



# Exercise 6.2



1. Find the common difference and write the next two terms of each arithmetic sequence.

i)  $9, 16, 23, \dots$

ii)  $5, 5 + \sqrt{2}, 5 + 2\sqrt{2}, \dots$

Sol:

i)  $9, 16, 23, \dots$

- Common difference:  $d = 16 - 9 = 7$
- Next two terms:
  - $23 + 7 = 30$
  - $30 + 7 = 37$

ii)  $5, 5 + \sqrt{2}, 5 + 2\sqrt{2}, \dots$

- Common difference:  $d = x + \sqrt{2} - x = \sqrt{2}$
- Next two terms:
  - $5 + 2\sqrt{2} + \sqrt{2} = 5 + 3\sqrt{2}$
  - $5 + 3\sqrt{2} + \sqrt{2} = 5 + 4\sqrt{2}$

2. Write the first three terms of each arithmetic sequence, with given information.

i)  $a_1 = 2, d = 13$

ii)  $a_1 = 12, d = -13$

Sol:

i)  $a_1 = 2, d = 13$

First term ( $a_1$ ) = 2

Second term  $a_2 = a_1 + d = 2 + 13 = 15$

Third term ( $a_3$ ) =  $a_2 + d = 15 + 13 = 28$

The first three terms are 2, 15, 28.

ii)  $a_1 = 12, d = -13$

First term ( $a_1$ ) = 12

Second term ( $a_2$ ) =  $a_1 + d = 12 + (-13) = -1$

Third term  $a_3 = a_2 + d = -1 + (-13) = -1 - 13 = -14$

The first three terms are 12, -1, -14

3. Find  $a_{n+1}$  and  $a_{2n}$  if  $a_n = 4 + 3n$

Sol:  $a_n = 4 + 3n$

We need to find  $a_{n+1}$  and  $a_{2n}$ .

To find  $a_{n+1}$ , replace  $n+1$  by  $n$  in the formula for  $a_n$ .

$$a_{n+1} = 4 + 3(n+1)$$

$$a_{n+1} = 4 + 3n + 3$$

$$a_{n+1} = 3n + 7$$

To find  $a_{2n}$ , replace  $2n$  by  $n$  in the formula for  $a_n$ :

$$a_{2n} = 4 + 3(2n)$$

$$a_{2n} = 4 + 6n$$



4. Find the indicated term of each of the following arithmetic sequences:

i)  $a_1 = 3, d = 7, a_{14}$  , ii)  $8, 3, -2, \dots, a_{12}$

Sol:

i)  $a_1 = 3, d = 7$ , find  $a_{14}$

Here,  $a_1 = 3, d = 7$ , and  $n = 14$ .

Substitute these values into the formula:  $a_n = a_1 + (n-1)d$

$$a_{14} = 3 + (14-1)7 = 3 + (13)7$$

$$a_{14} = 3 + 91 = 94$$

ii)  $8, 3, -2, \dots$ , find  $a_{12}$

Here,  $a_1 = 8, d = 3 - 8 = -5$ , and  $n = 12$

Substitute these values into the formula:  $a_n = a_1 + (n-1)d$

$$a_{12} = 8 + (12-1)(-5)$$

$$a_{12} = 8 + (11)(-5) = 8 - 55 = -47$$

5. The 18<sup>th</sup> and 30<sup>th</sup> terms of an arithmetic sequence are 367 and 499 respectively. How many terms of this sequence are less than 1000?

Sol: Given:  $a_{18} = 367$  and  $a_{30} = 499$

The formula for the  $n$ th term of an arithmetic sequence is  $a_n = a_1 + (n-1)d$ .

$$a_{18} = a_1 + (18-1)d \Rightarrow 367 = a_1 + 17d \text{ -----(1)}$$

$$a_{30} = a_1 + (30-1)d \Rightarrow 499 = a_1 + 29d \text{ -----(2)}$$

Subtract (1) and (2):

$$(a_1 + 29d) - (a_1 + 17d) = 499 - 367$$

$$12d = 132$$

$$d = 11$$

Substitute  $d = 11$  into equation (1):

$$367 = a_1 + 17(11) \Rightarrow 367 = a_1 + 187$$

$$a_1 = 367 - 187 \Rightarrow a_1 = 180$$

$$a_n = a_1 + (n-1)d$$

$$a_n = 180 + (n-1)11$$

We want to find how many terms of this sequence are less than 1000.

$$180 + (n-1)11 < 1000 \Rightarrow (n-1) < \frac{1000-180}{11}$$

$$n-1 < \frac{820}{11}$$

$$n-1 < 74.545\dots$$

Add 1 to both sides:

$$n < 74.545\dots + 1$$

$$n < 75.545\dots$$

Since  $n$  must be an integer (the term number), the largest integer value of  $n$  that is less than 75.545... is 75.

Therefore, there are 75 terms of this sequence that are less than 1000.

6. Is 301 a term of the A.P. 5, 11, 17, ...?

Sol: Given A.P.: 5, 11, 17, ...

Here

$$a_1 = 5, d = 11 - 5 = 6$$

Use the formula for the  $n$ th term of an A.P.:

$$a_n = a_1 + (n-1)d$$

Take  $a_n = 301$  and solve for  $n$ :

$$301 = 5 + (n-1)6$$

$$301 - 5 = (n-1)6$$

$$296 = (n-1)6$$

$$\frac{296}{6} = (n-1)$$

$$\frac{148}{3} = (n-1)$$

$$49.33... = n-1$$

$$n = 50.33...$$

Since  $n$  is not an integer, 301 is not a term of the A.P. 5, 11, 17, ...

7. If  $2x, x+8, 3x+1$  are in A.P., then find the value of  $x$ .

Sol: Given that  $2x, x+8$  and  $3x+1$  are in A.P., we use the property:

$$3x+1 - (x+8) = x+8 - 2x$$

$$3x+1 - x - 8 = 8 - x \Rightarrow 2x - 7 = 8 - x$$

$$\Rightarrow 2x + x = 15 \Rightarrow 3x = 15 \Rightarrow x = 5$$

8. Which term of the A.P., 3, 8, 13, ... is 123?

Sol: Given that 3, 8, 13, ...

Here,  $a_1 = 3$ .  $d = 8 - 3 = 5$ .  $a_n = 123$ .

Using the formula for the  $n$ th term of an A.P.:

$$a_n = a_1 + (n-1)d$$

$$123 = 3 + (n-1)5$$

$$123 - 3 = (n-1)5$$

$$120 = 5(n-1)$$

$$\frac{120}{5} = n-1$$

$$24 = n-1$$

$$n = 24 + 1$$

$$n = 25$$

123 is the 25<sup>th</sup> term of the sequence.

9. Which term of the A.P., 30, 29.5, 29, ... is the first negative term?

Sol: Given A.P.: 30, 29.5, 29, ...

Here,  $a_1 = 30$ . and  $d = 29.5 - 30 = -0.5$ .

We need to find the first term  $a_n$  such that  $a_n < 0$ .

The formula for the  $n$ th term of an A.P. is

$$a_n = a_1 + (n-1)d$$

Set  $a_n < 0$ :

$$30 + (n-1)(-0.5) < 0$$

$$30 - 0.5(n-1) < 0$$

$$30 < 0.5(n-1)$$

$$\frac{30}{0.5} < n-1 \Rightarrow 60 < n-1 \Rightarrow 60+1 < n \Rightarrow 61 < n$$

Since  $n$  must be an integer, the smallest integer value for  $n$  satisfying  $n > 61$  is 62.

The first negative term is the 62<sup>nd</sup> term.

10. The 7<sup>th</sup> and 21<sup>st</sup> term of an A.P., are 37 and 107 respectively. Find the A.P. and its 100<sup>th</sup> term.

Sol: Given  $a_7 = 37$  and  $a_{21} = 107$

$$\text{By using } a_n = a_1 + (n-1)d$$

$$a_7 = 37 \Rightarrow a_1 + 6d = 37 \quad (1)$$

$$a_{21} = 107 \Rightarrow a_1 + 20d = 107 \quad (2)$$

Subtract (1) from (2):

$$(a_1 + 20d) - (a_1 + 6d) = 107 - 37$$

$$14d = 70$$

$$d = \frac{70}{14} \Rightarrow d = 5$$

Substitute  $d = 5$  into (1):

$$a_1 + 6(5) = 37 \Rightarrow a_1 + 30 = 37$$

$$a_1 = 37 - 30 \Rightarrow a_1 = 7$$

The A.P. is  $a_1, a_1 + d, a_1 + 2d, \dots$

$$\text{A.P.} = 7, 7+5, 7+2(5), \dots$$

A.P is 7, 12, 17, .....

Now, find the 100<sup>th</sup> term ( $a_{100}$ ):

$$a_{100} = a + (100-1)d = 7 + (99)5$$

$$a_{100} = 7 + 495 = 502$$

The 100<sup>th</sup> term is 502.

11. If  $\frac{1}{a-c}, \frac{1}{b-c}, \frac{1}{b-a}$  are in A.P.,

The show that  $\frac{a-b}{a-c} = \frac{a-c}{b-a}$ .

Sol: Given  $\frac{1}{a-c}, \frac{1}{b-c}$  and  $\frac{1}{b-a}$  are in AP then

$$\begin{aligned} \frac{1}{b-c} - \frac{1}{a-c} &= \frac{1}{b-a} - \frac{1}{b-c} \\ \frac{a-c-(b-c)}{(b-c)(a-c)} &= \frac{b-c-(b-a)}{(b-a)(b-c)} \\ \frac{a-b}{(b-c)(a-c)} &= \frac{a-c}{(b-a)(b-c)} \end{aligned}$$

$$\frac{a-b}{a-c} = \frac{a-c}{b-a}$$

12. How many number of three digits are divisible by 7?

Sol: There digit numbers divisible by 7 between 100 and 999 are 105, 112, 119, ..., 994

$$a_1 = 105, d = 7, a_n = 994, n = ?$$

$$a_n = a_1 + (n-1)d$$

$$994 = 105 + (n-1)7 \Rightarrow 994 - 105 = (n-1)7$$

$$\Rightarrow (n-1) = \frac{889}{7} \Rightarrow n-1 = 127 \Rightarrow n = 128$$

Hence 128 three digits numbers are divisible by 7 between 100 and 999

13. Find the 8<sup>th</sup> term from the end of the A.P., 8, 11, 14, ..., 185

Sol': To find the 8<sup>th</sup> term from the end of the A.P., 8, 11, 14, ..., 185:

Here  $d = 11 - 8 = 3$ ,  $a_1 = 8, a_n = 185$  from start

$$185 = 8 + (n-1)3$$

$$177 = (n-1)3 \Rightarrow 59 = n-1$$

$$n = 60$$

Total terms are 60 so 8<sup>th</sup> term from the end is 53<sup>th</sup> term.

$$a_{53} = 8 + (53-1)3$$

$$a_{53} = 8 + (52)3$$

$$a_{53} = 8 + 156 = 164$$

Here 164 is eight term from the end.

14. Find the nth term of progression  $\left(\frac{3}{7}\right)^{10}, \left(\frac{10}{7}\right)^{10}, \left(\frac{17}{7}\right)^{10}, \dots$ . Is the progression an A.P.

Sol: Given progression:  $\left(\frac{3}{7}\right)^{10}, \left(\frac{10}{7}\right)^{10}, \left(\frac{17}{7}\right)^{10}, \dots$

Consider the bases:  $\frac{3}{7}, \frac{10}{7}, \frac{17}{7}, \dots$

$$\text{Here } a_1 = \frac{3}{7}, d = \frac{10}{7} - \frac{3}{7} = \frac{7}{7} = 1, n = n$$

$$a_n = a_1 + (n-1)d$$

$$a_n = \frac{3}{7} + (n-1)(1) = \frac{3}{7} + n - 1$$

$$= \frac{3 + 7n - 7}{7} = \frac{7n - 4}{7}$$

$$\text{Hence nth term} = \left(\frac{7n-4}{7}\right)^{10}$$

To check given progression is an A.P.

$$d (\text{Difference of 1st \& 2nd}) = \left(\frac{10}{7}\right)^{10} - \left(\frac{3}{7}\right)^{10} = \frac{10^{10} - 3^{10}}{7^{10}}$$

$$d (\text{Difference of 2nd \& 3rd}) = \left(\frac{17}{7}\right)^{10} - \left(\frac{10}{7}\right)^{10} = \frac{17^{10} - 10^{10}}{7^{10}}$$

$$\text{Clearly } \frac{10^{10} - 3^{10}}{7^{10}} \neq \frac{17^{10} - 10^{10}}{7^{10}}$$

Hence it is not an A.P.

15. If the arithmetic progressions 3, 10, 17, ... and 63, 65, 67, ... are such that their  $n$ th terms are equal, then find the value of  $n$ .

Sol: First A.P.: 3, 10, 17, ...

$$a_1 = 3, d_1 = 10 - 3 = 7, n = n$$

$$a_n = a_1 + (n-1)d = 3 + (n-1)7$$

Second A.P.: 63, 65, 67, ...

$$a_2 = 63, d_2 = 65 - 63 = 2, n = n$$

$$N\text{th term: } b_n = 63 + (n-1)2$$

Given  $a_n = b_n$  :

$$3 + (n-1)7 = 63 + (n-1)2$$

$$3 + 7n - 7 = 63 + 2n - 2$$

$$7n - 4 = 2n + 61$$

$$7n - 2n = 61 + 4 \Rightarrow 5n = 65 \Rightarrow n = 13$$

The value of  $n$  is 13.

16. If the  $p$ th term of an A.P. is  $q$  and the  $q$ th term is  $p$ , prove that its  $n$ th term is  $(p + q - n)$ .

Sol: Given an A.P. where:

$$a_p = q \Rightarrow a + (p-1)d = q \quad (1)$$

$$a_q = p \Rightarrow a + (q-1)d = p \quad (2)$$

Subtract (2) from (1)

$$(a + (p-1)d) - (a + (q-1)d) = q - p$$

$$pd - d - qd + d = q - p$$

$$d(p - q) = -(p - q)$$

$$d = -1 \text{ (assuming } p \neq q \text{)}$$

Substitute  $d = -1$  into (1):

$$a + (p-1)(-1) = q$$

$$a - p + 1 = q$$

$$a = p + q - 1$$

$$a_n = a + (n-1)d$$

$$a_n = (p + q - 1) + (n-1)(-1)$$

$$a_n = p + q - 1 - n + 1$$

$$a_n = p + q - n \text{ Hence proved.}$$

17. If  $\frac{1}{a}, \frac{1}{b}$  and  $\frac{1}{c}$  are in A.P., show that  $b = \frac{2ac}{a+c}$ .

Sol: Given  $\frac{1}{a}, \frac{1}{b}$  and  $\frac{1}{c}$  are in A.P.

$$\text{So, } \frac{1}{c} - \frac{1}{b} = \frac{1}{b} - \frac{1}{a}$$

$$\text{Or } \frac{1}{c} + \frac{1}{a} = \frac{1}{b} + \frac{1}{b}$$

$$\frac{a+c}{ac} = \frac{2}{b} \Rightarrow \frac{ac}{a+c} = \frac{b}{2}$$

$$\Rightarrow b = \frac{2ac}{a+c} \text{ Hence Proved.}$$

18. If  $\frac{1}{a}, \frac{1}{b}$  and  $\frac{1}{c}$  are in A.P., show that the common difference is  $\frac{a-c}{2ac}$ .

Sol: Given  $\frac{1}{a}, \frac{1}{b}$  and  $\frac{1}{c}$  are in A.P.

$$\text{So, } d = \frac{1}{c} - \frac{1}{b} \quad (3rd - 2nd) \quad (I)$$

$$d = \frac{1}{b} - \frac{1}{a} \quad (2nd - 1st) \quad (II)$$

$$I + II \Rightarrow 2d = \frac{1}{c} - \frac{1}{a}$$

$$2d = \frac{a-c}{ac} \Rightarrow d = \frac{a-c}{2ac} \text{ Hence Proved}$$

19. If  $a_k$  and  $a_m$  denotes two different terms of an A.P., show that its  $n$ th term is  $a_k + (n-k) \left( \frac{a_k - a_m}{k-m} \right)$ .

Sol: Given an A.P. with terms  $a_k$  and  $a_m$ .

We know the general form of the  $j$ th term:

$$a_j = a + (j-1)d$$

So, we have:

$$a_k = a + (k-1)d \quad (1)$$

$$a_m = a + (m-1)d \quad (2)$$

Subtracting (2) from (1)

$$a_k - a_m = [a + (k-1)d] - [a + (m-1)d]$$

$$a_k - a_m = a + kd - d - a - md + d$$

$$a_k - a_m = (k-m)d$$

Assuming  $k \neq m$ :

$$d = \frac{a_k - a_m}{k-m}$$

Now, consider the  $n$ th term,  $a_n$ . we can express  $a_n$  in relation to  $a_k$ :

$$a_n = a_k + (n-k)d$$

Substitute the expression for  $d$

$$a_n = a_k + (n-k) \left( \frac{a_k - a_m}{k-m} \right)$$

20. If  $a_1, a_2, a_3, \dots, a_n$  are positive and in A.P., prove that  $\frac{1}{\sqrt{a_1} + \sqrt{a_2}} + \frac{1}{\sqrt{a_2} + \sqrt{a_3}} + \dots + \frac{1}{\sqrt{a_{n-1}} + \sqrt{a_n}} = \frac{n-1}{\sqrt{a_1} + \sqrt{a_n}}$ .

Sol: Let  $d$  be the common difference of the A.P.

$$\frac{1}{\sqrt{a_i} + \sqrt{a_i + 1}} = \frac{1}{\sqrt{a_i} + \sqrt{a_i + 1}} \cdot \frac{\sqrt{a_i} - \sqrt{a_i + 1}}{\sqrt{a_i} - \sqrt{a_i + 1}} = \frac{\sqrt{a_i} - \sqrt{a_i + 1}}{(a_i) - (a_i + 1)}$$

Since  $a_i + 1 = a_i + d$ , then  $a_i - a_i - 1 = -d$

$$\text{So, } \frac{\sqrt{a_i} - \sqrt{a_i + 1}}{-d} = \frac{\sqrt{a_i - 1} - \sqrt{a_i}}{d}$$

Sum of L.H.S:

$$\sum_{i=1}^{n-1} \frac{\sqrt{a_i + 1} - \sqrt{a_i}}{d} = \frac{1}{d} [(\sqrt{a_2} - \sqrt{a_1}) + (\sqrt{a_3} - \sqrt{a_2}) + \dots + (\sqrt{a_n} - \sqrt{a_{n-1}})]$$

$$= \frac{1}{d}(\sqrt{a_n} - \sqrt{a_1}) \text{ -----(I)}$$

$$= \text{RHS: } \frac{n-1}{\sqrt{a_1} + \sqrt{a_n}} = \frac{n-1}{\sqrt{a_1} + \sqrt{a_n}} \cdot \frac{\sqrt{a_n} - \sqrt{a_1}}{\sqrt{a_n} - \sqrt{a_1}} = \frac{(n-1)(\sqrt{a_n} - \sqrt{a_1})}{a_n - a_1}$$

$$a_n = a_1 + (n-1)d,$$

$$a_n - a_1 = (n-1)d \Rightarrow \frac{a_n - a_1}{n-1} = d,$$

$$= \frac{\cancel{(n-1)}(\sqrt{a_n} - \sqrt{a_1})}{\cancel{(n-1)}d} = \frac{\sqrt{a_n} - \sqrt{a_1}}{d} \text{ -----(II)}$$

From I & II L.H.S = R.H.S

21. If the roots of the equation  $(b-c)x^2 + (c-a)x + (a-b) = 0$  are equal. Show that  $a, b, c$  are in A.P.

Sol: Given quadratic equation:  $(b-c)x^2 + (c-a)x + (a-b) = 0$

For equal roots, the discriminant must be zero.

$$\Delta = B^2 - 4AC = 0$$

$$\text{Here, } A = (b-c), B = (c-a), C = (a-b).$$

$$(c-a)^2 - 4(b-c)(a-b) = 0$$

$$c^2 - 2ac + a^2 - 4(ab - b^2 - ac + bc) = 0$$

$$c^2 - 2ac + a^2 - 4ab + 4b^2 + 4ac - 4bc = 0$$

$$c^2 - 2ac + a^2 - 4ab + 2ac - 4bc = 0$$

$$a^2 + 4b^2 + c^2 - 4ab + 2ac - 4bc = 0$$

Rearrange to form a perfect square:

$$a^2 + (-2b)^2 + c^2 + 2(a)(-2b) + 2(a)(c) + 2(-2b)(c) = 0$$

$$\text{So, } (a - 2b + c)^2 = 0$$

Taking the square root of both sides:

$$a - 2b + c = 0$$

$$a + c = 2b$$

$$\Rightarrow a + c = b + b \Rightarrow c - b = b - a$$

Hence  $a, b, c$  are in A.P.

22. If the sides of a right angled triangle are in A.P., find the ratio of its sides.

Sol: Let sides be  $x-d, x, x+d$

Hypotenuse:  $x+d$

$$(x-d)^2 + x^2 = (x+d)^2$$

$$x^2 - 2xd + d^2 + x^2 = x^2 + 2xd + d^2$$

$$2x^2 - 2xd + d^2 = x^2 + 2xd + d^2$$

$$x^2 - 4xd = 0$$

$$x(x - 4d) = 0$$

Since  $x \neq 0$  (side length):

$$x - 4d = 0 \Rightarrow x = 4d$$

Sides are:

$$x - d = 4d - d = 3d$$

$$x = 4d$$

$$x + d = 4d + d = 5d$$

Ratio of sides  $3d : 4d : 5d = 3 : 4 : 5$

23. If the  $n$ th term of a progression is a linear expression in  $n$ , then prove that this progression is an A.P.

Sol: Let the  $n$ th term be  $T_n$

Given that  $T_n = An + B$  for constant  $A, B$

$$T_{n+1} = A(n+1) + B = An + A + B.$$

$$T_{n+1} - T_n = (An + A + B) - (An + B) = An + A + B - An - B = A.$$

Also  $T_{n-1} = A(n-1) + B = A_n - A + B$

$$T_n - T_{n-1} = A_n + B - A_n + A - B = A$$

$$T_{n+1} - T_n = T_n - T_{n-1} \quad \text{Hence progression is an A.P.}$$

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