Exercise 7.2

Q1. Evaluate the following:

Formula for permutations $^{n}p_r = \frac{n!}{(n-r)!}$

i. $^{10}P_5$

Sol. $^{10}P_5 = \frac{10!}{(10-5)!} = \frac{10.9.8.7.6.\cancel{5!}}{\cancel{5!}} = 30,240$

ii. $^{5}P_2$

Sol. $^{5}P_2 = \frac{5!}{(5-2)!} = \frac{5.4.\cancel{3!}}{\cancel{3!}} = 20$

iii. $^{7}P_7$

Sol. $^{7}P_7 = \frac{7!}{(7-7)!} = \frac{7.6.5.4.3.2.1}{1} = 5040$

iv. $^{10}P_3$

Sol. $^{10}P_3 = \frac{10!}{(10-3)!} = \frac{10.9.8.\cancel{7!}}{\cancel{7!}} = 720$

Q.2. Find the value of *n* when:

i. $^{n}P_3 = 504$

Sol. $\frac{n!}{(n-3)!} = 504$

$\frac{n(n-1)(n-2)\cancel{(n-3)!}}{\cancel{(n-3)!}} = 504$

$n(n-1)(n-2) = 9.8.7 \Rightarrow n = 9$

ii. $^{15}P_n = 15.14.13.12.11$

Sol. $^{15}P_n = 15.14.13.12.11 \quad ('\times' and \div by\, 10!)$

$\frac{15!}{(15-n)!} = \frac{15.14.13.12.11.10!}{10!}$

$\frac{15!}{(15-n)!} = \frac{15!}{10!} \Rightarrow (15-n)! = 10!$

$\Rightarrow 15 - n = 10 \Rightarrow 15 - 10 = n$

$\Rightarrow n = 5$

iii. $^{n}P_5 : {}^{n-2}P_2 = 540 : 1$

Sol. $^{n}P_5 : {}^{n-2}P_2 = 540 : 1$

$\frac{^{n}P_5}{^{n-2}P_2} = \frac{540}{1}$

$\frac{\frac{n!}{(n-5)!}}{\frac{(n-2)!}{(n-2-2)!}} = \frac{540}{1}$

$\frac{n!}{(n-5)!} \times \frac{(n-4)!}{(n-2)!} = \frac{540}{1}$

$\frac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-5)!}} \times \frac{(n-4)\cancel{(n-5)!}}{\cancel{(n-2)!}} = 540$

$n(n-1)(n-4) = 540$

$n(n-1)(n-4) = 10.9.6 \Rightarrow n = 10$

Q.3. Prove from the first principle that:

i. $^{n}P_r = n\,.\,{}^{n-1}P_{r-1}$

Sol. $\text{R.H.S} = n\,.\,{}^{n-1}P_{r-1}$

$= n.\frac{(n-1)!}{(n-\cancel{1}-r+\cancel{1})!} = n.\frac{(n-1)!}{(n-r)!}$

$= \frac{n.(n-1)!}{(n-r)!} = \frac{n!}{(n-r)!} = {}^{n}p_r = \text{L.H.S}$

ii. $^{n}P_r = {}^{n-1}P_r + r\,.\,{}^{n-1}P_{r-1}$

Sol. $^{n}P_r = {}^{n-1}P_r + r\,.\,{}^{n-1}P_{r-1}$

$\text{R.H.S} = {}^{n-1}P_r + r\,.\,{}^{n-1}P_{r-1}$

$= \frac{(n-1)!}{(n-1-r)!} + \frac{r.(n-1)!}{(n-\cancel{1}-r+\cancel{1})!}$

$= \frac{(n-1)!}{(n-r-1)!} + \frac{r.(n-1)!}{(n-r)!}$

$= \frac{(n-1)!}{(n-r-1)!} + \frac{r.(n-1)!}{(n-r)(n-r-1)!}$

$= \frac{(n-1)!}{(n-r-1)!}\left[1 + \frac{r}{n-r}\right] = \frac{(n-1)!}{(n-r-1)!}\left[\frac{n-\cancel{r}+\cancel{r}}{n-r}\right]$

$= \frac{n(n-1)!}{(n-r)(n-r-1)!} = \frac{n!}{(n-r)!} = {}^{n}P_r = \text{L.H.S}$

Q.4. How many words can be formed from the letters of the following words using all letters when no letter is to be repeated?

i. **PYTHON**

Sol. Total letters $= 6$

Total words $= 6!$

$= 6.5.4.3.2.1 = 720$

ii. **NETWORK**

Sol. Total letters $=7$

Total words $=7!$

$=7.6.5.4.3.2.1=5040$

iii. **COMPUTER**

Sol. Total letters $=8$

Total words $=8!$

$=8.7.6.5.4.3.2.1=40320$

Q.5. How many signals can be given by 6 flags of different colors, using 2 flags at a time?

Sol. Total flag $=6 \Rightarrow n=6$

Flag that can be used at a time $=2 \Rightarrow r=2$

Total signals $= {}^nP_r = {}^6P_2 = \frac{6!}{(6-2)!}$

$$=\frac{6.5.4.3.2.1}{4!}=\frac{6.5.\cancel{4!}}{\cancel{4!}}=30$$

Q.6. How many signals can be given by 5 flags of different colors, when any number of flags are used at a time?

Sol. Total flag $=5$

When any number of flags is used at a time then

Total Signals $= {}^5P_5 + {}^5P_4 + {}^5P_3 + {}^5P_2 + {}^5P_1$ ____ (A)

$${}^5P_5=\frac{5!}{(5-5)!}=\frac{5.4.3.2.1}{0!}=120 \quad \text{\_\_\_\_ (1)}$$

$${}^5P_4=\frac{5!}{(5-4)!}=\frac{5.4.3.2.1}{1!}=120 \quad \text{\_\_\_\_ (2)}$$

$${}^5P_3=\frac{5!}{(5-3)!}=\frac{5.4.3.\cancel{2!}}{\cancel{2!}}=60 \quad \text{\_\_\_\_ (3)}$$

$${}^5P_2=\frac{5!}{(5-2)!}=\frac{5.4.\cancel{3!}}{\cancel{3!}}=20 \quad \text{\_\_\_\_ (4)}$$

$${}^5P_1=\frac{5!}{(5-1)!}=\frac{5.\cancel{4!}}{\cancel{4!}}=5 \quad \text{\_\_\_\_ (5)}$$

Put (1), (2), (3), (4), (5) in (A)

$=120+120+60+20+5=325$

Q.7. How many 4 digit *numbers* can be formed, with distinct digits, with each digit odd?

Sol. The odd digits are 1, 3, 5, 7, 9 there are 5 odd digits

For the first digit we have 5 options

For 2nd digit we have 4 options

For 3rd digit we have 3 *options*

For 4th *digit* we have 2 options

So,

Total number of 4-digit distinct odd numbers $=5\times4\times3\times2=120$

Q.8. How many numbers between 100 and 1000 can be formed by using the digits 0,1,2,3,4,5 without repetition? How many of them are divisible by 5?

Sol. Numbers between 100 and 1000 are 3 – digits numbers

i) For hundred place we have 5 options

ii) For tens place we have 5 options (including 0, but excluding the digit used at hundreds place)

iii) For the units place we have 4 options (excluding the digits used at hundreds and tens place)

Total 3-digit number $=5\times5\times4=100$

A numbers Divisible by '5' if its units digit is either 0 or 5

i) if unit digit is 0 hundreds place has 5 options (1,2,3,4,5) if unit digit is 5 hundreds place has 4 options

ii) if unit digit is 0 tens place has 4 options. Also if unit digit is 5 tens place has 4 options

Total numbers divisible by 5

$$=(5\times4)+(4\times4)=20+16=36$$

The final answers are

1. Total 3 digit number $=100$
2. Numbers divisible by 5 $=36$

Q.9. Find the numbers greater than 35000 that can be formed from the digits 1,2,3,4,5,6, without repeating any digit.

Sol. *Digits* $=1,2,3,4,5,6$

Total digits $=6$

There are two options for number greater than 35000

i) If first digit has 4,5,6 all combinations with be greater than 35,000.

Total digit $=4\times5\times6=120$

There are 3 options for first digit options so

Total numbers $=120\times3=360$

ii) If first digit is 3

the second must be 5 or 6

Total combinations $=4!+4!=24+24=48$

Total numbers greater than

$35000=360+48=408$

Q.10. **Find the number of 5-digit numbers that can be formed from the digits 1,2,4,6,8 (when no digit is repeated), but**

i. **The digit 2 and 8 are next to each other's**

ii. **The digits 2 and 8 are not next to each other.**

Sol. $Digits = 1, 2, 4, 6, 8$

$Total\ Digits = 5$

$Total\ arrangements = 5! = 5.4.3.2.1 = 120$

i. The digits 2 and 8 are when 2 and 8 are next to each other we consider it as a single unit '28' or '82'

Then , $Total\ Digits = 4$

$Total\ numbers = 2 \times 4! = 2 \times 24 = 48$

ii. The digits 2 and 8 are not next to each other $= 120 - 48 = 72$

Q.11. **How many 6-digit numbers can be formed without repeating any digit from the digits 0,1,2,3,4,5? In how many of them will 0 be at the tens place?**

Sol: $Digits = 0, 1, 2, 3, 4, 5$

$Total\ 6\ Digits = 6!$

$= 6.5.4.3.2.1 = 720$

When '0' fixed at tens place then total digits Becomes $= 5!$

$\Rightarrow Total\ numbers = 5! = 5.4.3.2.1 = 120$

Q.12. **How many 5-digit multiples of 5 can be formed from the digits 2,3,5,7,9, when no digit is repeated.**

Sol: $Digits = 2, 3, 5, 7, 9$

Total five digits

For multiple of '5'

5 is fixed at units place

$\Rightarrow Total\ digit = 4$

Then, $Total\ multiples\ of\ 5 = 4!$

$= 4.3.2.1 = 24$

Q.13. **In how many ways can 8 different books including 2 on English be arranged on a shelf in such a way that the English books are never together?**

Sol: Suppose

E_1, E_2 are two English books and

$B_1, B_2, B_3, B_4, B_5, B_6$ remaining books.

$$\text{Total} = {}^8P_8 = \frac{8!}{(8-8)!} = \frac{8.7.6.5.4.3.2.1}{0!} = 40320$$

English books are together

Case I $B_1, B_2, B_3, B_4, B_5, B_6\ ,\ \boxed{E_1E_2}$

$$= {}^7P_7 = \frac{7!}{(7-7)!} = \frac{7.6.5.4.3.2.1}{0!} = 5040$$

Case II $B_1, B_2, B_3, B_4, B_5, B_6\ ,\ \boxed{E_2E_1}$

$= {}^7P_7 = 5040$

English books together $= 5040 + 5040 = 1080$

English books are not together = Total – Together

$= 40320 - 10080 = 30240$

> نوٹ:
>
> E_1E_2 کو ملا کر ایک کتاب بنالیں اس میں دو آپشن بنیں گے۔ E_1 کو پہلے اور
>
> E_2 بعد اور E_2 کو پہلے اور E_1 کو بعد میں رکھیں۔

Q.14 **Find the number of arrangements of 3 different books on English and 5 different books on Urdu for placing them on a shelf such that the books on the same subject are together.**

Sol: E_1, E_2, E_3 are English and U_1, U_2, U_3, U_4, U_5 are Urdu books.

Case I $E_1, E_2, E_3 \times U_1, U_2, U_3, U_4, U_5$

$= {}^3P_3 \times {}^5P_5$

$$= \frac{3!}{(3-3)!} \times \frac{5!}{(5-5)!} = 3.2.1 \times 5.4.3.2.1$$

$= 6 \times 120 = 720$

Case II $U_1, U_2, U_3, U_4, U_5 \times E_1, E_2, E_3$

$$= {}^5P_5 \times {}^3P_3 = \frac{5!}{(5-5)!} \times \frac{3!}{(3-3)!}$$

$$= \frac{5.4.3.2.1}{0!} \times \frac{3.2.1}{0!}$$

$= 120 \times 6 = 720$

Total $= 720 + 720 = 1440$

Q.15 **In how many ways can 5 boys and 4 girls be seated on a bench so that the girls and the boys occupy alternate seats?**

Sol: B represents boys and G girls.

Total number of ways:

$= {}^5P_5 \times {}^4P_4$

$$= \frac{5!}{(5-5)!} \times \frac{4!}{(4-4)!} = \frac{5.4.3.2.1}{0!} \times \frac{4.3.2.1}{0!}$$

$= 120 \times 24 = 2880$