Exercise 7.1

Q.1. Evaluate each of the following:

i. $\dfrac{10!}{0!8!}$

Sol: $\dfrac{10.9.\cancel{8!}}{1.\cancel{8!}}=90 \qquad (\because\ 0!=1)$

ii. $\dfrac{12!}{3!(12-3)!}$

Sol: $\dfrac{12!}{3!.9!}=\dfrac{\cancel{12}^{2}.11.10.\cancel{9!}}{\cancel{3}.\cancel{2}.1.\cancel{9!}}=2.11.10=220$

iii. $\dfrac{1440}{3!4!}+\dfrac{2400}{5!2!}$

Sol: $\dfrac{1440}{3!.4!}+\dfrac{2400}{5!2!}=\dfrac{1440}{3.2.1.4.3.2.1}+\dfrac{2400}{5.4.3.2.1.2.1}$

$=\dfrac{1440}{6\times 24}+\dfrac{2400}{120\times 2}=\dfrac{1440}{144}+\dfrac{2400}{240}$

$=10+10=20$

iv. $\dfrac{(n+2)!}{(n+1)!}$

Sol: $\dfrac{(n+2)!}{(n+1)!}=\dfrac{(n+2)(n+1)!}{(n+1)!}$

$=\dfrac{(n+2)\cancel{(n+1)}!}{\cancel{(n+1)}!}=(n+2)$

Q.2. Write each of the following in the factorial form:

i. n^3-n

Sol: n^3-n

$n(n^2-1)=n(n+1)(n-1)$

$\dfrac{(n+1).n.(n-1).(n-2)!}{(n-2)!}=\dfrac{(n+1)!}{(n-2)!}$

ii. $n(n-1)(n-2)\dots\dots(n-r+1)$

Sol: $\dfrac{n(n-1)(n-2)\dots\dots(n-r+1)\times(n-r)!}{(n-r)!}=\dfrac{n!}{(n-r)!}$

Q.3. Find n, if $(n+4)!=3024.n!$

Sol: $(n+4)!=3024.n!$

$(n+4)(n+3)(n+2)(n+1)n!=3024.n!$

Divide both sides by $n!$

$(n+4)(n+3)(n+2)(n+1)=3024$

$(n+4)(n+3)(n+2)(n+1)=9.8.7.6$

$\Rightarrow n+4=9 \Rightarrow \ n=5$

نوٹ:
Equation کو ڈائریکٹ حل کرنے کے لیے جیسا کہ Q.3 کے آخر میں کیا گیا ہے۔ دونوں سائیڈ پر 4،4 Factors ہیں اور ترتیب سے لکھے ہوئے ہیں۔ ہم نے دونوں طرف کے پہلے Factor کو برابر لیا ہے۔ تو n کی قیمت آجاتی ہے۔

Q.4. If $\dfrac{1}{7!}+\dfrac{1}{8!}=\dfrac{x}{9!}$, find x.

Sol: $\dfrac{1}{7!}+\dfrac{1}{8!}=\dfrac{x}{9!}$

$\dfrac{1}{7!}+\dfrac{1}{8.7!}=\dfrac{x}{9!}$

$\dfrac{1}{7!}\left(1+\dfrac{1}{8}\right)=\dfrac{x}{9!}$

$\dfrac{1}{7!}\left(\dfrac{8+1}{8}\right)=\dfrac{x}{9!}$

$\dfrac{9}{8.7!}=\dfrac{x}{9!}$

To make $9!$ in denominator of L.H.S multiply and divide L.H.S by 9

$\dfrac{9}{7!.8}\times\dfrac{9}{9}=\dfrac{x}{9!}$

$\dfrac{81}{9.8.7!}=\dfrac{x}{9!}$

$\dfrac{81}{9!}=\dfrac{x}{9!}$

$\Rightarrow\ x=81$

Q.5. Prove that $\dfrac{(2n+1)!}{n!}=[1.3.5\dots(2n-1)(2n+1)]2^n$

Sol: L.H.S $=\dfrac{(2n+1)!}{n!}=\dfrac{(2n+1)(2n)(2n-1)\dots.5.4.3.2.1}{n!}$

$=\dfrac{\{(2n+1)(2n-1)\dots.5.3.1\}\{2n\dots.6.4.2\}}{n!}$

Even اور odd کو الگ کیا

$$=\frac{2^n\{1.2.3,......n\}.\{1.3.5......(2n-1)(2n+1)\}}{n!}$$

(Take 2 common from all even terms)

$$=\frac{2^n.\cancel{n!}\{1.3.5......(2n-1)(2n+1)\}}{\cancel{n!}}$$

$$=2^n\{1.3.5......(2n-1)(2n+1)\}$$

$$=\{1.3.5......(n-1)(n+1)\}2^n = \text{R.H.S}$$

Q.6. **Express as a single fraction:** $\frac{(n+2)!}{(r+2)!}+\frac{(n+1)!}{(r+1)!}$

Sol: $\frac{(n+2)!}{(r+2)!}+\frac{(n+1)!}{(r+1)!}$

$$\frac{(n+2)(n+1)!}{(r+2)(r+1)!}+\frac{(n+1)!}{(r+1)!}=\frac{(n+1)!}{(r+1)!}\left[\frac{n+2}{r+2}+1\right]$$

$$=\frac{(n+1)!}{(r+1)!}\left[\frac{n+2+r+2}{r+2}\right]$$

$$=\frac{(n+1)!.(n+r+4)}{(r+2)(r+1)!}$$

$$=\frac{(n+1)!}{(r+2)!}(n+r+4)$$

Q.7. **There are four distinct colored balls and four boxes of same colors as those of the balls. Determine the number of possible ways the balls, one each in a box, can be placed such that a ball does not go to a box of its own color?**

Sol: Key concept arrangement of n things

$B_1 \longrightarrow C_1$

$B_2 \longrightarrow C_2$

$B_3 \longrightarrow C_3$

$B_4 \longrightarrow C_4$

$$D_n=n!\left[1-\frac{1}{1!}+\frac{1}{2!}......n\right]$$

Arrangement of 4 balls in 4 boxes

$$=4!\left[1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}\right]$$

$$=24\left[1-1+\frac{1}{2}-\frac{1}{3.2.1}+\frac{1}{4.3.2.1}\right]$$

$$=24\left[\frac{1}{2}-\frac{1}{6}+\frac{1}{24}\right]=24\left[\frac{12-4+1}{24}\right]$$

$$=\cancel{24}\times\frac{9}{\cancel{24}}=9\text{ Ways}$$