Exercise 6.7

Q.1. **Find the sum of first 15 terms of the G.P., $1, \frac{1}{3}, \frac{1}{9}, \ldots$**

Sol: $1, \frac{1}{3}, \frac{1}{9}, \ldots$

$$S_n = 1 + \frac{1}{3} + \frac{1}{9} + \ldots + a_{15}$$

$$a_1 = 1, r = \frac{\frac{1}{3}}{1} = \frac{1}{3} < 1, n = 15$$

$$S_n = \frac{a_1(1-r^n)}{1-r} = \frac{\left(1-\left(\frac{1}{3}\right)^{15}\right)}{1-\frac{1}{3}} = \frac{1-\frac{1}{14348907}}{\frac{2}{3}} = \frac{14348907-1}{14348907} \times \frac{3}{2}$$

$$\frac{14348906}{14348907} \times \frac{3}{2} = \frac{7172453}{4782969}$$

Q.2. **The 3rd term of a G.P. is 16 and the 6th term is -128. Find the first term and the sum of the first seven terms.**

Sol: Given that

$a_3 = 16 \quad \Rightarrow \quad ar^2 = 16 \quad (i)$

$a_6 = -128 \quad \Rightarrow \quad ar^5 = -128 \quad (ii)$

Divide (ii) by (i)

$$\frac{ar^5}{ar^2}=\frac{-128}{16}$$

$r^3=-8\Rightarrow r^3=(-2)^3\Rightarrow r=-2$

Put $r=-2$ in (i)

$a(-2)^2=16\Rightarrow 4a=16\Rightarrow a=4$

To find the sum of first 7 terms

$$S_7=\frac{a(1-r^7)}{1-r}=\frac{4(1-(-2)^7)}{1-(-2)}=\frac{4(1+128)}{3}=\frac{4(129)}{3}=\frac{516}{3}=172$$

The sum of first 7 terms is 172.

Q.3. Sum to n terms the series:

i) $\mathbf{0.2+0.22+0.222+\dots}$

Sol: $S_n=.2+.22+.222+\dots+n\ terms$

$$=2(.1+.11+.111+\dots+n\ terms)$$

$$=\frac{2}{9}\left[\left(1-\frac{1}{10}\right)+\left(1-\frac{1}{100}\right)+\left(1-\frac{1}{1000}\right)+\dots to\ n\ terms\right]$$

$$=\frac{2}{9}\left[(1+1+1+\dots to\ n\ terms)-\left(\frac{1}{10}+\frac{1}{100}+\frac{1}{1000}+\dots\dots n\ term\right)\right]$$

$$=\frac{2}{9}\left[n-\left(\frac{1}{10}+\frac{1}{100}+\frac{1}{1000}+\dots\dots n\ term\right)\right]$$

$$a_1=\frac{1}{10},r=\frac{\frac{1}{100}}{\frac{1}{10}}=\frac{1}{100}\times\frac{10}{1}=\frac{1}{10}<1,n=n$$

$$=\frac{2}{9}\left[n-\frac{a_1(1-r^n)}{1-r}\right]=\frac{2}{9}\left[n-\frac{\frac{1}{10}\left(1-\left(\frac{1}{10}\right)^n\right)}{1-\frac{1}{10}}\right]=\frac{2}{9}\left[n-\frac{\frac{1}{10}\left(1-\frac{1}{10^n}\right)}{\frac{9}{10}}\right]=\frac{2}{9}\left[n-\frac{10}{9}\times\frac{1}{10}\left[1-\frac{1}{10^n}\right]\right]$$

$$=\frac{2}{9}\left[n-\frac{1}{9}\left(1-\frac{1}{10^n}\right)\right]=\frac{2}{9}\left[n-\frac{1}{9}\left(\frac{10^n-1}{10^n}\right)\right]$$

ii) $\mathbf{3+33+333+\dots}$

Sol: $3+33+333+\dots+to\ n\ terms$

$3(1+11+111+\dots+to\ n\ terms)$

$$=\frac{3}{9}(9+99+999+\dots+to\ n\ terms)=\frac{1}{3}[((10-1)+(100-1)+(1000-1)+\dots+to\ n\ terms)]$$

$$=\frac{1}{3}[(10+100+1000+\dots.+n\ terms)-(1+1+1+\dots+n\ terms)]$$

$$=\frac{1}{3}[10+100+1000+\dots.+n\ terms-n]$$

$$a=10,r=\frac{100}{10}=10>1,n=n$$

$$=\frac{1}{3}\left[10\left(\frac{10^n-1}{10-1}\right)-n\right]=\frac{1}{3}\left[\frac{10}{9}(10^n-1)-n\right]$$

Q.4. Sum to n terms the series:

i. $1+(a+b)+(a^2+ab+b^2)+(a^3+a^2b+ab^2+b^3)+\ldots$

Sol: $1+(a+b)+(a^2+ab+b^2)+(a^3+a^2b+ab^2+b^3)+\ldots to\ n\ terms$

$'\times'\&'\div' by\ (a-b)$

$$=\frac{1}{(a-b)}\left[(a-b)+(a^2-b^2)+(a^3-b^3)+\ldots n\ terms\right]$$

$$=\frac{1}{(a-b)}\left[a+a^2+a^3+\ldots n\ terms-(b+b^2+b^3+\ldots.n\ terms)\right]$$

$$=\frac{1}{(a-b)}\left[\frac{a(a^n-1)}{a-1}-\frac{b(b^n-1)}{b-1}\right]=\frac{1}{(a-b)}\left[\frac{a(b-1)(a^n-1)-b(a-1)(b^n-1)}{(a-1)(b-1)}\right]$$

$$=\left[\frac{a(b-1)(a^n-1)-b(a-1)(b^n-1)}{(a-b)(a-1)(b-1)}\right]$$

ii. $r+(1+k)r^2+(1+k+k^2)r^3+\ldots$

Sol: $r+(1+k)r^2+(1+k+k^2)r^3+\ldots n\ terms$

$\times'\&'\div'$ by $(1-k)$ we get

$$=\frac{1}{(1-k)}\left[(1-k)r+(1-k^2)r^2+(1-k^3)r^3+\ldots.n\ terms\right]$$

$$=\frac{1}{(1-k)}\left[r+r^2+r^3+\ldots n\ terms\right]-\left[kr+k^2r^2+k^3r^3+\ldots.\ n\ term\right]$$

First series $a_1=r, r=\frac{r^2}{r}, n=n$, second series $a_1=kr, r=\frac{k^2r^2}{kr}=kr$

$$S_n=\frac{1}{(1-k)}\left[\frac{r(r^n-1)}{r-1}-\frac{kr((kr)^n-1)}{kr-1}\right]$$

Q.5. Sum the series $2+(1-i)+\left(\frac{1}{i}\right)+\ldots$ **to 8 terms.**

Sol: $2+(1-i)+\frac{1}{i}+\ldots+8terms$

$a_1=2, r=\frac{1-i}{2}, n=8 |r|<1$

$$(1+i)^8=\left[(1-i)^2\right]^4$$
$$=(1+i^2-2i)^4=(1-1-2i)^4=(-2i)^4$$
$$=16i^4=16i^2.i^2=16(-1)(-1)=16$$

$$S_n=\frac{a_1(1-r^n)}{1-r}=\frac{2\left(1-\left(\frac{1-i}{2}\right)^8\right)}{1-\frac{1-i}{2}}=\frac{2(2^8-(1-i)^8)}{2^8\left(\frac{2-1+i}{2}\right)}=\frac{256-((1-i)^2)^4}{64(1+i)}=\frac{256-(1+i^2-2i)^4}{64(1+i)}=\frac{256-(1-1-2i)^4}{64(1+i)}$$

$$S_8=\frac{256-16}{64(1+i)}=\frac{240}{64(1+i)}=\frac{15}{4(1+i)}\times\frac{1-i}{1-i}=\frac{15(1-i)}{4(1-i^2)}=\frac{15(1-i)}{4(1+1)}=\frac{15}{8}(1-i)$$

Q.6. **Show that the ratio of the sum of first n terms of a G.P. to the sum of terms from $(n+1)^{th}$ to $(2n)^{th}$ term is $\frac{1}{r^n}$. Where r is the common ratio of the G.P.**

Sol: Let the geometric progression be

$a, ar, ar^2, ...$

Where a is the first term and r is the common ratio, then

Sum of first n term= $S_n = \frac{a(1-r^n)}{1-r}$

Sum of terms from $(n+1)^{th}$ to $(2n)^{th}$ terms:

$$S' = ar^n + ar^{n+1} + ... + ar^{2n-1} = \frac{ar^n(1-r^n)}{1-r}$$

Now let's find the ratio:

$$\frac{S_n}{S'} = \frac{\frac{a(1-r^n)}{1-r}}{\frac{ar^n(1-r^n)}{1-r}} = \frac{a(1-r^n)}{1-r} \times \frac{1-r}{ar^n(1-r^n)} = \frac{1}{r^n}$$

The ratio of the sum of first n terms to the sum of terms from $(n+1)^{th}$ to $(2n)^{th}$ term is indeed $\frac{1}{r^n}$.