Q.1. How many arrangements of the letters of the following words, taken all together can be made

i. **PAKISTAN**

Sol: PAKISTAN

Total Letters = 8

P repeated time = 1
A repeated time = 2
K repeated time = 1
I repeated time = 1
S repeated time = 1
T repeated time = 1
A repeated time = 1
N repeated time = 1

Total arrangements $= \dfrac{8!}{1!.2!.1!.1!.1!.1!.1!}$

$= \dfrac{8.7.6.5.4.3.\cancel{2}.\cancel{1}}{\cancel{2}.\cancel{1}} = 20160$

ii. **CURRICULUM**

Sol: CURRICULUM

Total Letters = n = 10

C repeated time = 2
U repeated time = 3
R repeated time = 2
I repeated time = 1
L repeated time = 1
M repeated time = 1

Total arrangements $= \dfrac{10!}{2!3!.2!.1!.1!.1!}$

$= \dfrac{10.9.8.7.6.5.4.3.\cancel{2}.\cancel{1}}{2.1.\cancel{3}2.1\cancel{2}.1.1.1.1}$

$= \dfrac{10.9.8.7.6.5.\cancel{4}}{\cancel{4}} = 15,1200$

iii. **PROBABILITY**

Sol: PROBABILITY

Total Letters = 11

P repeated time = 1
R repeated time = 1
O repeated time = 1
B repeated time = 2
A repeated time = 1
I repeated time = 2
L repeated time = 1
T repeated time = 1
Y repeated time = 1

$$\text{Total arrangements} = \frac{11!}{1!.1!.1!.2!.1!.2!.1!.1!.1!}$$

$$= \frac{11.10.9.8.7.6.5.4.3.2.1}{2.1.2.1} = 9,979,200$$

Q.2. How many permutations of the letters of the word "BANANA" can be made, if B must be the first letter in each arrangement?

Sol: Total letters = 6
Word = BANANA
When "B" must be the study letter, $n = 5$
A is repeated time = 3
N is repeated times = 2

$$\text{Total words} = \frac{5!}{3!.2!} = \frac{5.4.3!}{3!.2.1} = 10$$

نوٹ
اس میں B کو پہلے Fix کیا ہے تو اسے count نہیں کرتے

Q.3. How many arrangements of the letters of the word TRIGONOMETRY can be made, if each arrangement begins with T and ends with Y?

Sol: Word = TRIGONOMETRY
Total Letters = 12
When T and Y are fixed than
Total Letters = $12 - 2 = 10 \Rightarrow n = 10$
R is repeated time = 2
I is repeated time = 1
G is repeated time = 1
O is repeated time = 2
N is repeated time = 1
M is repeated time = 1
E is repeated time = 1
T is repeated time = 1

$$\text{Total words} = \frac{10!}{2!.1!.1!.2!.1!.1!.1!.1!}$$

$$= \frac{10.9.8.7.6.5.4.3.2.1}{2.1.2.1}$$

$$= 907200$$

Q.4. Abdullah has a collection of 9 marbles consisting of 4 identical red marbles, 3 identical blue marble and 2 identical green marbles. If he wants to arrange all of them are a straight, how many distinct arrangements are possible?

Sol: Total marbles = 9
Red marbles = 4
Blue marbles = 3
Green marbles = 2

$$\text{Total arrangement in a row} = \frac{9!}{4!.3!.2!}$$

$$= \frac{9.8.7.6.5.\cancel{4!}}{\cancel{4!}.3.2.1.2.1} = \frac{9.^2\cancel{8}.7.\cancel{6}.5}{\cancel{6}.\cancel{2}_1} = 9.4.7.5 = 1260$$

Q.5. In how many different ways can the following persons sit around a round table?

a. 8 Persons

Sol: Case I $\quad n = 8$

$$\text{Total ways} = (n-1)!$$

$$= (8-1)! = 7! = 7.6.5.4.3.2.1 = 5040$$

b. 7 Persons

Sol: $n = 7$

$$\text{Total arrangements} = (n-1)!$$

$$= (7-1)! = 6! = 6.5.4.3.2.1 = 720$$

c. 6 Persons

Sol: $n = 6$

$$\text{Total arrangements} = (n-1)!$$

$$= (6-1)! = 5! = 5.4.3.2.1 = 120$$

Q.6. In how many ways can 5 couples sit around a round table if no two women are sitting together?

Sol: Couples = 5
Men = 5
Women = 5
Total sitting arrangement on round tables
$$= 5! \times (5-1)!$$
$$= 5! \times 4!$$
$$= 5.4.3.2.1 \times 4.3.2.1 = 2880.$$

Q.7. How many 6-digit numbers can be formed from the digits 7,7,8,8,9,9?

Sol: $digits = 7,7,8,8,9,9 \quad \Rightarrow n = 6$

$$\text{Total numbers} = \frac{1}{2} \times 6!$$

$$= \frac{1}{\cancel{2}}.6.5.4.3.\cancel{2}.1 = 6.5.4.3.1 = 360$$

Q.8. 15 members of a club form 4 committees of 3,4,5 and 3 members so that no member is a member of more than one committee. Find the number of committees.

Sol: $Total\ members = 15$

$n = 15$

4 Committee have 3,5,4,3 members

$No\ of\ committees = \dfrac{15!}{3!.5!.4!.3!}$

$= \dfrac{15.14.13.12.11.10.9.8.7.6.\cancel{5!}}{3.2.1.\cancel{5!}.4.3.2.1.3.2.1} = 12,612,600$

Q.9. The D.C. Os of 11 districts meet to discuss the law and order situation in their districts. In how many ways can they be seated at a round table, when two particular D.C.Os insist on sitting together?

Sol: $Numbers\ of\ ways = {}^{9}P_9 \times {}^{2}P_2$

(when 2 particular D.C.Os sit together)

$= \dfrac{9!}{(9-9)!} \times \dfrac{2!}{(2-2)!} = \dfrac{9!}{0!} \times \dfrac{2!}{0!}$

$= 9! \times 2! = 727560$

Q.10. The Governor of the Punjab calls a meeting of 14 officers. In how many ways can they be seated at a round table?

Sol: $n = 14$

Total arrangements on a round table

$= (14-1)! = 13!$

$= 6,227,020,800$

Q.11. Fatima invites 14 people for a dinner. There are 9 males and 5 female who are seated at two different tables. Guests of one sex sit at one round table and the guests of the other sex sit at the second table. Find the number of ways in which all guests can be seated.

Sol: $Total\ people = 14$

$Male = 9$

$Female = 5$

Total arrangements of on two different tables

$= (9-1)! \times (5-1)! = 8! \times 4!$

$= 8.7.6.5.4.3.2.1 \times 4.3.2.1 = 40320 \times 24$

$= 967680$

Q.12. Find the number of ways in which 5 men and 5 women can be seated at a round table in such a way that no two persons of the same sex sit together.

Sol: $Total\ men = 5$

$Women = 5$

Total arrangements on a

Round table $= 5! \times (5-1)!$

$\qquad\qquad = 5.4.3.2.1 \times 4.3.2.1 = 120 \times 24$

$\qquad\qquad = 2880$

Q.13. In how many ways can 8 keys be arranged in a circular key ring?

Sol: $n = 8$

Total arrangements $= \dfrac{1}{2}(n-1)!$

$= \dfrac{1}{2}(8-1)! = \dfrac{1}{2}.7! = \dfrac{1}{2} \times 7.6.5.4.3.2.1 = 2520$

Q.14. How many necklaces can be made from 10 beads of different colors?

Sol: $n = 10$

Total arrangements $= \dfrac{1}{2}(n-1)!$

$= \dfrac{1}{2}(10-1)! = \dfrac{1}{2}(9!)$

$= \dfrac{9.8.7.6.5.4.3.2.1}{2}$

$= 181440$