}{2x} = \frac{2400}{x+2}$$

$$\Rightarrow (x+2)(4800 - x) = 2400(2x)$$

$$\Rightarrow 4800x - x^2 + 9600 - 2x = 4800x$$

$$x^2 + 2x + \cancel{4800x} - \cancel{4800x} - 9600$$

$$\Rightarrow x^2 + 2x - 9600 = 0$$

$$= \frac{-2 \pm \sqrt{(2)^2 - 4(1)(-9600)}}{2(1)} = \frac{-2 \pm \sqrt{4 + 38400}}{2}$$

$$x = \frac{-2 \pm \sqrt{38404}}{2}$$

$$x = \frac{-2 \pm 196 \, (about)}{2}$$

$$\Rightarrow \frac{196 - 2}{2} \quad and \quad x = \frac{-196 - 2}{2}$$

(Ignore -ve) not possible

$$x = \frac{194}{2} = 97 \quad and \quad x = \frac{-198}{2} = -99$$

He sold 97 dozen eggs.

Q4. A cyclist travelled 48km at a uniform speed. If he had travelled 2 km/hour slower, he would have taken 2 hours more to perform the journey. How long did he take to cover 48km?

Sol Let speed $= V$; Time $= t$

According to the given condition equation is:

$$Distance = S = vt = 48 - I \ \& \ (v-2)(t+2) = 48 - II$$

(two km slow $= v - 2$ two hour more $= t + 2$)

$$II \Rightarrow vt + 2v - 2t - 4 = 48$$

(put vt = 48) $\cancel{48} + 2v - 2t - 4 - \cancel{48} = 0$

$$2v - 2t - 4 = 0 \quad ((\div) \ by \ 2)$$

$v - t - 2 = 0 \implies v = t + 2 \text{——III}$

Put III in I $\implies vt = 48 \implies (t+2)t = 48$

$t^2 + 2t - 48 = 0$

$t^2 + 8t - 6t - 48 = 0$

$t(t+8) - 6(t+8) = 0$

$(t+8)(t-6) = 0$

$(ignore, t = -8) \implies \boxed{t = 6}$

Q5. To do a piece of work, Abdullah takes 10 days more than Abdul Hadi. Together they finish the work in 12 days. How long would Abdul Hadi take to finish it alone?

Sol Let Abdul Hadi finishes in $= x$ days.

Let Abdullah finishes in $= x + 10$

Hadi's one days work $= \dfrac{1}{x}$

Abdullah's one days work $= \dfrac{1}{x+10}$

Abdullah's one days work $= \dfrac{1}{x+10} + \dfrac{1}{x}$

According to the given conditions equation is

$\dfrac{1}{x+10} + \dfrac{1}{x} = \dfrac{1}{12}$

$\implies \dfrac{x + x + 10}{x(x+10)} = \dfrac{1}{12} \implies \dfrac{2x+10}{x(x+10)} = \dfrac{1}{12}$

(By cross multiplication) $12(2x+10) = x(x+10)$

$\implies 24x + 120 = x^2 + 10x$

$\implies x^2 + 10x - 24x - 120 = 0$

$\implies x^2 - 14x + 120 = 0$

$\implies x^2 - 20x + 6x - 120 = 0$

$\implies x(x-20) + 6(x-20) = 0$

$\implies (x-20)(x+6) = 0$

$x - 20 = 0 \ or \ x + 6 = 0$

$\implies x = 20 \ or \ x = -6$

$\implies x = 20$ Abdul Hadi finish work

-6 not possible

So in 20 days Abdullah Hadi finish his work.

Q6. The braking distance (in metres) of a car is modeled by: $d(s) = 0.02s^2 + 0.1s$, where s is the speed of car in km/h. if the maximum safe braking distance is 50 metres, find the range of speed where braking is safe.

Sol: According to the given conditions

$d(s) \le 50 \implies 0.02s^2 + 0.1s \le 50$

$s^2 + 5s - 2500 \le 0 (\div \text{ by } 0.02)$

To find values of s

$\implies s = \dfrac{-5 \pm \sqrt{(5)^2 - 4(1)(-2500)}}{2(1)}$

$s = \dfrac{-5 \pm \sqrt{25 + 10000}}{2}$

$s = \dfrac{-5 \pm \sqrt{10025}}{2} \implies s = \dfrac{-5 \pm 100.12}{2}$

(ignore due to +ve)

$s = 47.56 < 0 \quad and \quad s = \dfrac{-115.12}{2}$

$(0 \le s \le 47.560)$

Q7. A rocket follows the height function $h(t) = -5t^2 + 20t + 30$, where h(t) is the height in metres and t is the time in seconds. Find the time interval during which the rocket is at least 40 metres above the ground.

Sol: According to the given conditions

$h(t) \ge 40 \implies -5t^2 + 20t + 30 \ge 40$

$\implies -5t^2 + 20t + 30 - 40 \ge 0$

$\implies -5t^2 + 20t - 10 \ge 0$

$\implies 5t^2 - 20t + 10 \le 0 (\text{×'by} -1)$

$\implies t^2 - 4t + 2 \le 0 (\div \text{by } 5)$

here a=1 , b=-4 , c=2

$t = \dfrac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(2)}}{2(1)} \le 0$

$\implies t = \dfrac{4 \pm \sqrt{16 - 8}}{2(1)} \le 0$

$t = \dfrac{4 \pm \sqrt{8}}{2} \le 0 \implies t = \dfrac{4 \pm 2.828}{2} \le 0$

$t = \dfrac{4 + 2.828}{2} \le 0 \quad and \quad t = \dfrac{4 - 2.828}{2} \le 0$

$t = \dfrac{6.828}{2} \le 0 \quad and \quad t = \dfrac{1.172}{2} \le 0$

$t = 3.414 \le 0 \quad and \quad t = 0.586 \le 0$

$[0.586 \sec , 3.414 \sec]$