



Exercise 7.4



Q.1. Evaluate the following:

$$\text{Formula } {}^n C_r = \frac{n!}{(n-r)!r!}$$

i. ${}^{50} C_{50}$

Sol: ${}^{50} C_{50} = \frac{50!}{(50-50)! \cdot 50!} = \frac{1}{0! \cdot 1} = 1$

ii. ${}^{1000} C_0$

Sol: ${}^{1000} C_0 = \frac{1000!}{(1000-0)! \cdot 0!} = \frac{1000!}{1000! \cdot 1} = 1$

iii. ${}^{10} C_7$

Sol: ${}^{10} C_7 = \frac{10!}{(10-7)! \cdot 7!} = \frac{10!}{3! \cdot 7!} = \frac{10 \cdot 9 \cdot 8 \cdot 7!}{3 \cdot 2 \cdot 1 \cdot 7!}$
 $= \frac{10 \cdot 9 \cdot 8}{6} = 120$

iv. ${}^{20} C_{17}$

Sol: ${}^{20} C_{17} = \frac{20!}{(20-17)! \cdot 17!} = \frac{20 \cdot 19 \cdot 18 \cdot 17!}{3! \cdot 17!}$
 $= \frac{20 \cdot 19 \cdot 18}{3 \cdot 2 \cdot 1} = \frac{20 \cdot 19 \cdot 18}{6} = 1140$

Q.2. (i) If ${}^{3n} C_2 : {}^n C_2 = 15 : 1$, Find n

(ii) If ${}^n P_r = 120$ and ${}^n C_r = 20$, Find r

Sol: ${}^{3n} C_2 : {}^n C_2 = 15 : 1$

$$\frac{{}^{3n} C_2}{{}^n C_2} = \frac{15}{1} = {}^{3n} C_2 \div {}^n C_2 = 15$$

$$\frac{3n!}{(3n-2)! \cdot 2!} \div \frac{n!}{(n-2)! \cdot 2!} = 15$$

$$\frac{3n(3n-1)(\cancel{3n-2})!}{(\cancel{3n-2})! \cdot 2!} \times \frac{(n-2)! \cdot 2!}{n!} = 15$$

$$\frac{3n(3n-1)}{2!} \times \frac{(n-2)! \cdot 2!}{n!} = 15$$

$$3 \cancel{n} (3n-1) \times \frac{(n-2)!}{\cancel{n} (n-1) (n-2)!} = 15$$

$$\frac{3(3n-1)}{(n-1)} = 15$$

$$9n-3 = 15(n-1)$$

$$9n-3 = 15n-15$$

$$15n-9n = -3+15 = 6n = 12 \Rightarrow n = 2$$

Q.3. Find the values of n and r , when

i. ${}^n C_r = 56, {}^n P_r = 336$

ii. ${}^{n-1} C_{r-1} : {}^n C_r : {}^n C_{r+1} = 1 : 3 : 7$

i. ${}^n C_r = 56, {}^n P_r = 336$

Sol: ${}^n C_r = 56, {}^n P_r = 336$

$$\frac{n!}{(n-r)!r!} = 56 \quad \text{--- (I)}$$

$$\frac{n!}{(n-r)!} = 336 \quad \text{--- (II)}$$

Divide (II) by (I)

$$\frac{n!}{(n-r)!} \div \frac{n!}{(n-r)!r!} = \frac{336}{56}$$

$$\frac{\cancel{n!}}{(\cancel{n-r})!} \times \frac{(n-r)!r!}{\cancel{n!}} = 6$$

$$r! = 6 \Rightarrow r! = 3! \Rightarrow \boxed{r=3}$$

Put $r=3$ in (II)

$$\frac{n!}{(n-3)!} = 336$$

$$\frac{n(n-1)(n-2)(\cancel{n-3})!}{(\cancel{n-3})!} = \frac{336}{56}$$

$$n(n-1)(n-2) = 8 \cdot 7 \cdot 6 \Rightarrow n = 8$$

$r=3$ and $n=8$

ii. ${}^{n-1} C_{r-1} : {}^n C_r : {}^{n+1} C_{r+1} = 1 : 3 : 7$

Sol: ${}^{n-1} C_{r-1} : {}^n C_r : {}^{n+1} C_{r+1} = 1 : 3 : 7$

$${}^{n-1} C_{r-1} : {}^n C_r = 1 : 3$$

$${}^{n-1} C_{r-1} \div {}^n C_r = 1 \div 3$$

$$\frac{(n-1)!}{(n-1-r+1)!(r-1)!} \div \frac{n!}{(n-r)!r!} = \frac{1}{3}$$

$$\frac{(n-1)!}{(\cancel{n-r})!(r-1)!} \times \frac{(\cancel{n-r})!r!}{n!} = \frac{1}{3}$$

$$\frac{(n-1)!}{(r-1)!} \times \frac{r(r-1)}{n(n-1)!} = \frac{1}{3}$$

$$\frac{r}{n} = \frac{1}{3} \Rightarrow 3r = n \text{-----I}$$



Now

$${}^n C_r : {}^{n+1} C_{r+1} = 3:7$$

$${}^n C_r \div {}^{n+1} C_{r+1} = 3 \div 7$$

$$\frac{n!}{r!(n-r)!} \div \frac{(n+1)!}{(r+1)!(n-r-1)!} = 3 \div 7$$

$$\frac{n!}{r!(n-r)!} \times \frac{(r+1) \cdot n! \times (n-r)!}{(n+1) \cdot n!} = \frac{3}{7}$$

$$\frac{r+1}{n+1} = \frac{3}{7} \Rightarrow 7(r+1) = 3(n+1) \text{-----II}$$

Put I in II

$$7(r+1) = 3(3r+1) \Rightarrow 7r+7 = 9r+3$$

$$9r-7r = 7-3 \Rightarrow 2r = 4 \Rightarrow \boxed{r=2}$$

$$\text{Put in I} \Rightarrow n = 3(2) \Rightarrow \boxed{n=6}$$

Q.4. Prove that

i. ${}^n C_r + {}^n C_{r-1} = {}^{n+1} C_r$

Sol. ${}^n C_r + {}^n C_{r-1} = {}^{n+1} C_r$

L.H.S

$$= {}^n C_r + {}^n C_{r-1}$$

$$= \frac{n!}{(n-r)! \cdot r!} + \frac{n!}{(n-r+1)! \cdot (r-1)!}$$

$$= \frac{n!}{(n-r)! \cdot r(r-1)!} + \frac{n!}{(n-r+1)(n-r)! \cdot (r-1)!}$$

$$= \frac{n!}{(n-r)! \cdot (r-1)!} \left[\frac{1}{r} + \frac{1}{n-r+1} \right]$$

$$= \frac{n!}{(n-r)! \cdot (r-1)!} \left[\frac{n-r+1+r}{r(n-r+1)} \right]$$

$$= \frac{(n+1)n!}{(n-r+1)(n-r)! \cdot r(r-1)!} = \frac{(n+1)!}{(n-r+1)! \cdot r!}$$

$$= {}^{n+1} C_r = R.H.S$$

ii. $r \cdot {}^n C_r = (n-r+1) \cdot {}^n C_{r-1}$

Sol. $r \cdot {}^n C_r = (n-r+1) \cdot {}^n C_{r-1}$

$$\text{L.H.S} = r \cdot {}^n C_r = r \cdot \frac{n!}{(n-r)! \cdot r!} \text{----- (I)}$$

$$\text{R.H.S} = (n-r+1) \cdot {}^n C_{r-1}$$

$$= (n-r+1) \cdot \frac{n!}{(n-r+1)! \cdot (r-1)!}$$

$$= \left((n-r+1) \times \frac{(n-r)!}{(n-r)!} \right) \left(\frac{n!}{(n-r+1)! \cdot (r-1)!} \times \frac{r}{r} \right)$$

$$= \frac{(n-r+1)!}{(n-r)!} \times \frac{r \cdot n!}{(n-r+1)! \cdot r(r-1)!}$$

$$= r \cdot \frac{n!}{(n-r)! \cdot r!} = r \cdot {}^n C_r = \text{L.H.S}$$

Q.5. Prove that product of r consecutive integers is divisible by $r!$.

Sol. Proof:

Let the r consecutive integers be

$(n+1), (n+2), (n+3), \dots, (n+r)$ then, their

product

$$= (n+1)(n+2)(n+3) \dots (n+r)$$

$$= \frac{n!(n+1)(n+2)(n+3) \dots (n+r)}{n!}$$

(multiply and divide by $n!$)

$$= \frac{(n+r)!}{n!} = \frac{(n+r)!}{n!} \times \frac{r!}{r!} = r! \cdot \frac{(n+r)!}{r!(n+r-r)!}$$

$$= r! \times {}^{n+r} C_r \text{ which is Divisible by } r!$$

Since ${}^{n+r} C_r$ is positive integer.

Q.6. In how many ways can five subjects be selected out of eight subjects to select a course program?

Sol. Total subjects: $= n = 8$

Subjects to be selected: $= r = 5 \Rightarrow r = 5$

Total ways: $= {}^n C_r = {}^8 C_5$

$$= \frac{8!}{(8-5)! \cdot 5!} = \frac{8 \cdot 7 \cdot 6 \cdot 5!}{3! \cdot 5!} = \frac{8 \cdot 7 \cdot 6}{3!} = 56$$

Q.7. Find the number of possible arrangements of vowel letters from the English alphabet?

Sol: Total letters of English alphabets = 26

$$\Rightarrow n = 26$$

Total vowels = 5 $\Rightarrow r = 5$

Total ways = $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$

Q.8. In how many ways 3 dishes of Desi food and 2 dishes of Chinese food be selected from 6 dishes of Desi food and 8 dishes of Chinese food?

Sol. Total dishes of Desi food = 6 $\Rightarrow n = 6$

To be selected = 3 $\Rightarrow r = 3$

Total dishes of Chinese food = 8 $\Rightarrow n = 8$

To be selected = 2 $\Rightarrow r = 2$

$$\text{Total ways} = {}^6 C_3 \cdot {}^8 C_2$$

$$= \frac{6!}{(6-3)!3!} \cdot \frac{8!}{(8-2)!2!} = \frac{6 \cdot 5 \cdot 4 \cdot \cancel{3!}}{3! \cdot \cancel{3!}} \cdot \frac{8 \cdot 7 \cdot \cancel{6!}}{\cancel{6!} \cdot 2!}$$

$$= \frac{6 \cdot 5 \cdot 4}{6} \times \frac{8 \cdot 7}{2} = 20 \times 28 = 560$$

Q.9. From a standard deck of 52 playing cards, there are 26 black cards and 26 red cards. How many different ways can eight cards be selected if 3 are black and the remaining 5 are red?

Sol.

| | |
|----------------------------|-------------|
| Total black cards | = 26 |
| Total red cards | = 26 |
| To be arranged cards | = 8 |
| To be arranged black cards | = 3 |
| To be arranged red cards | = 8 - 3 = 5 |

Total possible ways = ${}^{26}C_3 \cdot {}^{26}C_5$

$$= \frac{26!}{(26-3)!3!} \cdot \frac{26!}{(26-5)!5!}$$

$$= \frac{26 \cdot 25 \cdot 24 \cdot \cancel{23!}}{\cancel{23!} \cdot 6} \times \frac{26 \cdot 25 \cdot 24 \cdot 23 \cdot 22 \cdot \cancel{21!}}{\cancel{21!} \cdot 120}$$

$$= 171,028,00$$

Q.10. A bag contains 8 red balls, 7 green balls. Find the total number of possible ways in which five balls are selected in a way:

| | |
|----------------------|-----|
| Total Red balls | = 8 |
| Total Green balls | = 7 |
| Balls to be selected | = 5 |

i. 3 red and 2 green

Sol. Total ways = ${}^8C_3 \cdot {}^7C_2$

$$= \frac{8!}{(8-3)!3!} \times \frac{7!}{(7-2)!2!} = \frac{8!}{5!3!} \cdot \frac{7!}{5!2!}$$

$$= \frac{8 \cdot 7 \cdot 6 \cdot \cancel{5!}}{\cancel{5!} \cdot 3 \cdot 2 \cdot 1} \cdot \frac{7 \cdot 6 \cdot \cancel{5!}}{\cancel{5!} \cdot 2 \cdot 1} = 56 \cdot 21 = 1176$$

ii. 1 red and 4 green

Sol. Total ways = ${}^8C_1 \cdot {}^7C_4$

$$= \frac{8!}{(8-1)!1!} \times \frac{7!}{(7-4)!4!}$$

$$= \frac{8 \cdot \cancel{7!}}{\cancel{7!} \cdot 1!} \times \frac{7 \cdot 6 \cdot 5 \cdot \cancel{4!}}{3! \cdot \cancel{4!}}$$

$$= 8 \times \frac{7 \cdot 6 \cdot 5}{6} = 8 \times 7 \times 5 = 280$$

iii. 4 red and 1 green

Sol. Total ways = ${}^8C_4 \cdot {}^7C_1$

$$= \frac{8!}{(8-4)!4!} \times \frac{7!}{(7-1)!1!}$$

$$= \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot \cancel{4!}}{4! \cdot \cancel{4!}} \times \frac{7 \cdot \cancel{6!}}{\cancel{6!} \cdot 1} = \frac{8 \cdot 7 \cdot 6 \cdot 5}{4 \cdot 3 \cdot 2 \cdot 1} \times 7 = 490$$

iv. All the red balls

Sol. Total ways = 8C_5

$$= \frac{8!}{(8-5)!5!} = \frac{8 \cdot 7 \cdot 6 \cdot \cancel{5!}}{3! \cdot \cancel{5!}} = \frac{8 \cdot 7 \cdot 6}{6} = 56$$

Q.11. How many diagonals and triangles can be formed by joining the vertices of the polygon having 15 sides.

Sol. For diagonal = ${}^nC_2 - n$

For triangle = nC_3

Total Side of polygon = 15 $\Rightarrow n = 15$

Numbers of Diagonals = ${}^{15}C_2 - 15$

$$= \frac{15!}{(15-2)!2!} - 15 = \frac{15 \cdot 14 \cdot \cancel{13!}}{\cancel{13!} \cdot 2 \cdot 1} - 15$$

$$= 105 - 15 = 90$$

Numbers of Triangles = ${}^nC_3 = {}^{15}C_3$

$$= \frac{15!}{(15-3)!3!} = \frac{15 \cdot 14 \cdot 13 \cdot \cancel{12!}}{\cancel{12!} \cdot 3 \cdot 2 \cdot 1} = 455$$

Q.12. Find the number of sides of a polygon if the number of its diagonals is 104

Sol: Let Number of sides of polygon = n

Given Diagonals = 104

Number of Diagonals = ${}^nC_2 - n$

$$104 = {}^nC_2 - n$$

$$104 = \frac{n!}{(n-2)!2!} - n$$

$$104 = \frac{n(n-1)(\cancel{n-2})!}{(\cancel{n-2})!2!} - n$$

$$104 = \frac{n(n-1)}{2} - n$$

$$104 = n \left[\frac{n-1}{2} - 1 \right]$$

$$104 = n \left[\frac{n-1-2}{2} \right]$$

$$104 = \frac{n}{2} [n-3]$$

$$2 \times 104 = \cancel{2} \times \frac{n}{\cancel{2}} [n-3]$$

$$208 = n^2 - 3n$$

$$n^2 - 3n - 208 = 0$$

$$n^2 - 16n + 13n - 208 = 0$$

$$n(n-16) + 13(n-16) = 0$$

$$(n-16)(n+13) = 0$$

$$n-16=0, n+13=0$$

$$\boxed{n=16}, n=-13 \text{ Impossible}$$

Q.13 How many triangles can be formed by joining 15 points, 6 of which lie on the same straight line?

Sol: Total points = 15 $\Rightarrow n = 15$

Total triangles formed by

$$15 \text{ points} = {}^n C_3 = {}^{15} C_3$$

$$\frac{15!}{(15-3)! \cdot 3!} = \frac{15 \cdot 14 \cdot 13 \cdot \cancel{12!}}{\cancel{12!} \cdot 3!} = \frac{15 \cdot 14 \cdot 13}{6} = 455$$

Since 6 points lie on the straight line. Choosing any '3' of these

6 points will not form a triangle. The number of such combinations = ${}^6 C_3$

$$\frac{6!}{(6-3)! \cdot 3!} = \frac{6 \cdot 5 \cdot 4 \cdot \cancel{3!}}{3! \cdot \cancel{3!}} = \frac{6 \cdot 5 \cdot 4}{6} = 20$$

$$\text{Total triangles} = 455 - 20 = 435$$

Q.14. The members of a club are 10 boys and 8 girls. In how many ways can a committee of 6 boys and 3 girls be formed?

Sol. Boys: $n = 10$, $r = 6$

Girls: $n = 8$, $r = 3$

$$\text{Number of ways of committee} = {}^{10} C_6 \times {}^8 C_3$$

$$= \frac{10!}{(10-6)! \cdot 6!} \times \frac{8!}{(8-3)! \cdot 3!} = \frac{10!}{4! \cdot 6!} \times \frac{8!}{5! \cdot 3!}$$

$$= \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot \cancel{6!}}{4 \cdot 3 \cdot 2 \cdot 1 \cdot \cancel{6!}} \times \frac{8 \cdot 7 \cdot 6 \cdot \cancel{5!}}{\cancel{5!} \cdot 3 \cdot 2 \cdot 1} = \frac{10 \cdot 9 \cdot 8 \cdot 7}{4 \cdot 3 \cdot 2 \cdot 1} \times \frac{8 \cdot 7 \cdot 6}{3 \cdot 2 \cdot 1}$$

$$= 210 \times 56 = 11760$$

Q.15. How many committees of 7 members can be chosen from a group of 10 persons when each committee must include 2 particular persons?

Sol. $n = 10$; $r = 7$

For two particular persons

$$\Rightarrow n = 8; r = 5$$

$$\text{Total committees} = {}^n C_r = {}^8 C_5$$

$$= \frac{8!}{(8-5)! \cdot 5!} = \frac{8 \cdot 7 \cdot 6 \cdot \cancel{5!}}{3! \cdot \cancel{5!}} = \frac{8 \cdot 7 \cdot 6}{3 \cdot 2 \cdot 1} = 56$$

Q.16. In how many ways can a cricket team of 11 players be selected out of 17 players? How many of them will include a particular player?

Sol. Total Players = 17

To be selected = 11

$$n = 17; r = 11$$

$$\text{Total ways} = {}^{17} C_{11}$$

$$= \frac{17!}{(17-11)! \cdot 11!}$$

$$= \frac{17 \cdot 16 \cdot 15 \cdot 14 \cdot 13 \cdot 12 \cdot \cancel{11!}}{6! \cdot \cancel{11!}}$$

$$= \frac{8,910,720}{720}$$

$$= 12,376$$

When a particular player is include

$$\Rightarrow n = 16 = 17 - 1; r = 10 = 11 - 1$$

$$\text{Total ways} = {}^{16} C_{10}$$

$$= \frac{16!}{(16-10)! \cdot 10!}$$

$$= \frac{16 \cdot 15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot \cancel{10!}}{6! \cdot \cancel{10!}}$$

$$= \frac{5765,760}{720} = 8008$$

Q.17. There are 6 men and 8 women members of a club. How many committees of seven can be formed?

Sol. Men = 6 , Women = 8

Total members = 7

i. **With 3 women**

Total committees = ${}^8C_3 \times {}^6C_4$

$$= \frac{8!}{(8-3)! \cdot 3!} \times \frac{6!}{(6-4)! \cdot 4!} = \frac{8 \cdot 7 \cdot 6 \cdot \cancel{5!}}{\cancel{5!} \cdot 3 \cdot 2 \cdot 1} \times \frac{6 \cdot 5 \cdot \cancel{4!}}{2! \cdot \cancel{4!}} = \frac{8 \cdot 7 \cdot \cancel{6}}{\cancel{6}} \times \frac{\cancel{3} \cdot 6 \cdot 5}{\cancel{2}} = 56 \times 15 = 840$$

ii. **with at most 3 women**

Sol. Total committees = $({}^8C_1 \times {}^6C_6) + ({}^8C_2 \times {}^6C_5) + ({}^8C_3 \times {}^6C_4)$

$$= \frac{8!}{(8-1)! \cdot 1!} \times \frac{6!}{(6-6)! \cdot 6!} + \frac{8!}{(8-2)! \cdot 2!} \times \frac{6!}{(6-5)! \cdot 5!} + \frac{8!}{(8-3)! \cdot 3!} \times \frac{6!}{(6-4)! \cdot 4!}$$

$$= \frac{8 \cdot \cancel{7!}}{\cancel{7!} \cdot 1!} \times \frac{\cancel{6!}}{(6-6)! \cdot \cancel{6!}} + \frac{8 \cdot 7 \cdot \cancel{6!}}{\cancel{6!} \cdot 2!} \times \frac{6 \cdot \cancel{5!}}{1! \cdot \cancel{5!}} + \frac{8 \cdot 7 \cdot 6 \cdot \cancel{5!}}{\cancel{5!} \cdot 3!} \times \frac{6 \cdot 5 \cdot \cancel{4!}}{2! \cdot \cancel{4!}}$$

$$= \left(8 \times \frac{1}{0!}\right) + \left(\frac{8 \cdot 7}{2} \times 6\right) + \left(\frac{8 \cdot 7 \cdot 6}{3 \cdot 2 \cdot 1} \times \frac{6 \cdot 5}{2 \cdot 1}\right) = 8 + 168 + 840 = 1016$$

iii. **with at least 5 women**

Sol. Total ways = ${}^8C_5 \times {}^6C_2 + {}^8C_6 \times {}^6C_1 + {}^8C_7 \times {}^6C_0$

$$= \left(\frac{8!}{(8-5)! \cdot 5!} \times \frac{6!}{(6-2)! \cdot 2!}\right) + \left(\frac{8!}{(8-6)! \cdot 6!} \times \frac{6!}{(6-1)! \cdot 1!}\right) + \left(\frac{8!}{(8-7)! \cdot 7!} \times \frac{6!}{(6-0)! \cdot 0!}\right)$$

$$= \left(\frac{8 \cdot 7 \cdot 6 \cdot \cancel{5!}}{3! \cdot \cancel{5!}} \times \frac{6 \cdot 5 \cdot \cancel{4!}}{\cancel{4!} \cdot 2!}\right) + \left(\frac{8 \cdot 7 \cdot \cancel{6!}}{2! \cdot \cancel{6!}} \times \frac{6 \cdot \cancel{5!}}{\cancel{5!} \cdot 1!}\right) + \left(\frac{8 \cdot \cancel{7!}}{1! \cdot \cancel{7!}} \times \frac{\cancel{6!}}{\cancel{6!} \cdot 0!}\right)$$

$$= \left(\frac{8 \cdot 7 \cdot 6}{6} \times \frac{6 \cdot 5}{2}\right) + \left(\frac{8 \cdot 7}{2} \times 6\right) + (8 \times 1) = 840 + 168 + 8 = 1016$$

Q.18. There are three section in a question paper, each section has 3 questions. A student has to solve all 5 questions, choosing at least one question from each section. In how many ways can the student make his choice?

Sol: Total section = 3 \Rightarrow Total Options = $3! = 6$

Each section has questions = 3

Total number of ways:

(Option - I)

$${}^3C_1 \times {}^3C_1 \times {}^3C_3 = \frac{3!}{(3-1)! \cdot 1!} \times \frac{3!}{(3-1)! \cdot 1!} \times \frac{3!}{(3-3)! \cdot 3!} = \frac{3 \cdot \cancel{2!}}{\cancel{2!} \cdot 1!} \times \frac{3 \cdot \cancel{2!}}{\cancel{2!} \cdot 1!} \times \frac{\cancel{3!}}{0! \cdot \cancel{3!}} = 3 \times 3 \times 1 = 9$$

(Option - II)

$${}^3C_1 \times {}^3C_2 \times {}^3C_2 = \frac{3!}{(3-1)! \cdot 1!} \times \frac{3!}{(3-2)! \cdot 2!} \times \frac{3!}{(3-2)! \cdot 2!} = \frac{3 \cdot \cancel{2!}}{\cancel{2!} \cdot 1!} \times \frac{3 \cdot \cancel{2!}}{1! \cdot \cancel{2!}} \times \frac{3 \cdot \cancel{2!}}{1! \cdot \cancel{2!}} = 3 \times 3 \times 3 = 27$$

(Option - III)

$${}^3C_1 \times {}^3C_3 \times {}^3C_1 = \frac{3!}{(3-1)! \cdot 1!} \times \frac{3!}{(3-3)! \cdot 3!} \times \frac{3!}{(3-1)! \cdot 1!} = 3 \times 1 \times 3 = 9$$

(Option - IV)

$${}^3C_2 \times {}^3C_1 \times {}^3C_2 = \frac{3!}{(3-2)!2!} \times \frac{3!}{(3-1)!1!} \times \frac{3!}{(3-2)!2!} = \frac{3 \cdot 2!}{1! \cdot 2!} \times \frac{3 \cdot 2!}{2! \cdot 1!} \times \frac{3 \cdot 2!}{1! \cdot 2!} = 3 \times 3 \times 3 = 27$$

(Option - V)

$${}^3C_3 \times {}^3C_1 \times {}^3C_1 = \frac{3!}{(3-3)!3!} \times \frac{3!}{(3-1)!1!} \times \frac{3!}{(3-1)!1!} = \frac{3!}{0! \cdot 3!} \times \frac{3 \cdot 2!}{2! \cdot 1!} \times \frac{3 \cdot 2!}{2! \cdot 1!} = 1 \times 3 \times 3 = 9$$

Total Ways: $9 + 27 + 9 + 27 + 9 = 108$

Q.19. Consider a cryptographic system that generates an 8 characters password. Each character in the password can be either a lowercase letter (a - f) or a digit (0 - 5). How many passwords can be generated if each password must contain exactly 5 lowercase letters and 3 digits?

a. with repetition allowed

Sol. Lower case letters = a - f

Total options = 6

Digits = (0 - 5)

Total options = 6

Letters to be selected = 5

Digits to be selected = 3

Total Characters in Password = 8

Options for letters = 8C_5

$$= \frac{8!}{(8-5)!5!} = \frac{8 \cdot 7 \cdot 6 \cdot 5!}{3! \cdot 5!} = \frac{8 \cdot 7 \cdot 6}{3!} = 56$$

Options for digits = 8C_3

$$= \frac{8!}{(8-3)!3!} = \frac{8 \cdot 7 \cdot 6 \cdot 5!}{3! \cdot 3!} = \frac{8 \cdot 7 \cdot 6}{3!} = 56 \Rightarrow {}^8C_5 = {}^8C_3 = 56$$

For each of '5' letters there are 6 options so,

$$\text{Number of Password} = 56 \times 6^5 \times 6^3 = 56 \times 7776 \times 216$$

$$= 56 \times 1679616 = 94018596$$

b. Without repetition

Sol. $56 \times {}^6C_5 \cdot {}^6C_3$

$$= 56 \times \frac{6!}{(6-5)!} \times \frac{6!}{(6-3)!} = 56 \times \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{1!} \times \frac{6 \cdot 5 \cdot 4 \cdot 3!}{3!} = 56 \times 72 \times 120 = 4838400$$

Q.20. On Defense Day, Teacher I compiles a list of 10 distinct national songs, while Teacher II prepares a separate list of 10 different national songs (with no overlap between the two lists). The principal needs to select 3 songs from Teacher I's list, and 3 songs from Teacher II's list.

Determine the number of possible selection methods when:

i. The order/sequence of the selected songs is important.

Sol. Teachers I one prepares songs = 10

To be selected by principle = 3

Teacher II prepares songs = 10

To be selected by principle = 3

$$\text{Total ways} = {}^{10}P_3 \cdot {}^{10}P_3$$

$$= \frac{10!}{(10-3)!} \cdot \frac{10!}{(10-3)!} = \frac{10 \cdot 9 \cdot 8 \cdot \cancel{7!}}{\cancel{7!}} \times \frac{10 \cdot 9 \cdot 8 \cdot \cancel{7!}}{\cancel{7!}} = 10 \times 9 \times 8 \times 10 \times 9 \times 8 = 518,400$$

ii. **The order/sequence of the selected songs is not important.**

Sol. Teachers I one prepares songs = 10

To be selected by principle = 3

Teacher II prepares songs = 10

To be selected by principle = 3

Total ways when the sequence of the selected songs does not matter

$$= {}^{10}C_3 \cdot {}^{10}C_3 = \frac{10!}{(10-3)! \cdot 3!} \cdot \frac{10!}{(10-3)! \cdot 3!} = \frac{10 \cdot 9 \cdot 8 \cdot \cancel{7!}}{\cancel{7!} \cdot 3 \cdot 2 \cdot 1} \times \frac{10 \cdot 9 \cdot 8 \cdot \cancel{7!}}{\cancel{7!} \cdot 3 \cdot 2 \cdot 1} = 720 + 720 = 1440$$

studyplusplus.com (study++)