Exercise 6.3

Q1: Find A.M between the given numbers:

i. $2+\sqrt{3}i, 2-\sqrt{3}i$

Sol: We use formula $A.M=\frac{x_1+x_2}{2}$

Substituting the given values

$$A.M=\frac{(2+\sqrt{3}i)+(2-\sqrt{3}i)}{2}$$

$$A.M=\frac{2+\sqrt{3}i+2-\sqrt{3}i}{2}=\frac{4}{2}=2$$

Thus the *A.M* between given complex number is 2

ii. $(a+b)^2, (a-b)^2$

Sol: We use formula $A.M=\frac{x_1+x_2}{2}$

Substituting the given values

$$A.M=\frac{(a+b)^2+(a-b)^2}{2}=\frac{a^2+2ab+b^2+a^2-2ab+b^2}{2}$$

$$=\frac{2a^2+2b^2}{2}=\frac{2(a^2+b^2)}{2}$$

A.M. $=a^2+b^2$

So, A.M between $(a+b)^2$ and $(a-b)^2$ is a^2+b^2

Q2: If 6, 11, 16 are three A.Ms. between *a* and *b*, find *a* and *b*.

Sol: Given 6, 11, 16 are A.M. in between a and b

Then $a, 6, 11, 16, b$ are in A.P.

$b-16=16-11$ and $11-6=6-a$

$b-16=5$ and $5=6-a \Rightarrow a=6-5=1$

$\boxed{b=21}$ $\boxed{a=1}$

Q3: Insert five A. Ms. between $\sqrt{2}$ and $\frac{15}{\sqrt{2}}$.

Sol: Suppose A_1, A_2, A_3, A_4, A_5 are five A.Ms between $\sqrt{2}$ and $\frac{15}{\sqrt{2}}$

Then $\sqrt{2}, A_1, A_2, A_3 A_4, A_5, \frac{15}{\sqrt{2}}$ are in A.P.

$\boxed{a=\sqrt{2}}$, $a_7=a+6d=\frac{15}{\sqrt{2}} \Rightarrow \sqrt{2}+6d=\frac{15}{\sqrt{2}}$

$6d=\frac{15}{\sqrt{2}}-\sqrt{2}=\frac{15-2}{\sqrt{2}}=\frac{13}{\sqrt{2}} \Rightarrow d=\frac{13}{6\sqrt{2}}$

$$d=\frac{13}{6\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}=\frac{13\sqrt{2}}{6\times 2}\Rightarrow \boxed{d=\frac{13\sqrt{2}}{12}}$$

Five inserted terms

$$A_1=a_2=a+d=\sqrt{2}+\frac{13\sqrt{2}}{12}=\frac{25\sqrt{2}}{12}$$

$$A_2=a_3=a+2d=\sqrt{2}+2.\frac{13\sqrt{2}}{12}=\frac{38\sqrt{2}}{12}=\frac{19\sqrt{2}}{6}$$

$$A_3=a_4=a+3d=\sqrt{2}+3.\frac{13\sqrt{2}}{12}=\frac{51\sqrt{2}}{12}=\frac{17\sqrt{2}}{4}$$

$$A_4=a_5=a+4d=\sqrt{2}+4.\frac{13\sqrt{2}}{12}=\frac{64\sqrt{2}}{12}=\frac{16\sqrt{2}}{3}$$

$$A_5=a_6=a+5d=\sqrt{2}+5.\frac{13\sqrt{2}}{12}=\frac{77\sqrt{2}}{12}$$

$$A_1=\frac{25\sqrt{2}}{12}\quad,\quad A_2=\frac{19\sqrt{2}}{6}\quad,\quad A_3=\frac{17\sqrt{2}}{4}$$

$$A_4=\frac{16\sqrt{2}}{3}\quad,\quad A_5=\frac{77\sqrt{2}}{12}$$

Q4: The A.M of two numbers is 7 and their product is 45. Find the numbers.

Sol: $A.M=\frac{x+y}{2}=7$

$x+y=14\Rightarrow y=14-x$ ------------**(I)**

Their product is given by

$x\cdot y=45$ ---------**(II)**

Put I in II

$x(14-x)=45\Rightarrow 14x-x^2=45\Rightarrow x^2-14x+45=0$

$x^2-5x-9x+45=0\Rightarrow x(x-5)-9(x-5)=0$

$(x-5)(x-9)=0\Rightarrow x-5=0$

or $x-9=0\Rightarrow x=5$ or $x=9$

When $x=5$ then $y=14-5=9$

When $x=9$ then $y=14-9=5$

So, required numbers are 9 and 5.

Q5: If n arithmetic means are inserted between a and b, prove that $d=\frac{b-a}{n+1}$, where d is common difference.

Sol: Let $A_1, A_2, A_3, \dots, A_n$ are n A.M between a and b then

a, $A_1, A_2, A_3, \dots, A_n$, b are in A.P.

Here $a_1=a$ and $a_{n+2}=b$

We know that formula: $a_n=a_1+(n-1)d$

Put $n=n+2$

$a_{n+2}=a+(n+2-1)d$

$\Rightarrow d=\frac{b-a}{n+1}$ Hence proved

Q6: If A is the A.M between a and b, prove that $(a-A)^2+(A-b)^2=\frac{1}{2}(a-b)^2$

Sol: $A=\frac{a+b}{2}$

Now let's compute $(a-A)^2+(A-b)^2$

$a-A=a-\frac{a+b}{2}$ and $A-b=\frac{a+b}{2}-b$

$a-A=\frac{2a-a-b}{2}$ and $A-b=\frac{a+b-2b}{2}$

$a-A=\frac{a-b}{2}$ and $A-b=\frac{a-b}{2}$

$$(a-A)^2+(A-b)^2=\left(\frac{a-b}{2}\right)^2+\left(\frac{a-b}{2}\right)^2$$

$$=\frac{(a-b)^2}{4}+\frac{(a-b)^2}{4}=\frac{2(a-b)^2}{4}=\frac{(a-b)^2}{2}$$

Hence prove that $(a-A)^2+(A-b)^2=\frac{1}{2}(a-b)^2$

Q7: For what ... , a^n+b^n is the A.M. between a and b where $a\neq b$

Sol: If $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$ is A.M. in between a & b

then $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}=\frac{a+b}{2}$

$a^{n+1}+b^{n+1}=\frac{a+b}{2}(a^n+b^n)$

$2(a^{n+1}+b^{n+1})=(a+b)(a^n+b^n)$

$2a^{n+1}+2b^{n+1}=a^{n+1}+a^nb+ab^n+b^{n+1}$

$2a^{n+1}-a^{n+1}+2b^{n+1}-b^{n+1}=a^nb+ab^n$

$a^{n+1}+b^{n+1}=a^nb+ab^n\Rightarrow a^{n+1}-a^nb=ab^n-b^{n+1}$

$a^n(a-b)=b^n(a-b)$

$\frac{a^n}{b^n}=\frac{a-b}{a-b}=1\Rightarrow\left(\frac{a}{b}\right)^n=\left(\frac{a}{b}\right)^0$

$\boxed{\Rightarrow n=0}$