Q1: **Find the 6th term of the G.P: $-6,-3,\frac{-3}{2},\ldots$**

Sol: Here Given G.P. is $-6,-3,-3/2$

$a_1=-6$ and $r=\frac{-3}{-6}=\frac{1}{2}$, $n=6$

$a_n=a_1r^{n-1}$

$a_6=-6\cdot\left(\frac{1}{2}\right)^{6-1}=-6\cdot\left(\frac{1}{2}\right)^5=-6\cdot\frac{1}{32}=-\frac{6}{32}=-\frac{3}{16}$

Q2: **Find the 8th term of the sequence, $3,3^2,3^3,\ldots$**

Sol: Given G.P. is $3,3^2,3^3,\ldots\ldots$

Here $a_1=3$ and $r=\frac{3^2}{3}=3, n=8$

$a_n=a_1r^{n-1}$

$a_8=3\cdot3^{8-1}=3\cdot3^7=3^8=6561$

Q3: **The n^{th} term of the sequence, $1,2,4,8,\ldots$ and $256,128,64,\ldots$ are equal. Find the value of n.**

Sol: For sequence 1: $a_1, r=2, n=n$ so

$a_n=a_1r^{n-1}=(1)2^{n-1}$

For sequence 2: $b_1=256, r=\frac{1}{2}, n=n$ so

$b_n=b_1r^{n-1}=(256)\left(\frac{1}{2}\right)^{n-1}$

Given $a_n=b_n \Rightarrow 2^{n-1}=256\cdot\left(\frac{1}{2}\right)^{n-1}$

$2^{n-1}=2^8\cdot\left(2^{-1}\right)^{n-1} \Rightarrow 2^{n-1}=2^8\cdot2^{-1(n-1)}$

$2^{n-1}=2^{8-n+1} \Rightarrow 2^{n-1}=2^{9-n}$

$n-1=9-n \Rightarrow 2n=10$

$\Rightarrow n=5$

Q4: **Find the first five terms of each sequence described:**

i. $a_1=243, r=\frac{1}{3}$

Sol: $a_1=243$

$a_2=a_1.r=243\cdot\frac{1}{3}=81$

$a_3=a_2r=81\cdot\frac{1}{3}=27$

$a_4=a_3r=27\cdot\frac{1}{3}=9$

$a_5=a_4r=9\cdot\frac{1}{3}=3$

ii. $a_1=579, r=-\frac{1}{2}$

Sol: $a_1=579$

$a_2=a_1r=579\cdot\left(-\frac{1}{2}\right)=-\frac{579}{2}$

$a_3=a_2r=\left(-\frac{579}{2}\right)\cdot\left(-\frac{1}{2}\right)=\frac{579}{4}$

$a_4=a_3r=\frac{579}{4}\cdot\left(-\frac{1}{2}\right)=\frac{-579}{8}$

$a_5=a_4r=\left(-\frac{579}{8}\right)\cdot\left(-\frac{1}{2}\right)=\frac{579}{16}$

Q5: **Find the 12th term of the sequence, $1+i, 2i, -2+2i, \ldots$**

Sol: Here $1+i, 2i, -2+2i, \ldots a_{12}=?$

$a_1=1+i, r=\frac{2i}{1+i}, n=12$

$a_n=a_1r^{n-1}$

$a_{12}=(1+i)\frac{(2i)^{11}}{(1+i)^{11}}=\frac{2^{11}\times i^{11}}{(1+i)^{10}}=\frac{2048\times i\times\left(i^2\right)^5}{\left[(1+i)^2\right]^5}$

$a_{12}=\frac{2048\times i\times(-1)^5}{(2i)^5}=\frac{-2048i}{32i^5}=\frac{-64i}{i^2\cdot i^2\cdot i}=\frac{-64i}{(-1)(-1)i}=-64$

Q6: **If the 4^{th} and 9^{th} terms of a G.P. are 54 and 13122 respectively. Find the G.P. Also find its general term.**

Sol: Given $a_4 = 54$ and $a_9 = 13122$

The formula for the n^{th} term of a G.P. is

$$a_n = a_1 \cdot r^{n-1}$$

From the given information, we can set up two equations:

$$a_4 = a_1 \cdot r^{4-1} = 54$$

$$\Rightarrow \quad a_4 = a_1 \cdot r^3 = 54 ------(I)$$

$$a_9 = a_1 \cdot r^{9-1} = 13122$$

$$\Rightarrow \quad a_9 = a_1 \cdot r^8 = 13122 -------(II)$$

Divide the second equation by the first equation:

$$\frac{a_1 \cdot r^8}{a_1 \cdot r^3} = \frac{13122}{54} \Rightarrow r^5 = 243 \Rightarrow r^5 = (3)^5 \Rightarrow r = 3$$

Substitute the value of $r = 3$ in I:

$$a_1 \cdot (3)^3 = 54 \Rightarrow a_1 \cdot 27 = 54$$

$$a_1 = \frac{54}{27} \Rightarrow a_1 = 2$$

The G.P. starts with $a_1 = 2$ and has a common ratio $r = 3$.

The terms of the G.P. are: $2, 2\cdot3, 2\cdot3^2, 2\cdot3^3, \ldots$

The G.P. is $2, 6, 18, 54, \ldots$

The general term of the G.P. is $a_n = a_1 \cdot r^{n-1}$.

Substituting the values of a_1 and r: $a_n = 2\cdot3^{n-1}$

Q7: **If *a, b, c, d* are in G.P, prove that**

i. **$a-b, b-c, c-d$ are in G.P**

Sol: Given *a, b, c, d* are in G.P then

$$\frac{b}{a} = \frac{c}{b} = \frac{d}{c} \text{ or } \frac{b}{a} = \frac{c}{b}, \frac{c}{b} = \frac{d}{c}, \frac{d}{c} = \frac{b}{a}$$

$$\Rightarrow b^2 = ac \;\&\; c^2 = bd \;\&\; bc = ad \longrightarrow I$$

Now if $a-b, b-c, c-d$ are in G.P then

$$\frac{c-d}{b-c} = \frac{b-c}{a-b}$$

$$\Rightarrow (a-b)(c-d) = (b-c)(b-c) = (b-c)^2$$

L.H.S $= (a-b)(c-d) = ac - ad - bc + bd$

$$= b^2 - bc - bc + c^2 \,(\text{use} I)$$

$$= b^2 - 2bc + c^2 = (b-c)^2$$

So, L.H.S = R.H.S

Hence $(a-b), (b-c), (c-d)$ are in G.P

ii. **$a^2-b^2, b^2-c^2, c^2-d^2$ are in G.P**

Sol: If $a^2-b^2, b^2-c^2, c^2-d^2$ are in G.P

Then $\frac{c^2-d^2}{b^2-c^2} = \frac{b^2-c^2}{a^2-b^2}$

$$\Rightarrow (b^2-c^2)^2 = (a^2-b^2)(c^2-d^2)$$

R.H.S $= (a^2-b^2)(c^2-d^2) = a^2c^2 - a^2d^2 - b^2c^2 + b^2d^2$

$$= (ac)^2 - (ad)^2 - b^2c^2 + (bd)^2$$

$$= (b^2)^2 - (bc)^2 - b^2c^2 + (c^2)^2 \text{ Use I}$$

$$= (b^2)^2 - b^2c^2 - b^2c^2 + (c^2)^2$$

$$= (b^2)^2 - 2b^2c^2 + (c^2)^2$$

$$= (b^2-c^2)^2 = \text{L.H.S}$$

Hence Proved.

iii. **$a^2+b^2, b^2+c^2, c^2+d^2$ are in G.P**

Sol: If $a^2+b^2, b^2+c^2, c^2+d^2$ are in G.P

Then $\frac{c^2+d^2}{b^2+c^2} = \frac{b^2+c^2}{a^2+b^2}$

$$\Rightarrow (b^2+c^2)^2 = (a^2+b^2)(c^2+d^2)$$

R.H.S

$$= (a^2+b^2)(c^2+d^2) = a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2$$

$$= (ac)^2 + (ad)^2 + b^2c^2 + (bd)^2$$

$$= (b^2)^2 + (bc)^2 + b^2c^2 + (c^2)^2 \text{ Use } I$$

$$= (b^2)^2 + b^2c^2 + b^2c^2 + (c^2)^2$$

$$= (b^2)^2 + 2b^2c^2 + (c^2)^2$$

$$= (b^2+c^2)^2 = \text{L.H.S}$$

Hence,

$a^2+b^2, b^2+c^2, c^2+d^2$ are in G.P

Q8: **If $(p+q)^{th}$ term of a G.P. is *m* and $(p-q)^{th}$ term is *n*, then find the p^{th} term.**

Sol:

$$a_{p+q} = ar^{(p+q)-1} = m$$

$$a_{p-q} = ar^{(p-q)-1} = n$$

$$\left(ar^{(p+q)-1}\right)\cdot\left(ar^{(p-q)-1}\right) = mn$$

$$a^2r^{(p+q)-1+(p-q)-1} = mn$$

$$a^2r^{2p-2} = mn$$

$$a^2r^{2(p-1)} = mn$$

$$\left(ar^{p-1}\right)^2 = mn$$

$a_p = ar^{p-1} = \sqrt{mn}$ Hence Pth term a_p is $\sqrt{mn}$

Q9: **Find three, consecutive numbers in G.P whose sum is 26 and their product is 216 .**

Sol: Suppose three numbers in G.P. are $\frac{a_1}{r}, a_1, a_1r$

Condition II $\Rightarrow \left(\frac{a_1}{r}\right)(a_1)(a_1r)=216$

$a_1^3=216 \Rightarrow (6)^3 \Rightarrow a_1=6$

Condition I $\Rightarrow \frac{a_1}{r}+a_1+a_1r=26$

$a_1\left(\frac{1}{r}+1+r\right)=26$

$a_1\left(\frac{1+r+r^2}{r}\right)=26 \Rightarrow 6(r^2+r+1)=26r$

$\Rightarrow 6r^2+6r+6-26r=0 \Rightarrow 6r^2-20r+6=0$

$'\div' by 2 \Rightarrow 3r^2-10r+3=0$

$\Rightarrow 3r^2-9r-r+3=0 \Rightarrow 3r(r-3)-1(r-3)=0$

$(r-3)(3r-1)=0$

$r-3=0 \text{ or } 3r-1=0$

$r=3 \text{ or } r=\frac{1}{3}$

When $r=\frac{1}{3} \ \& \ a_1=6$

$\frac{a}{r}=\frac{6}{1/3} 6\times3=18$

$a_1=6 \text{ and } a_1r=6\left(\frac{1}{3}\right)=2$

When $a_1=6, r=3$

$\frac{a_1}{r}=\frac{6}{3}=2$

$a_1=6$

$a_1r=(6)(3)=18$

Hence numbers are 18, 6, 2 or 2, 6, 18

Q10: **The 3rd term of a G.P. Is the square of 1st term. If the 2nd term is 9 then find the 6th term.**

Sol: Given $a_3=a_1^2$ and $a_2=9$, $a_6=?$

$a_1r^2=a_1^2$ and $a_1r=9$

$\Rightarrow a_1=r^2 \text{ and } (r^2)(r)=9 \Rightarrow r^3=9 (put\ a_1=r^2)$

$a_6=a_1r^5=(r^2)r^5=r^7$

$r^7=r^3\cdot r^3\cdot r=9\cdot9\cdot r=81r \Rightarrow r^7=81.r$

$\Rightarrow r^6.r=81.r$

$\Rightarrow r^6=(9)^2 \Rightarrow r=9^{2\times\frac{1}{6}}=9^{\frac{1}{3}}$ also $a_1=r^2=9^{2/3}$

Now $a_6=a_1r^5=9^{2/3}(9^{1/3})^5=9^{2/3}.9^{5/3}=9^{\frac{2}{3}+\frac{5}{3}}$

$=9^{\frac{7}{3}}=9^{2+\frac{1}{3}}=9^2.9^{\frac{1}{3}}=81.\sqrt[3]{9}$

Q11: **If $\frac{1}{a},\frac{1}{b}$ and $\frac{1}{c}$ are in G.P.**

Show that the common ration is $\pm\sqrt{\frac{a}{c}}$.

Sol: If $\frac{1}{a},\frac{1}{b},\frac{1}{c}$ are in G.P

Then $r=\frac{Second}{First}=\frac{\frac{1}{b}}{\frac{1}{a}}=\frac{1}{b}\times\frac{a}{1}=\frac{a}{b} \longrightarrow I$

$r=\frac{Third}{Second}=\frac{\frac{1}{c}}{\frac{1}{b}}=\frac{1}{c}\times\frac{b}{1}=\frac{b}{c} \longrightarrow II$

$I\times II$

$r^2=\frac{a}{b}\times\frac{b}{c}=\frac{a}{c} \Rightarrow \pm\sqrt{\frac{a}{c}}$

Q12: **If the numbers 1, 4 and 3 are subtracted from three consecutive terms of an A.P., the resulting numbers are in G.P. Find the original numbers if their sum is 21.**

Sol: Suppose three now in A.P are a_1-d, a_1, a_1+d

Condition II $a_1-d+a_1+a_1+d=21$

$3a=21 \Rightarrow a_1=\frac{21}{3}=7 \Rightarrow a_1=7$

Condition I $\Rightarrow a_1-d-1, a_1-4, a_1+d-3$ are in G.P

$7-d-1, 7-4, 7+d-3$ are in G.P

$6-d, 3, 4+d$ are in G.P

$\Rightarrow \frac{4+d}{3}=\frac{3}{6-d} \Rightarrow (4+d)(6-d)=9$

$\Rightarrow 24-4d+6d-d^2=9$

$\Rightarrow 24+2d-d^2-9=0 \Rightarrow -d^2+2d+15=0$

$\Rightarrow d^2-2d-15=0 \quad ('\times' by -1)$

$\Rightarrow d^2-5d+3d-15=0$

$d(d-5)+3(d-5)=0$

$(d-5)(d+3)=0$

$d-5=0 \text{ or } d+3=0$

$d=5 \text{ or } d=-3$

When $d=5 \ \& \ a_1=7$

$a_1-d=7-5=2=7$

$a_1+d=7+5=12$

When $d=-3$ and $a_1=7$

$a_1-d=7-(-3)=7+3=10=7$

$a_1+d=7+(-3)=7-3=4$

Required numbers are $4,7,10$ or $2,7,12$

Q13: **If three consecutive numbers in A.P. are increased by $1,4,15$ respectively, the resulting numbers are in G.P. Find the original numbers if their sum is 6.**

Sol: Let the A.P. terms be a_1-d,a_1,a_1+d

$(a_1-d)+a_1+(a_1+d)=6 \Rightarrow 3a_1=6 \Rightarrow a_1=2.$

A.P. terms are $2-d,2,2+d$

Now $(2-d)+1=3-d$, $2+4=6$

$(2+d)+15=17+d$

Are in G.P.

$\frac{17+d}{6}=\frac{6}{3-d} \Rightarrow 6^2=(3-d)(17+d)$

$36=51+3d-17d-d^2$

$36=51-14d-d^2$

$d^2+14d+36-51=0$

$d^2+14d-15=0$

Factor the quadratic equation:

$(d+15)(d-1)=0$

$d=1$ or $d=-15$

Case 1: If $d=1$

Original numbers: $2-1,2,2+1 \quad \Rightarrow \quad$ **1,2,3.**

Case 2: If $d=-15$

Original numbers: $2-(-15),2,2+(-15)$

$\Rightarrow$ **17,2,13.**

Q14: **If p^{th},q^{th} terms of a G.P. are q and p respectively, show that $(p+q)^{th}$ term is $\left(q^p \div p^q\right)^{\frac{1}{p-q}}$.**

Sol: *Given:*

$a_p=q \quad \Rightarrow \quad AR^{p-1}=q \longrightarrow$ (i)

$a_q=p \quad \Rightarrow \quad AR^{q-1}=p \longrightarrow$ (ii)

Divide (i) by (ii):

$\frac{AR^{p-1}}{AR^{q-1}}=\frac{q}{p}$

$R^{(p-1)-(q-1)}=\frac{q}{p}$

$R^{p-q}=\frac{q}{p}$

Raise both sides to the power of $\frac{1}{p-q}$:

$R=\left(\frac{q}{p}\right)^{\frac{1}{p-q}}$

Now, express A in terms of R from (i):

$A=\frac{q}{R^{p-1}}$ (From i)

Substitute the value of R:

$A=\frac{q}{\left(\left(\frac{q}{p}\right)^{\frac{1}{p-q}}\right)^{p-1}}=q\cdot\left(\frac{q}{p}\right)^{-\frac{p-1}{p-q}}$

$A=q\cdot\left(\frac{p}{q}\right)^{\frac{p-1}{p-q}}$

$A=q\cdot\frac{p^{\frac{p-1}{p-q}}}{q^{\frac{p-1}{p-q}}}$

$A=q^{1-\frac{p-1}{p-q}}\cdot p^{\frac{p-1}{p-q}}$

$A=q^{\frac{p-q-(p-1)}{p-q}}\cdot p^{\frac{p-1}{p-q}}$

$A=q^{\frac{p-q-p+1}{p-q}}\cdot p^{\frac{p-1}{p-q}}$

$A=q^{\frac{1-q}{p-q}}\cdot p^{\frac{p-1}{p-q}}$

Now find the $(p+q)th$ term, $a_{p+q}=AR^{(p+q)-1}$:

$a_{p+q}=\left(q^{\frac{1-q}{p-q}}\cdot p^{\frac{p-1}{p-q}}\right)\cdot\left(\left(\frac{q}{p}\right)^{\frac{1}{p-q}}\right)^{(p+q)-1}$

$a_{p+q}=q^{\frac{1-q}{p-q}}p^{\frac{p-1}{p-q}}\cdot\left(\frac{q}{p}\right)^{\frac{p+q-1}{p-q}}=q^{\frac{1-q}{p-q}}p^{\frac{p-1}{p-q}}\cdot\frac{q^{\frac{p+q-1}{p-q}}}{p^{\frac{p+q-1}{p-q}}}$

$a_{p+q}=q^{\frac{1-q+p+q-1}{p-q}}\cdot p^{\frac{p-1-(p+q-1)}{p-q}}$

$a_{p+q}=q^{\frac{p}{p-q}}\cdot p^{\frac{p-1-p-q+1}{p-q}}=q^{\frac{p}{p-q}}\cdot p^{\frac{-q}{p-q}}$

$a_{p+q}=\frac{q^{\frac{p}{p-q}}}{p^{\frac{q}{p-q}}}=\left(\frac{q^p}{p^q}\right)^{\frac{1}{p-q}}$

Q15: **If $a, 2a+2, 3a+3, \ldots$ are in G.P., then find the fifth term.**

Sol: $a, 2a+2, 3a+3$ are in G.P.

Then $\frac{3a+3}{2a+2} = \frac{2a+2}{a} \Rightarrow (2a+2)^2 = (3a+3)a$

$(2(a+1))^2 = 3(a+1).a$

$\frac{4(a+1)^{2}}{(a+1)} = 3a \Rightarrow 4a+4 = 3a \Rightarrow a = -4$

If $a = -4$:

$a_1 = -4$

$a_2 = 2(-4)+2 = -6$

$a_3 = 3(-4)+3 = -9$

G.P. is $-4, -6, -9, \ldots$

Common ratio $r = \frac{-6}{-4} = \frac{3}{2}$.

Fifth term $a_5 = a_1 r^4$:

$a_5 = -4 \cdot \left(\frac{3}{2}\right)^4$

$a_5 = -4 \cdot \frac{81}{16}$

$\Rightarrow a_5 = -\frac{81}{4}$