Exercise 6.1

1. **Find the next four terms of each sequence:**
   i) $12, 16, 20, \ldots$ ii) $3, 1, -1, \ldots$

Sol:

i) Sequence: $12, 16, 20, \ldots$

- Common difference: $d = 16 - 12 = 4$
- Next four terms:
- $20 + 4 = 24$
- $24 + 4 = 28$
- $28 + 4 = 32$
- $32 + 4 = 36$

ii) Sequence: $3, 1, -1, \ldots$

- Common difference: $d = 1 - 3 = -2$
- Next four terms:
- $-1 + (-2) = -3$
- $-3 + (-2) = -5$
- $-5 + (-2) = -7$
- $-7 + (-2) = -9$

2. **Write down the first three terms of each of the following sequence:**

Sol:

i) $a_n = 3n + 5$

- For the first term $(n = 1): a_1 = 3(1) + 5 = 3 + 5 = 8$
- For the second term $(n = 2): a_2 = 3(2) + 5 = 6 + 5 = 11$
- For the third term $(n = 3): a_3 = 3(3) + 5 = 9 + 5 = 14$

The first three terms are $8, 11, 14$

ii) $a_{n+1} = 4a_n - 7$ *and* $a_1 = 3$

- The first term is given $a_1 = 3$
- For the second term $(n = 1): a_{1+1} = a_2 = 4a_1 - 7 = 4(3) - 7 = 12 - 7 = 5$
- For the third term $(n = 2): a_{2+1} = a_3 = 4a_2 - 7 = 4(5) - 7 = 20 - 7 = 13$

The first three terms are $3, 5, 13$

iii) $a_n = (n - 3)(n + 1)$

- For the first term $(n = 1): a_1 = (1 - 3)(1 + 1) = (-2)(2) = -4$
- For the second term $(n = 2): a_2 = (2 - 3)(2 + 1) = (-1)(3) = -3$
- For the third term $(n = 3): a_3 = (3 - 3)(3 + 1) = (0)(4) = 0$

The first three terms are $-4, -3, 0$

iv) $a_1 = -1, a_{n+1} = \dfrac{3}{a_n + 2}$

- The first term is given: $a_1 = -1$
- For the second term $(n = 1): a_2 = \dfrac{3}{a_1 + 2} = \dfrac{3}{-1 + 2} = \dfrac{3}{1} = 3$
- For the third term $(n = 2): a_3 = \dfrac{3}{a_2 + 2} = \dfrac{3}{3 + 2} = \dfrac{3}{5}$

The first three terms are $-1, 3, \dfrac{3}{5}$

v) $a_n = 8 - \frac{20}{3+n}$

- The first term is given: $(n=1): a_1 = 8 - \frac{20}{3+1} = 8 - \frac{20}{4} = 8 - 5 = 3$
- For the second term $(n=2): a_2 = 8 - \frac{20}{3+2} = 8 - \frac{20}{5} = 8 - 4 = 4$
- For the third term $(n=3): a_3 = 8 - \frac{20}{3+3} = 8 - \frac{20}{6} = 8 - \frac{10}{3} = \frac{24-10}{3} = \frac{14}{3}$

The first three terms are $3, 4, \frac{14}{3}$

vi) $a_1 = 1, a_{n+1} = (3a_n + 2)^2$

- The first term is given: $a_1 = 1$
- For the second term $(n=1): a_2 = (3a_1+2)^2 = (3(1)+2)^2 = (3+2)^2 = 5^2 = 25$
- For the third term $(n=2): a_3 = (3a_2+2)^2 = (3(25)+2)^2 = (75+2)^2 = 77^2 = 5529$

The first three terms are $1, 25, 5929$

vii) $a_n = (-2n)^2$

- For the first term $(n=1): a_1 = (-2(1))^2 = (-2)^2 = 4$
- For the second term $(n=2): a_2 = (-2(2))^2 = (-4)^2 = 16$
- For the third term $(n=3): a_3 = (-2(3))^2 = (-6)^2 = 36$

The first three terms are $4, 16, 36$

viii) $a_n = (-1)^n 7n^2$

- For the first term $(n=1): a_1 = (-1)^1 .7(1)^2 = (-1).7(1) = -7$
- For the second term $(n=2): a_2 = (-1)^2 .7(2)^2 = (1).7(4) = 28$
- For the third term $(n=3): a_3 = (-1)^3 .7(3)^2 = (-1).7(9) = -63$

The first three terms are $-7, 28, -63$

3. **An expression for n^{th} triangular number is $\frac{n(n+1)}{2}$. Write down the 15th triangle number. Make triangle of dots by taking $n=5$**

Sol: The formula for the nth triangular number is given by:

$$T_n = \frac{n(n+1)}{2}$$

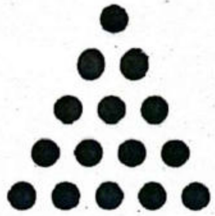
To find the 15th triangular number, we substitute $n = 15$ into the formula:

$$T_{15} = \frac{15(15+1)}{2} = \frac{15(16)}{2} = \frac{240}{2} = 120$$

15th triangular number is 120.

Now for $n=5$ $\quad T_5 = \frac{5(5+1)}{2} = 15$

Triangle is



4. **Write down the n^{th} term of each of the following sequences:**

Sol:

i) $1,4,9,...$

Notice that these are the squares of the natural numbers:

$1=1^2, 4=2^2, 9=3^2$

So, the nth term of this sequence is:

$a_n = n^2$

ii) $1, 1+2, 1+2+3, ...$

The terms of this sequence are the sums of the first n natural numbers. We know that the sum of the first n natural numbers is given by the formula $\frac{n(n+1)}{2}$.

Therefore, the nth term of this sequence is:

$a_n = \frac{n(n+1)}{2}$

iii) $a_1b_1, a_2b_2, a_3b_3, ...$

Following the pattern, the nth term of this sequence is simply the product of the nth terms of the two underlying sequences $\{a_n\}$ and $\{b_n\}$

$a_n = a_n b_n$

iv) $x, 2x^2, 3x^3, ...$

Here

$a_1 = (1)x^1$

$a_2 = (2)x^2$

$a_3 = (3)x^3$

$\vdots \quad \vdots$

$a_n = nx^n$

v) $a_1, a_1+d, a_1+2d, ...$

Here

$a_1 = a_1 + 0d = a_1 + (1-1)d$

$a_2 = a_1 + d = a_1 + (2-1)d$

$a_3 = a_1 + 2d = a_1 + (3-1)d$

$\vdots \quad \vdots$

$a_n = a_1 + (n-1)d$

vi) $a_1, a_1r, a_1r^2, ...$

Here

$a_1 = a_1r^0$

$a_2 = a_1r = a_1r^{(2-1)}$

$a_3 = a_1r^2 = a_1r^{(3-1)}$

$\vdots \quad \vdots \quad \vdots$

$a_n = a_1r^{n-1}$

Hence $a_n = a_1r^{n-1}$

vii) $\frac{a_1}{b_1+c_1}, \frac{2a_2}{b_2+c_2}, \frac{3a_3}{b_3+c_3}, ...$

Looking at the pattern in the numerators and denominators:

The numerator of the nth term is $n.a_n$.

The denominator of the nth term is $b_n + c_n$.

Therefore, the nth term of this sequence is: $a_n = \frac{na_n}{b_n+c_n}$

viii) $\frac{1}{a_1}, \frac{1}{a_1+d}, \frac{1}{a_1+2d}, \ldots$

Here $\frac{1}{a_1} = \frac{1}{a_1+0d} = \frac{1}{a_1+(1-1)d}$, $\frac{1}{a_2} = \frac{1}{a_1+d} = \frac{1}{a_1+(2-1)d}$

$\frac{1}{a_3} = \frac{1}{a_1+2d} = \frac{1}{a_1+(3-1)d} \ldots\ldots a_n = \frac{1}{a_1+(n-1)d}$