Resolve the following into partial fractions:

Q1. $\frac{2}{x^2-1}$

Sol: Let

$$\frac{2}{x^2-1}=\frac{2}{(x-1)(x+1)}=\frac{A}{x-1}+\frac{B}{x+1} \quad \text{\_\_\_\_\_ (i)}$$

Multiply both sides by $(x-1)(x+1)$ we get

$$2=A(x+1)+B(x-1) \quad \text{\_\_\_\_\_ (ii)}$$

Put $x-1=0 \Rightarrow x=1$ in (ii)

$$2=A(1+1)+B(1-1) \Rightarrow 2=2A \Rightarrow A=1$$

Now put $x+1=0 \Rightarrow x=-1$ in (ii)

$$2=A(-1+1)+B(-1-1)$$

$$2=A(0)+B(-2)$$

$$2=B(-2) \Rightarrow B=-1$$

Put values of A and B in (i)

Hence $\frac{2}{x^2-1}=\frac{1}{x-1}-\frac{1}{x+1}$

Q2. $\frac{a-b}{(x-a)(x-b)}, a \neq b$

Sol: Let $\frac{a-b}{(x-a)(x-b)}=\frac{A}{x-a}+\frac{B}{x-b}$ _____ (i)

Multiply both sides by $(x-a)(x-b)$ we get

$$a-b=A(x-b)+B(x-a) \quad \text{\_\_\_\_\_ (ii)}$$

Put $x-a=0 \Rightarrow x=a$ in (ii)

$$a-b=A(a-b)+B(a-a)$$

$$a-b=A(a-b)+0$$

$$a-b=A(a-b) \Rightarrow A=\frac{a-b}{a-b} \Rightarrow A=1$$

Put $x-b=0 \Rightarrow x=b$ in (ii)

$$a-b=A(b-b)+B(b-a)$$

$$a-b=A(0)+B(b-a)$$

$$a-b=-B(a-b) \Rightarrow B=\frac{-(a-b)}{(a-b)} \Rightarrow B=-1$$

Put values of A and B in (i)

Hence $\frac{a-b}{(x-a)(x-b)}=\frac{1}{x-a}+\frac{-1}{x-b}=\frac{1}{x-a}-\frac{1}{x-b}$

Q3. $\dfrac{x^2+1}{(x+1)(x-1)}$

Sol: $\dfrac{x^2+1}{(x+1)(x-1)}=\dfrac{x^2+1}{x^2-1}$ (Improper fraction)

$$\frac{x^2+1}{x^2-1}=1+\frac{2}{x^2-1} \text{ ——— (i)}$$

$$\begin{array}{r|l} & \quad 1 \\ \hline x^2-1 & \cancel{x^2}+1 \\ & \pm\cancel{x^2}\mp 1 \\ \hline & \quad\; 2 \end{array}$$

Now take $\dfrac{2}{x^2-1}=\dfrac{2}{(x-1)(x+1)}$

Let $\dfrac{2}{(x-1)(x+1)}=\dfrac{A}{x-1}+\dfrac{B}{x+1}$ ——— (ii)

Multiply both sides by $(x-1)(x+1)$ we get

$2=A(x+1)+B(x-1)$ ——— (iii)

Put $x-1=0 \Rightarrow x=1$ in (iii)

$2=A(1+1)+B(1-1)$

$2=A(2)+0 \Rightarrow A=1$

Put $x+1=0 \Rightarrow x=-1$ in (iii)

$2=A(-1+1)+B(-1-1)$

$2=A(0)+B(-2) \Rightarrow 2=-2B \Rightarrow B=-1$

Put values in (ii)

$\dfrac{2}{(x-1)(x+1)}=\dfrac{1}{x-1}+\dfrac{-1}{x+1}$

(i) become

Hence $\dfrac{x^2+1}{(x-1)(x+1)}=1+\dfrac{1}{x-1}-\dfrac{1}{x+1}$

Q4. $\dfrac{2x+3}{(x+1)(x+2)(x+3)}$

Sol: $\dfrac{2x+3}{(x+1)(x+2)(x+3)}$

Suppose

$\dfrac{2x+3}{(x+1)(x+2)(x+3)}=\dfrac{A}{x+1}+\dfrac{B}{x+2}+\dfrac{C}{x+3}$ ——— (i)

Multiply both sides by $(x+1)(x+2)(x+3)$ we get

$2x+3=A(x+2)(x+3)+B(x+1)(x+3)+C(x+1)(x+2)$ ——— (ii)

Put $x+1=0 \Rightarrow x=-1$ in (ii)

$$2(-1)+3=A(-1+2)(-1+3)+B(-1+1)(-1+3)+C(-1+1)(-1+2)$$

$$-2+3=A(1)(2)+B(0)+C(0)$$

$$1=\ 2A\Rightarrow A=\frac{1}{2}$$

Put $x+2=0\Rightarrow x=-2$ in (ii)

$$2(-2)+3=A(-2+2)(-2+3)+B(-2+1)(-2+3)+C(-2+1)(-2+2)$$

$$-4+3=A(0)+B(-1)(1)+C(0)$$

$$-1=-1B\Rightarrow B=1$$

Put $x+3=0\Rightarrow x=-3$ in (ii)

$$2(-3)+3=A(-3+2)(-3+3)+B(-3+1)(-3+3)+C(-3+1)(-3+2)$$

$$-6+3=A(0)+B(0)+C(-2)(-1)$$

$$-3=2C\Rightarrow\ -3/2=C$$

Equation (I) become

$$\frac{2x+3}{(x+1)(x+2)(x+3)}=\frac{1}{2(x+1)}+\frac{1}{(x+2)}+\frac{-3/2}{(x+3)}$$

Hence $\dfrac{2x+3}{(x+1)(x+2)(x+3)}=\dfrac{1}{2(x+1)}+\dfrac{1}{(x+2)}+\dfrac{-3}{2(x+3)}$

Q5. $\dfrac{x^2+4x+5}{(x+1)(x^2+5x+6)}$

Sol: $\dfrac{x^2+4x+5}{(x+1)(x^2+5x+6)}=\dfrac{x^2+4x+5}{(x+1)(x^2+3x+2x+6)}$

$$\frac{x^2+4x+5}{(x+1)(x^2+5x+6)}=\frac{x^2+4x+5}{(x+1)x(x+3)+2(x+3)}=\frac{x^2+4x+5}{(x+1)(x+2)(x+3)}$$

Suppose

$$\frac{x^2+4x+5}{(x+1)(x+2)(x+3)}=\frac{A}{x+1}+\frac{B}{x+2}+\frac{C}{x+3}\qquad\_\_\_\_\_\_\ \text{(i)}$$

Multiply by $(x+1)(x+2)(x+3)$ both sides

$$x^2+4x+5=A(x+2)(x+3)+B(x+1)(x+3)+C(x+1)(x+2)\ \_\_\_\_\_\_\ \text{(ii)}$$

Put $x+1=0\Rightarrow x=-1$ in (ii)

$$(-1)^2+4(-1)+5=A(-1+2)(-1+3)+B(-1+1)(-1+3)+C(-1+1)(-1+2)$$

$$1-4+5=A(1)(2)+B(0)+C(0)$$

$$2=\ 2A\Rightarrow A=1$$

put $x+2=0\Rightarrow x=-2$ in (ii)

$$(-2)^2+4(-2)+5=A(-2+2)(-2+3)+B(-2+1)(-2+3)+C(-2+1)(-2+2)$$

$$4-8+5=A(0)+B(-1)(1)+C(0)$$

$$1=\ -1B\Rightarrow B=-1$$

Put $x+3=0\Rightarrow x=-3$ in (ii)

$(-3)^2+4(-3)+5=A(-3+2)(-3+3)+B(-3+1)(-3+3)+C(-3+1)(-3+2)$

$9-12+5=A(0)+B(0)+C(-2)(-1)$

$2=(2)C\Rightarrow C=1$

I Become

Hence $\dfrac{x^2+4x+5}{(x+1)(x^2+5x+6)}=\dfrac{1}{x+1}-\dfrac{1}{x+2}+\dfrac{1}{x+3}$

Q6. $\dfrac{4x^3+5x^2-3x-2}{x^2-1}$

Sol: $\dfrac{4x^3+5x^2-3x-2}{x^2-1}$ (improper fraction)

$$\begin{array}{r|l} & 4x+5 \\ x^2-1 & 4x^3+5x^2-3x-2 \\ & \pm 4x^3 \qquad \mp 4x \\ \hline & 5x^2+x-2 \\ & \pm 5x^2 \qquad \mp 5 \\ \hline & x+3 \end{array}$$

$\dfrac{4x^3+5x^2-3x-2}{x^2-1}=4x+5+\dfrac{x+3}{x^2-1}\rightarrow I$

Now take $\dfrac{x+3}{x^2-1}=\dfrac{x+3}{(x+1)(x-1)}$

Suppose

$\dfrac{x+3}{(x-1)(x+1)}=\dfrac{A}{x-1}+\dfrac{B}{x+1}\rightarrow II$

Multiply by $(x-1)(x+1)$

$x+3=A(x+1)+B(x-1)\rightarrow III$

Put $x-1=0\Rightarrow x=1$ in eq. (iii)

$1+3=A(1+1)+B(1-1)$

$4=A(2)\Rightarrow \boxed{A=2}$

Put $x+1=0\Rightarrow x=-1$ in (iii)

$-1+3=A(-1+1)+B(-1-1)$

$2=A(0)+B(-2)\Rightarrow 2=-2B\Rightarrow \boxed{B=-1}$

Eq. (ii) Become

$\dfrac{x+3}{x^2-1}=\dfrac{2}{x-1}+\dfrac{-1}{x+1}$

Hence $\dfrac{4x^3+5x^2-3x-2}{x^2-1}=4x+5+\dfrac{2}{x-1}-\dfrac{1}{x+1}$ (equation (i) become)

Q7. $\dfrac{3x^2-12x+11}{(x-1)(x-2)(x-3)}$

Sol: Suppose

$$\frac{3x^2-12x+11}{(x-1)(x-2)(x-3)}=\frac{A}{(x-1)}+\frac{B}{(x-2)}+\frac{C}{x-3}\rightarrow I$$

Multiply by $(x-1)(x-2)(x-3)$ on both sides

$$3x^2-12x+11=A(x-2)(x-3)+B(x-1)(x-3)+C(x-1)(x-2)\rightarrow II$$

Put $x-1=0\Rightarrow x=1$ in eq. (ii)

$$3(1)^2-12(1)+11=A(1-2)(1-3)+B(1-1)(1-3)+C(1-1)(1-2)$$

$$3-12+11=A(-1)(-2)+B(0)+C(0)$$

$$2=2A\Rightarrow \boxed{A=1}$$

Put $x-2=0\Rightarrow x=2$ in (ii)

$$3(2)^2-12(2)+11=A(2-2)(2-3)+B(2-1)(2-3)+C(2-1)(2-2)$$

$$12-24+11=A(0)+B(1)(-1)+C(0)$$

$$-1=-B\Rightarrow \boxed{B=1}$$

Put $x-3=0\Rightarrow x=3$ in (ii)

$$3(3)^2-12(3)+11=A(3-2)(3-3)+B(3-1)(3-3)+C(3-1)(3-2)$$

$$27-36+11=A(0)+B(0)+C(2)(1)$$

$$2=2C\Rightarrow \boxed{C=1}$$

Put values of A,B and C in eq. (i)

Hence $$\frac{3x^2-12x+11}{(x-1)(x-2)(x-3)}=\frac{1}{x-1}+\frac{1}{x-2}+\frac{1}{x-3}$$

Q8. $$\frac{(x-1)(x-2)(x-3)}{(x-4)(x-5)(x-6)}$$

Sol: $$\frac{(x-1)(x-2)(x-3)}{(x-4)(x-5)(x-6)}=\frac{(x-1)(x^2-3x-2x+6)}{(x-4)(x^2-6x-5x+30)}$$

$$\frac{(x-1)(x^2-5x+6)}{(x-4)(x^2-11x+30)}=\frac{x^3-5x^2+6x-x^2+5x-6}{x^3-11x^2+30x-4x^2+44x-120}=\frac{x^3-6x^2+11x-6}{x^3-15x^2+74x-120}$$

$$=1+\frac{9x^2-63x+114}{x^3-15x^2+74x-120}=1+\frac{9x^2-63x+114}{(x-4)(x-5)(x-6)}\rightarrow (I)$$

$$\begin{array}{r|l} & 1 \\ x^3-15x^2+74x-120 & x^3-6x^2+11x-6 \\ & \underline{\underset{-}{x^3}\ \underset{+}{-15x^2}\ \pm 74x\ \underset{+}{-120}} \\ & 9x^2-63x+114 \end{array}$$

Now we take

Let $$\frac{9x^2-63x+114}{(x-4)(x-5)(x-6)}=\frac{A}{(x-4)}+\frac{B}{(x-5)}+\frac{C}{(x-6)}\rightarrow (II)$$

Multiply by both sides $(x-4)(x-5)(x-6)$ we get

$$9x^2-63x+114=A(x-5)(x-6)+B(x-4)(x-6)+C(x-4)(x-5)\rightarrow (III)$$

Put $x-4=0\Rightarrow x=4$ in eq. (iii)

$$9(4)^2-63(4)+114=A(4-5)(4-6)+B(0)+C(0)$$

$$144-252+114=A(-1)(-2)$$

$$6=2A\Rightarrow \boxed{A=3}$$

Put $x-5=0\Rightarrow x=5$ in Eq. (iii)

$$9(5)^2-63(5)+114=A(0)+B(5-4)(5-6)+C(0)$$

$225-315+114=B(1)(-1)$

$24=-B\Rightarrow \boxed{B=-24}$

Put $x-6=0\Rightarrow x=6$ in Eq. (iii)

$9(6)^2-63(6)+114=A(0)+B(0)+C(6-4)(6-5)$

$324-378+114=0+0+C(2)(1)$

$60=2C\Rightarrow \boxed{C=30}$

Eq. (i) Become

$$\frac{(x-1)(x-2)(x-3)}{(x-4)(x-5)(x-6)}=1+\frac{3}{(x-4)}+\frac{-24}{x-5}+\frac{30}{x-6}=1+\frac{3}{x-4}-\frac{24}{x-5}+\frac{30}{x-6}$$

Q9. $$\frac{x^2}{(x^2+a)(x^2+b)(x^2+c)}$$

Sol: Replace x^2 by y

$$\frac{x^2}{(x^2+a)(x^2+b)(x^2+c)}=\frac{y}{(y+a)(y+b)(y+c)}$$

Suppose $$\frac{y}{(y+a)(y+b)(y+c)}=\frac{A}{(y+a)}+\frac{B}{(y+b)}+\frac{C}{(y+c)}\rightarrow(1)$$

Multiply by $(y+a)(y+b)(y+c)$ we get

$y=A(y+b)(y+c)+B(y+a)(y+c)+C(y+a)(y+b)\rightarrow(II)$

Put $y+a=0\Rightarrow y=-a$ in eq. (ii)

$-a=A(-a+b)(-a+c)+B(0)+C(0)$

$-a=A(b-a)(c-a)$

$$A=\frac{-a}{(b-a)(c-a)}=\frac{\not{-}a}{\not{-}(a-b)(c-a)}\Rightarrow A=\frac{a}{(a-b)(c-a)}$$

Put $y+b=0\Rightarrow y=-b$ in eq. (ii)

$-b=A(0)+B(-b+a)(-b+c)+c(0)$

$-b=+B(a-b)(c-b)$

$$-b=-B(a-b)(b-c)\Rightarrow B=\frac{b}{(a-b)(b-c)}$$

Put $y+c=0\Rightarrow y=-c$ in eq. (ii)

$-c=A(0)+B(0)+C(-c+a)(-c+b)$

$-c=C(a-c)(b-c)$

$$-c=-C(c-a)(b-c)\Rightarrow C=\frac{c}{(c-a)(b-c)}$$

Eq. (i) Become

$$\frac{y}{(y+a)(y+b)(y+c)}=\frac{a}{(a-b)(c-a)(y+a)}+\frac{b}{(a-b)(b-c)(y+b)}+\frac{c}{(c-a)(b-c)(y+c)}$$

Replace y by x^2

$$\frac{x^2}{(x^2+a)(x^2+b)(x^2+c)}=\frac{a}{(a-b)(c-a)(x^2+a)}+\frac{b}{(a-b)(b-c)(x^2+b)}+\frac{c}{(c-a)(b-c)(x^2+c)}$$

Q10. $\frac{x+1}{(x-1)^2}$

Sol: Suppose $\frac{x+1}{(x-1)^2}=\frac{A}{(x-1)}+\frac{B}{(x-1)^2}\rightarrow(I)$

Multiply by $(x-1)^2$; we get

$x+1=A(x-1)+B\rightarrow(II)$

Put $x-1=0\Rightarrow x=1$ in eq. (ii)

$1+1=A(1-1)+B\Rightarrow 2=A(0)+B\Rightarrow B=2$

Rearrange (II)

$x+1=Ax-A+B$

Comparing co-efficient

x; $\boxed{A=1}$

Eq. (I) Become

Hence $\frac{x+1}{(x-1)^2}=\frac{1}{(x-1)}+\frac{2}{(x-1)^2}$

Q11. $\frac{x^2+x}{(x^2-1)^2}$

Sol: $\frac{x^2+x}{(x^2-1)^2}=\frac{x^2+x}{((x-1)(x+1))^2}$ $\quad\because a^2-b^2=(a-b)(a+b)$

Let $\frac{x^2+x}{(x-1)^2(x+1)^2}=\frac{A}{x-1}+\frac{B}{(x-1)^2}+\frac{C}{(x+1)}+\frac{D}{(x+1)^2}\rightarrow(I)$

Multiply by $(x-1)^2(x+1)^2$, we get

$x^2+x=A(x-1)(x+1)^2+B(x+1)^2+C(x+1)(x-1)^2+D(x-1)^2\rightarrow(II)$

Put $x-1=0\Rightarrow x=1$ in eq. (ii)

$(1)^2+(1)=A(1-1)(1+1)^2+B(1+1)^2+C(1+1)(1-1)^2+D(1-1)^2$

$2=A(0)+B(2)^2+C(0)+D(0)$

$2=4B\Rightarrow B=\frac{1}{2}$

Put $x+1=0\Rightarrow x=-1$ in Eq. (ii)

$(-1)^2+(-1)=A(0)+B(0)+C(0)+D(-1-1)^2$

$1-1=D(-2)^2\Rightarrow 0=4D\Rightarrow\boxed{D=0}$

Re-arrange (II)

$x^2+x=A(x-1)(x^2+1+2x)+B(x^2+1+2x)+C(x+1)(x^2+1-2x)+D(x^2+1-2x)$

$x^2+x=A(x^3+x+2x^2-x^2-1-2x)+B(x^2+1+2x)+C(x^3+x-2x^2+x^2+1-2x)+D(x^2+1-2x)$

$x^2+x=A(x^3+x^2-x-1)+B(x^2+1+2x)+C(x^3-x^2-x+1)+D(x^2+1-2x)$

$x^2+x=(A+C)x^3+(A+B-C+D)x^2+(-A+2B-C-2D)x+(-A+B+C+D)x^0$

Comparing co-efficient

x^3; $\quad 0=A+C\rightarrow(III)$

$x^2;\quad 1 = A + B - C + D \rightarrow (IV)$

$x;\quad 1 = -A + 2B - C - 2D \rightarrow (V)$

$x^0;\quad 0 = -A + B + C + D \rightarrow (VI)$

Put values $B = \frac{1}{2}, D = 0$ in eq. (IV)

$$1 = A + \frac{1}{2} - C + 0$$

$$1 - \frac{1}{2} = A - C$$

$$\frac{1}{2} = A - C \rightarrow (VII)$$

Adding eq. (iii) and (vii)

$$0 = A + \not{C}$$

$$\frac{1}{2} = A - \not{C}$$

$$\frac{1}{2} = 2A$$

$$\boxed{A = \frac{1}{4}}$$

Put $A = \frac{1}{4}$ in eq. (iii)

$$\frac{1}{4} + C = 0 \Rightarrow \boxed{C = -\frac{1}{4}}$$

Put values A, B, C and D in eq. (i)

$$\frac{x^2 + x}{(x^2 - 1)^2} = \frac{1}{4(x-1)} + \frac{1}{2(x-1)^2} - \frac{1}{4(x+1)} + \frac{0}{(x+1)^2} = \frac{1}{4(x-1)} + \frac{1}{2(x-1)^2} - \frac{1}{4(x+1)}$$

Q12. $\dfrac{3x^2 + 4x - 5}{(x-1)^3}$

Sol: Suppose

$$\frac{3x^2 + 4x - 5}{(x-1)^3} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(x-1)^3} \rightarrow (I)$$

Multiply by $(x-1)^3$, we get

$$3x^2 + 4x - 5 = A(x-1)^2 + B(x-1) + C \rightarrow (II)$$

Put $x - 1 = 0 \Rightarrow x = 1$ in eq. (II)

$$3(1)^2 + 4(1) - 5 = A(1-1)^2 + B(1-1) + C$$

$$3 + 4 - 5 = A(0) + B(0) + C \Rightarrow \boxed{2 = C}$$

Re-Arrange eq. (II)

$$3x^2 + 4x - 5 = A(x^2 + 1 - 2x) + B(x-1) + C$$

$$3x^2 + 4x - 5 = Ax^2 + A - 2Ax + Bx - B + C$$

$$3x^2 + 4x - 5 = Ax^2 + (-2A + B)x + (A - B + C)x^o$$

Comparing co-efficient

$x^2;$ $\boxed{3=A}$

$x;$ $4=-2A+B\rightarrow(iii)$

Put $A=3$ in eq. (iii)

$4=-2(3)+B\Rightarrow B=4+6=10\Rightarrow\boxed{B=10}$

Put values A, B and C in eq. (i)

Hence $\frac{3x^2+4x-5}{(x-1)^3}=\frac{3}{x-1}+\frac{10}{(x-1)^2}+\frac{2}{(x-1)^3}$

Q13. $\frac{1}{x(x+1)^3}$

Sol: Suppose

$\frac{1}{x(x+1)^3}=\frac{A}{x}+\frac{B}{(x+1)}+\frac{C}{(x+1)^2}+\frac{D}{(x+1)^3}\rightarrow(i)$

'X' by both sides $x(x+1)^3$ we get

$1=A(x+1)^3+B(x)(x+1)^2+C(x)(x+1)+D(x)\rightarrow(ii)$

Put $x=0$ in eq. (ii)

$1=A(0+1)^3+0+0+0\Rightarrow\boxed{A=1}$

Put $x+1=0\Rightarrow x=-1$ in eq. (ii)

$1=A(-1+1)^3+B(-1)(-1+1)^2+C(-1)(-1+1)+D(-1)$

$1=A(0)+B(0)+C(0)+D(-1)\Rightarrow\boxed{D=-1}$

Re-arrange eq. (ii)

$1=A(x^3+3x^2+3x+1)+Bx(x^2+1+2x)+C(x^2+x)+Dx$

$1=Ax^3+3Ax^2+3Ax+A+Bx^3+Bx+2Bx^2+Cx^2+Cx+Dx$

Comparing co-efficients

$x^3;$ $0=A+B\rightarrow(iii)$

Put $A=1$ in eq. (iii)

$0=1+B\Rightarrow\boxed{B=-1}$

$x^2;$

$3A+2B+C=0\rightarrow(iv)$

Put values $A\,\&\,B$ in eq. (iv)

$3(1)+2(-1)+C=0\Rightarrow 3-2+C=0\Rightarrow\boxed{C=-1}$

Put values A, B, C and D in eq. (i)

Hence $\frac{1}{x(x+1)^3}=\frac{1}{x}-\frac{1}{x+1}-\frac{1}{(x+1)^2}-\frac{1}{(x+1)^3}$

Q14. $\frac{4x^2-3x+1}{(x+1)(x-1)^2}$

Sol: Suppose

$\frac{4x^2-3x+1}{(x+1)(x-1)^2}=\frac{A}{x+1}+\frac{B}{(x-1)}+\frac{C}{(x-1)^2}\rightarrow(I)$

Multiply by $(x+1)(x-1)^2$, we get

$$4x^2-3x+1=A(x-1)^2+B(x+1)(x-1)+C(x+1)\rightarrow(II)$$

Put $x+1=0\Rightarrow x=-1$ in eq. (ii)

$$4(-1)^2-3(-1)+1=A(-1-1)^2+B(0)+C(0)$$

$$4+3+1=4A\Rightarrow 8=4A\Rightarrow \boxed{A=2}$$

Put $x-1=0\Rightarrow x=1$ in eq. (ii)

$$4(1)^2-3(1)+1=A(1-1)^2+B(1+1)(1-1)+C(1+1)$$

$$4-3+1=A(0)+B(0)+C(2)$$

$$2=2C\Rightarrow \boxed{C=1}$$

Re-arrange

$$4x^2-3x+1=A(x^2+1-2x)+B(x^2-1)+C(x+1)$$

Comparing co-efficient

$$x^2;\ 4=A+B\rightarrow(III)$$

$$4=2+B\Rightarrow 4-2=B\Rightarrow \boxed{B=2}$$

Eq. (i) Become

Hence $\dfrac{4x^2-3x+1}{(x+1)(x-1)^2}=\dfrac{2}{x+1}+\dfrac{2}{x-1}+\dfrac{1}{(x-1)^2}$

Q15. $\dfrac{12x^2-48}{(x-2)^2(x+2)^2}$

Sol: Suppose

$$\frac{12x^2-48}{(x-2)^2(x+2)^2}=\frac{A}{(x-2)}+\frac{B}{(x-2)^2}+\frac{C}{(x+2)}+\frac{D}{(x+2)^2}\rightarrow(i)$$

Multiply by $(x-2)^2(x+2)^2$ both sides we get

$$12x^2-48=A(x-2)(x+2)^2+B(x+2)^2+C(x+2)(x-2)^2+D(x-2)^2\rightarrow(ii)$$

Put $x-2=0\Rightarrow x=2$ in eq. (ii)

$$12(2)^2-48=A(0)+B(2+2)^2+C(0)+D(0)$$

$$12(4)-48=B(4)^2$$

$$48-48=16B\Rightarrow \boxed{B=0}$$

Put $x+2=0\Rightarrow x=-2$ in eq. (ii)

$$12(-2)^2-48=A(0)+B(0)+C(0)+D(-2-2)^2$$

$$48-48=16D\Rightarrow \boxed{D=0}$$

Rearrange eq. (II)

$$12x^2-48=A(x-2)(x^2+4x+4)+B(x^2+4x+4)$$

$$+C(x+2)(x^2-4x+4)+D(x^2-4x+4)$$

$$12x^2-48=A(x^3+4x^2+4x-2x^2-8x-8)+B(x^2+4x+4)$$

$$+C(x^3-4x^2+4x+2x^2-8x+8)+D(x^2-4x+4)$$

$$12x^2-48=A(x^3+2x^2-4x-8)+B(x^2+4x+4)$$

$$+C(x^3-2x^2-4x+8)+D(x^2-4x+4)$$

Comparing co-efficient

$x^3;\quad A+C=0\rightarrow(III)$

$x^2;\quad 12=2A+B-2C+D\rightarrow(IV)$

$x;\quad 0=-4A+4B-4C-4D\rightarrow(V)$

$x^0;\quad -48=-8A+4B+8C+4D\rightarrow(VI)$

Put values B and D in eq. (V) and (VI)

$0=-4A+4(0)-4C-4(0)$

$-48=-8A+4(0)+8C+4(0)$

$0=-4A-4C\rightarrow(VII)$

$-48=-8A+8C\rightarrow(VIII)$

Eq. (VII) X by 2 and add in eq. (VII)

$0=-8A-8C$

$-48=-8A+8C$

$-48=-16A \quad \Rightarrow \boxed{A=3}$

Put $A=3$ in Eq. (iii)

$A+C=0$

$3+C=0\Rightarrow \boxed{C=-3}$

Put values A, B, C and D in eq. (i)

Hence $\frac{12x^2-48}{(x-2)^2(x+2)^2}=\frac{3}{(x-2)}+\frac{0}{(x-2)^2}+\frac{-3}{(x+2)}+\frac{0}{(x+2)^2}=\frac{3}{(x-2)}-\frac{3}{(x+2)}$