Exercise 9.1

1. Find remainder and quotient by simplify the following:

i. $(3x^2 - x + 2) \div (x - 1)$

Sol:

$$\begin{array}{r|l} & 3x+2 \\ \hline x-1 & 3x^2 - x + 2 \\ & \pm 3x^2 \mp 3x \\ \hline & 2x + 2 \\ & \pm 2x \mp 2 \\ \hline & 4 \end{array}$$

Quotient $= 3x + 2$; Remainder $= 4$

ii. $(x^3 + 12x^2 - 3x + 4) \div (x - 2)$

Sol:

$$\begin{array}{r|l} & x^2 + 14x + 25 \\ \hline x-2 & x^3 + 12x^2 - 3x + 4 \\ & \pm x^3 \mp 2x^2 \\ \hline & 14x^2 - 3x + 4 \\ & \pm 14x^2 \mp 28x \\ \hline & 25x + 4 \\ & \pm 25x \mp 54 \\ \hline & 58 \end{array}$$

Quotient $= x^2 + 14x + 25$; Remainder $= 58$

iii. $(x^4 - 5x^3 - 8x^2 + 13x + 12) \div (x - 6)$

Sol:

$$\begin{array}{r|l} & x^3 + x^2 - 2x + 1 \\ \hline x-6 & x^4 - 5x^3 - 8x^2 + 13x + 12 \\ & \pm x^4 \mp 6x^3 \\ \hline & x^3 - 8x^2 + 13x + 12 \\ & \pm x^3 \mp 6x^2 \\ \hline & -2x^2 + 13x + 12 \\ & \mp 2x^2 \pm 12x \\ \hline & x + 12 \\ & \pm x \mp 6 \\ \hline & 18 \end{array}$$

Quotient $= x^3 + x^2 - 2x + 1$; Remainder $= 18$

iv. $(5x^4 - 3x^3 + 2x^2 - 1) \div (x^2 + 4)$

Sol:

$$\begin{array}{r|l} & 5x^2 - 3x - 18 \\ \hline x^2+4 & 5x^4 - 3x^3 + 2x^2 - 1 \\ & \pm 5x^4 \qquad \pm 20x^2 \\ \hline & -3x^3 - 18x^2 \qquad -1 \\ & \mp 3x^3 \qquad \mp 12x \\ \hline & -18x^2 + 12x - 1 \\ & \mp 18x^2 \qquad \mp 72 \\ \hline & 12x + 71 \end{array}$$

Quotient $= 5x^2 - 3x - 18$; Remainder $= 12x + 71$

v. $\left(3x^4-5x^3+4x-6\right)\div\left(x^2-3x+5\right)$

Sol:

$$
\begin{array}{r}
3x^2+4x-3 \\
x^2-3x+5\,\overline{)\,3x^4-5x^3+4x-6} \\
\underline{\pm 3x^4 \mp 9x^3 \pm 15x^2} \\
4x^3-15x^2+4x-6 \\
\underline{\pm 4x^3 \mp 12x^2 \pm 20x} \\
-3x^2-16x-6 \\
\underline{\mp 3x^2 \pm 9x \mp 15} \\
-25x+9
\end{array}
$$

Quotient $=3x^2+4x-3$; Remainder $=-25x+9$

2. Use the remainder theorem to find the remainder when the first polynomial is divided by the second polynomial.

i) $\boldsymbol{x^2+5x+6\ ,\ x-2}$

Sol: Let $f(x)=x^2+5x+6$ & $x-a=x-2 \Rightarrow a=2$

By remainder theorem:

Remainder $=f(2)=(2)^2+5(2)+6$

$=4+10+6=20$

> نوٹ:
>
> Remainder معلوم کرنے کے لیے دیے گئے function کو f(x) کے
>
> برابر کر کے اس میں x کی قیمت put کریں گے تو remainder آ جائے گا۔

ii) $\boldsymbol{x^3+5x^2+6\ ,\ x+1}$

Sol: Let $f(x)=x^3+5x^2+6$

$x-a=x+1 \Rightarrow a=-1$

By using remainder theorem:

Remainder $=f(-1)=(-1)^3+5(-1)^2+6$

$=-1+5+6=10$

iii) $\boldsymbol{x^4+x^3+x^2+x+1\ ,\ x-1}$

Sol: Let $f(x)=x^4+x^3+x^2+x+1$

$x-a=x-1 \Rightarrow a=1$

By using remainder theorem:

Remainder$=f(1)=(1)^4+(1)^3+(1)^2+(1)+1=5$

iv) $\boldsymbol{x^4+x^2+1\ ,\ x+3}$

Sol: Let $f(x)=x^4+x^2+1$

$x-a=x+3 \Rightarrow a=-3$

By using remainder theorem:

Remainder $=f(-3)=(-3)^4+(-3)^2+1$

$=81+9+1=91$

v) $\boldsymbol{x^4+x^3+2\ ,\ x+2}$

Sol: Let $f(x)=x^4+x^3+2$

$x-a=x+2 \Rightarrow a=-2$

By using remainder theorem:

Remainder $=f(-2)=(-2)^4+(-2)^3+2$

$=16-8+2=10$

3. Use the factor theorem to determine if the first polynomial is a factor of the second polynomial.

i) $\boldsymbol{x+1\ ,\ x^2-1}$

Sol: Here $f(x)=x^2-1$ and $a=-1$

$f(-1)=(-1)^2-1=1-1=0$

So, $x+1$ is factor of x^2-1

> نوٹ:
>
> یہ چیک کرنے کے لیے کہ دیا گیا polynomial اس کا factor ہے یا نہیں
>
> صرف remainder معلوم کریں اگر remainder=0 ہو تو factor ہے
>
> اور اگر remainder ≠ 0 ہو تو factor نہیں ہے۔

ii) $\boldsymbol{x-2\ ,\ x^2-5x+6}$

Sol: Here $f(x)=x^2-5x+6$ and $a=2$

$f(2)=(2)^2-5(2)+6$

$f(2)=4-10+6=0$

So, $x+2$ is factor of x^2-5x+6

iii) $\boldsymbol{x+1\ ,\ x^3+x^2+x-3}$

Sol: Here $f(x)=x^3+x^2+x-3$ and $a=-1$

$f(-1)=(-1)^3+(-1)^2+(-1)-3$

$f(-1)=-1+1-1-3=-4\neq 0$

So, $x+1$ is not factor of x^3+x^2+x-3.

iv) $\boldsymbol{x-2\ ,\ x^3+x^2-7x+2}$

Sol: Here $f(x)=x^3+x^2-7x+2$ and $a=2$

$f(2)=(2)^3+(2)^2-7(2)+2$

$f(2)=8+4-14+2=0$

So, $x-2$ is factor of x^3+x^2-7x+2

v) $\boldsymbol{x-3\ ,\ x^4-3x^3+x^2-x+1}$

Here $f(x)=x^4-3x^3+x^2-x+1$ and $a=3$

$f(3)=(3)^4-3(3)^3+(3)^2-3+1$

$f(3)=81-81+9-3+1=7\neq 0$

So, $x-3$ is not factor of $x^4-3x^3+x^2-x+1$

4. Use synthetic division to show that x is the zero of the polynomial and use the result to factorize the polynomial completely.

i) x^3-7x+6 , $x=2$

Sol: Let $f(x)=x^3-7x+6$

$=x^3+0x^2-7x+6$

Here $a=2$

2	1	0	-7	6
		2	4	-6
	1	2	-3	0

$x^2+2x-3=0$

$x^2+3x-x-3=0$

$x(x+3)-1(x+3)=0$

$(x-1)(x+3)=0$

Therefore, other two factors are $(x-1)$ and $(x+3)$

Hence $x^3-7x+6=(x-2)(x-1)(x+3)$

نوٹ:

ان سوالات میں جب $f(x)$ لکھا ہے تو اگر x کی کوئی power نہیں دی گئی تو اس کی جگہ 0 لگاتا ہے۔

ii) $x^3-28x-48$, $x=-4$

Let $f(x)=x^3-28x-48$

$=x^3+0x^2-28x-48$

Here $a=-4$

-4	1	0	-28	-48
		-4	16	48
	1	-4	-12	0

$x^2-4x-12=0$

$x^2-6x+2x-12=0$

$x(x-6)+2(x-6)=0$

$(x+2)(x-6)=0$

So, other two factors are $(x+2)$ and $(x-6)$

Hence $x^3-28x-48=(x+4)(x+2)(x-6)$

iii) $2x^4+7x^3-4x^2-27x-18$, $x=2$, $x=-3$

Let $f(x)=2x^4+7x^3-4x^2-27x-18$

Step 1: use synthetic division with $x=2$ or $a=2$

2	2	7	-4	-27	-18
		4	22	36	18
	2	11	18	9	0

As remainder $=0$ so, $(x-2)$ is factor of new polynomial $2x^3+11x^2+18x+9$.

Step 2: use synthetic division $x=-3$ or $a=-3$

Again

-3	2	11	18	9
		-6	-15	-9
	1	2	-3	0

Remainder $=0$, so $(x+3)$ is also factor of new polynomial.

$2x^2+5x+3=0 \Rightarrow 2x^2+2x+3x+3=0$

$2x(x+1)+3(x+1)=0 \Rightarrow (2x+3)(x+1)=0$

Other two factors are $(2x+3)$ and $(x+1)$

So, complete factors of

$2x^4+7x^3-4x^2-27x-18$

$=(x-2)(x+3)(x+1)(2x+3)$.

Q5. Use synthetic division to find the quotient and the remainder when the polynomial x^4-10x^2-2x+4 is divided by $x+3$.

Sol:

-3	1	0	-10	-2	4
		-3	9	3	-3
	1	-3	-1	1	1

Here $f(x)=x^4+0x^3-10x^2-2x+4$ and $a=-3$

So, when x^4-10x^2-2x+4 is divided by $x+3$ using synthetic division.

Quotient $=x^3-3x^2-x+1$ and remainder $=1$

6. If $x+1$ and $x-2$ are factors of x^3-px^2+qx+2 Using synthetic division, find the values of p and q.

Sol: As given that $x+1$ and $x-2$ are factors of the polynomial $f(x)=x^3-px^2+qx+2$

This means: $f(-1)=0$ and $f(2)=0$

By using synthetic division

-1	1	-p	q	2
		-1	P+1	-p-q-1
	1	-p-1	p+q+1	-p-q+1

As $x+1$ is factor so, $-p-q+1=0$-----(a)

2	1	-p	q	2
		2	-2p+4	-4p+2q+8
	1	-p+2	-2p+q+4	-4p+2q+10

As $x-2$ is factor so, $-4p+2q+10=0$-----(b)

By $(b)-4(a)$ we get

$-4p+2q+10=0$

$\mp 4p \mp 4q \pm 4=0$

$6q+6=0$

$6q=-6$

$\Rightarrow q=\frac{-6}{6}\Rightarrow q=-1$

By using $q=-1$ in (a) we get

$-p-(-1)+1=0$

$-p+1+1=0$

$-p+2=0$

$-p=-2\Rightarrow p=2$

So, we have $p=2$ and $q=-1$

Q7. **When the polynomial $4x^4+2x^3+kx^2+13$ is divided by $x+1$, the remainder is 16. Find the value of k.**

Sol: Let $f(x)=4x^4+2x^3+kx^2+13$

Substitute $x=-1$

$f(-1)=4(-1)^4+2(-1)^3+k(-1)^2+13$

Put $f(-1)=16$

$16=k+15$

$16-15=k\Rightarrow 1=k$ or $k=1$

Q8. **When the polynomial x^3+x^2+x+k is divided by $x+1$, the remainder is 7. Find the value of k.**

Sol: Let $f(x)=x^3+x^2+x+k$

Substitute $x=1$

$f(1)=(1)^3+(1)^2+(1)+k$

Put $f(1)=7$

$7=3+k$

$7-3=k\Rightarrow k=4$

9. **Use factor theorem to find the values of p and q if $x+1$ and $x-2$ are the factors of the polynomial x^3+px^2+qx+3.**

Sol: By using factor theorem if $x+1$ is a factor then $f(-1)=0$ & if $x-2$ is a factor then $f(2)=0$

So, $f(-1)=(-1)^3+p(-1)^2+q(-1)+3$

$f(-1)=-1+p-q+3$

Put $f(-1)=0$

$\Rightarrow p-q+2=0----(a)$

So, $f(2)=(2)^3+p(2)^2+q(2)+3$

$f(2)=8+4p+2q+3$

Put $f(2)=0$

$4p+2q+11=0----(b)$

From eq (a)

$p=q-2$

Substitute into eq (b)

$4(q-2)+2q+11=0$

$4q-8+2q+11=0$

$6q+3=0$

$6q=-3$

$q=\frac{-3}{6}\Rightarrow q=\frac{-1}{2}$

By using $p=q-2$

$p=\frac{-1}{2}-2\Rightarrow\frac{-1-4}{2}=\frac{-5}{2}$

So, $p=\frac{-5}{2}$

Q10. **Use factor theorem to find the values of a & b if -2 & 2 are the roots of the polynomial $2x^3+4x^2+ax+b$.**

Sol: Let $f(x)=2x^3+4x^2+ax+b$

$f(2)=2(2)^3+4(2)^2+a(2)+b$

$f(2)=2(8)+4(4)+2a+b$

$f(2)=16+16+2a+b$

$f(2)=32+2a+b$

Given 2 is a root so, $f(2)=0$

$\Rightarrow 32+2a+b=0----(a)$

Now $f(-2)=2(-2)^3+4(-2)^2+a(-2)+b$

$f(-2)=2(-8)+4(4)-2a+b$

$f(-2)=-16+16-2a+b$

$f(-2)=-2a+b$

Given -2 is a root so, $f(-2)=0$

$\Rightarrow -2a+b=0-----(b)$

by adding eq. (a) & (b)

we have

$-2a+b=0$

$2a+b+32=0$

$2b+32=0\Rightarrow 2b=-32$

$b=\frac{-32}{2}\Rightarrow b=-16$

By using eq. (b)

$-2a+b=0$

$-2a+(-16)=0$

$-2a=16\Rightarrow a=\frac{-16}{2}\Rightarrow a=-8$