Exercise 5.2

Resolve into Partial Fractions.

Q1. $\dfrac{2x^2+3x+3}{(x+1)(x^2+1)}$

Sol: Suppose

$\dfrac{2x^2+3x+3}{(x+1)(x^2+1)}=\dfrac{A}{x+1}+\dfrac{Bx+C}{x^2+1}$ ____________ (i)

Multiply by both sides $(x+1)(x^2+1)$, we get

$2x^2+3x+3=A(x^2+1)+(Bx+C)(x+1)$ ____________ (ii)

Put $x+1=0\Rightarrow x=-1$ in II

$2(-1)^2+3(-1)+3=A((-1)^2+1)+(B(-1)+C)(-1+1)$

$2-\cancel{3}+\cancel{3}=A(1+1)+0 \;=\; 2=2A\Rightarrow\boxed{A=1}$

Rearrange II

$2x^2+3x+3=Ax^2+A+Bx^2+Bx+Cx+C$

$2x^2+3x+3=(A+B)x^2+(B+C)x+(A+C)x^0$

Computing co-efficient

$x^2;\quad A+B=2$ ____________ (iii)

Put $A=1$ in equation (iii)

$1+B=2\;\Rightarrow\; B=2-1\;\Rightarrow B=1$

$x;\quad B+C=3$ ____________ (iv)

Put value $B=1$ in equation (iv)

$1+C=3\quad =C=3-1\Rightarrow 2=C$

Put values A,B and C in equation (i)

Hence $\dfrac{2x^2+3x+3}{(x+1)(x^2+1)}=\dfrac{1}{x+1}+\dfrac{x+2}{x^2+1}$

Q2. $\dfrac{2x+1}{(x-2)(x^2+3x+5)}$

Sol: Suppose

$\dfrac{2x+1}{(x-2)(x^2+3x+5)}=\dfrac{A}{x-2}+\dfrac{Bx+C}{x^2+3x+5}$ ____________ (i)

Multiply by $(x-2)(x^2+3x+5)$ we get

$2x+1=A(x^2+3x+5)+(Bx+C)(x-2)$ ____________ (ii)

Put $x-2=0\Rightarrow\; x=2$ in equation (ii)

$2(2)+1=A((2)^2+3(2)+5)+(B(2)+C)(2-2)$

$4+1=A(4+6+5)+0$

$5=\quad 15A\Rightarrow\dfrac{5}{15}=A\quad\Rightarrow A=\dfrac{1}{3}$

Rearrange equation II

$2x+1=Ax^2+3Ax+5A+Bx^2-2Bx+Cx-2C$

$2x+1=(A+B)x^2+(3A-2B+C)x+(5A-2C)x^0$

Comparing co-efficient

$x^2;\quad A+B=0$ __________ (iii)

$x;\quad 3A-2B+C=2$ __________ (iv)

$x;\quad 5A-2C=1$ __________ (v)

Put value $A=\frac{1}{3}$ in equation (iii)

$\frac{1}{3}+B=0 \Rightarrow B=\frac{-1}{3}$

Put value $A=\frac{1}{3}$ in equation (v)

$5\left(\frac{1}{3}\right)-2C=1 \Rightarrow \frac{5}{3}-2C=1$

$-2C=\frac{1-5}{3} \Rightarrow -2C=\frac{-2}{3}=C=\frac{1}{3}$

Put values A, B and C in equation (i)

Hence $\frac{2x+1}{(x-2)(x^2+3x+5)}=\frac{1}{3(x-2)}+\frac{\frac{-x}{3}+\frac{1}{3}}{(x^2+3x+5)}=\frac{1}{3(x-2)}-\frac{x-1}{3(x^2+3x+5)}$

Q3. $\frac{2x+32}{(x-2)(x^2+2)^2}$

Sol: Suppose

$\frac{2x+32}{(x-2)(x^2+2)^2}=\frac{A}{(x-2)}+\frac{Bx+C}{(x^2+2)}+\frac{Dx+E}{(x^2+2)^2}$ ________ (i)

Multiply by $(x-2)(x^2+2)^2$ we get

$2x+32=A(x^2+2)^2+(Bx+C)(x-2)(x^2+2)+(Dx+E)(x-2)$ ______ (ii)

Put $x-2=0 \Rightarrow x=2$ in equation (ii)

$2(2)+32=A((2)^2+2)^2+0+0$

$4+32=A(4+2)^2=36=36A \Rightarrow A=1$

Rearrange equation (ii)

$2x+32=A(x^4+4x^2+4)+(Bx+C)(x^3+2x-2x^2-4)+(Dx+E)(x-2)$

$2x+32=Ax^4+4Ax^2+4A+Bx^4+2Bx^2-2Bx^3-4Bx+Cx^3+2Cx-2Cx^2-4C+Dx^2-2Dx+Ex-2E$

$2x+32=(A+B)x^4+(-2B+C)x^3$

$(4A+2B-2C+D)x^2+(-4B+2C-2D+E)x+(4A-4C-2E)x^0$

Comparing co-efficient

$x^4; A+B=0$ ________ (iii)

$x^3;\ -2B+C=0$ ________ (iv)

$x^2;\ 4A+2B-2C+D=0$ ________ (v)

$x;\ -4B+2C-2D+E=2$ ________ (vi)

$x;\ 4A-4C-2E=32$ ________ (vii)

Put $A=1$ in equation (iii)

$$1+B=0 \quad \Rightarrow B=-1$$

Put value $B=-1$ in equation (iv)

$$-2(-1)+C=0 \quad \Rightarrow \quad 2+C=0 \Rightarrow C=-2$$

Put A,B and C in equation (v)

$$4(1)+2(-1)-2(-2)+D=0$$

$$4-2+4+D=0 \quad \Rightarrow \quad D=-6$$

Put A , and C in equation (vii)

$$4(1)-4(-2)-2E=32$$

$$4+8-2E=32 \quad =-2E=32-12$$

$$-2E=20 \quad \Rightarrow \quad E=-10$$

Put values A,B,C,D and E in equation (i)

$$\frac{2x+32}{(x-2)(x^2+2)^2}=\frac{1}{x-2}+\frac{-x-2}{(x^2+2)}+\frac{-6x-10}{(x^2+2)^2}$$

Hence $\dfrac{2x+32}{(x-2)(x^2+2)^2}=\dfrac{1}{x-2}-\dfrac{x+2}{(x^2+2)}-\dfrac{2(3x+5)}{(x^2+2)^2}$

Q4. $\dfrac{3x^2+3}{x^3+1}$

Sol: Suppose

$$\frac{3x^2+3}{x^3+1}=\frac{3x^2+3}{(x+1)(x^2-x+1)}=\frac{A}{x+1}+\frac{Bx+C}{x^2-x+1} \quad \_\_\_\_\_\_\_\_ \text{(i)} \qquad \left\{a^3+b^3=(a+b)(a^2-ab+b^2)\right\}$$

Multiply by $(x+1)(x^2-x+1)$ we get

$$3x^2+3=A(x^2-x+1)+(Bx+C)(x+1) \quad \_\_\_\_\_\_\_\_ \text{(ii)}$$

Put $x+1=0 \Rightarrow x=-1$ in equation (ii)

$$3(-1)^2+3=A((-1)^2-(-1)+1)+(B(-1)+C)(-1+1)$$

$$3+3=A(1+1+1)+0 \quad \Rightarrow 6=3A \Rightarrow A=2$$

Re-arrange equation ii

$$3x^2+3=Ax^2-Ax+A+Bx^2+Bx+Cx+C$$

$$3x^2+3=(A+B)x^2+(-A+B+C)x+(A+C)x^0$$

Comparing co-efficient

$x^2;\ A+B=3 \quad \_\_\_\_\_\_\_\_$ (iii)

$x;\ -A+B+C=0 \quad \_\_\_\_\_\_\_\_$ (iv)

$x;\ A+C=3 \quad \_\_\_\_\_\_\_\_$ (v)

Put $A=2$ in equation (iii)

$$2+B=3 \quad \Rightarrow B=3-2 \quad \Rightarrow B=1$$

Put values A in equation (v)

$$2+C=3 \quad =C=3-2 \quad \Rightarrow C=1$$

Put values A,B and C in equation (i)

Hence $\dfrac{3x^2+3}{x^3+1}=\dfrac{2}{x+1}+\dfrac{x+1}{x^2-x+1}$

Q5. $\dfrac{x^4}{x^4+2x^2+1}$

Sol: $\dfrac{x^4}{x^4+2x^2+1}$ (Improper Fraction)

$$\begin{array}{r|l} & \quad 1 \\ x^4+2x^2+1 & \cancel{x^4} \\ & \underline{\pm\cancel{x^4} \pm 2x^2 \pm 1} \\ & -2x^2-1 \end{array}$$

$=1+\dfrac{-2x^2-1}{x^4+2x^2+1}=1-\dfrac{2x^2+1}{(x^2+1)^2}$ ______ (i)

Now we take $\dfrac{2x^2+1}{(x^2+1)^2}$

Suppose $\dfrac{2x^2+1}{(x^2+1)^2}=\dfrac{Ax+B}{(x^2+1)}+\dfrac{Cx+D}{(x^2+1)^2}$ ______ (ii)

Multiply by $(x^2+1)^2$, we get

$2x^2+1=(Ax+B)(x^2+1)+(Cx+D)$ ______ (iii)

Re-arrange equation (iii)

$2x^2+1=Ax^3+Ax+Bx^2+B+Cx+D$

Comparing co-efficients

$x^3;\quad A=0$

$x^2;\quad B=2$

$x;\quad A+C=0$ ______ (iv)

$x^0;\quad B+D=1$ ______ (v)

Put $A=0$ in equation (iv)

$A+C=0 \Rightarrow 0+C=0 \Rightarrow C=0$

Put $B=2$ in equation (v)

$2+D=1 \Rightarrow D=1-2 \Rightarrow D=-1$

Put values A,B,C and D in equation (ii)

$\dfrac{2x^2+1}{(x^2+1)^2}=\dfrac{2}{x^2+1}-\dfrac{1}{(x^2+1)^2}$ put in equation (i)

Hence $\dfrac{x^4}{x^4+2x^2+1}=1-\dfrac{2}{x^2+1}+\dfrac{1}{(x^2+1)^2}$

Q6. $\dfrac{6x^4+40x^2}{(4-x^2)(x^2+4)^2}$

Sol: Suppose

$$\frac{6x^4+40x^2}{(4-x^2)(x^2+4)^2}=\frac{6x^4+40x^2}{(2-x)(2+x)(x^2+4)^2}$$

$$\frac{6x^4+40x^2}{(2-x)(2+x)(x^2+4)^2}=\frac{A}{(2-x)}+\frac{B}{(2+x)}+\frac{Cx+D}{(x^2+4)}+\frac{Ex+F}{(x^2+4)^2} \quad \text{—— (i)}$$

Multiply by $(2-x)(2+x)(x^2+4)^2$ we get

$$6x^4+40x^2=A(2+x)(x^2+4)^2+B(2-x)(x^2+4)^2+(Cx+D)(2-x)(2+x)(x^2+4)+(Ex+F)(2-x)(2+x) \quad \text{—— (ii)}$$

Put $2-x=0 \Rightarrow 2=x$ in equation (ii)

$$6(2)^4+40(2)^2=A(2+2)\left((2)^2+4\right)^2+B(2-2)\left((2)^2+4\right)^2+(C(2)+D)(2-2)(2+2)\left((2)^2+4\right)+(E(2)+F)(2-2)(2+2)$$

$$96+160=A(4)(64)+0+0+0\Rightarrow 256=256A \quad \Rightarrow A=1$$

Put $2+x=0 \quad \Rightarrow \quad x=-2$ in equation (ii)

$$6(-2)^4+40(-2)^2=0+B(+2+2)\left((-2)^2+4\right)^2+0+0$$

$$96+160=B(4)(64)\Rightarrow 256=256\ B \ \Rightarrow B=1$$

Re-arrange equation ii

$$6x^4+40x^2=A(2+x)\left(x^4+16+8x^2\right)+B(2-x)\left(x^4+16+8x^2\right)+(Cx+D)\left(4-x^2\right)\left(4+x^2\right)+(Ex+F)\left(4-x^2\right)$$

$$6x^4+40x^2=A\left(2x^4+32+16x^2+x^5+16x+8x^3\right)+B\left(2x^4+32+16x^2-x^5-16x-8x^3\right)+(Cx+D)\left(16-x^4\right)+(Ex+F)\left(4-x^2\right)$$

$$6x^4+40x^2=A\left(2x^4+32+16x^2+x^5+16x+8x^3\right)+B\left(2x^4+32+16x^2-x^5-16x-8x^3\right)+\left(16Cx-Cx^5+D16-Dx^4\right)+\left(4Ex-Ex^3+4F-Fx^2\right)$$

Comparing co-efficients

x^5; $A-B-C=0$ _________ (iii)

x^4; $2A+2B-D=6$ _________ (iv)

x^3; $8A-8B-E=0$ _________ (v)

x^2; $16A+16B-F=40$ _________ (vi)

x; $16A-16B+16C+4E=0$

$4A-4B+4C+E=0$ _________ (vii)

x^0; $32A+32B+16D+4F=0$

$8A+8B+4D+F=0$ _________ (viii)

Put values A and B in Equation (iii)

$A-B-C=0\Rightarrow 1-1-C=0 \quad \Rightarrow \quad C=0$

Put value A and B in Equation (iv)

$2(1)+2(1)-D=6\Rightarrow 4-D=6 \ \Rightarrow -D=6-4\Rightarrow D=-2$

Put value A and B in Equation (v)

$8(1)-8(1)-E=0 \ \Rightarrow E=0$

Put value A and B in equation (vi)

$16(1)+16(1)-F=40 \quad =-F=40-32\Rightarrow -F=8 \ \Rightarrow F=-8$

Equation (i) Become

Hence $\dfrac{6x^4+40x^2}{(2-x)(2+x)\left(x^2+4\right)^2}=\dfrac{1}{2-x}+\dfrac{1}{2+x}-\dfrac{2}{\left(x^2+4\right)}-\dfrac{8}{\left(x^2+4\right)^2}$