Exercise 9.2

1. **Consider a data set at monthly sales figures. A polynomial regression model $P(x)=x^3+2x^2+x-3$ is fitted to this data. If the observed sales in the 5th month are 240 units, find the percentage error.**

Sol: $P(x)=x^3+2x^2+x-3$

$P(5)=(5)^3+2(5)^2+5-3=125+2(25)+5-3=177$

$Absolute\ Error=|Observed\ Value-Predicted\ Value|=|240-177|=63$

$Percentage\ Error=\dfrac{Absolute\ Error}{Observed\ Value}\times 100\%=\dfrac{63}{240}\times 100\%=26.25\%$

2. **A retailer company has developed a polynomial regression model to predict weekly product demand: $D(w)=w^3-2w^2+5w-4$, where $D(w)$ represents predicted demand (in units) and w is the week number. Use the remainder theorem to predict demand for 3rd week. If the observe demand is 22 units, calculate the prediction error.**

Sol: Given the demand function $D(w)=w^3-2w^2+5w-4$, for the 3rd week $(w=3)$:

$D(3)=(3)^3-2(3)^2+5(3)-4$

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$D(3)=27-18+15-4 \Rightarrow D(3)=20$

Prediction Error:

Observed Demand $=22\ units$

Predicted Demand $=20\ units$

Prediction Error=Observed Demand-Prediction Demand

Prediction Error $=22-20=2$

So, the predicted demand is $20\ units$, and the prediction error is $2\ units$.

3. A digital signal processing system has a transfer function with a numerator $B(z)=z^2-z-2$. Use the factor theorem to find the zeros of the system.

Sol: Given $B(z)=z^2-z-2$. We want to find the values of z such that $B(z)=0$.

$z^2-z-2=0$

Factoring the quadratic:

$(z-2)(z+1)=0$

Setting each factor to zero:

$z-2=0 \Rightarrow z=2$

$z+1=0 \Rightarrow z=-1$

Therefore, the zeros of the system are $z=2,-1$.

4. A signal process system has a transfer function $H(z)=\dfrac{z^2+3z+2}{z^2-0.2z+0.9}$. Find the zero(s) of the transfer function by using factor theorem.

Sol: Given the numerator of the transfer function:

$B(z)=z^2+3z+2$

To find the zeros, we set $B(z)=0$:

$x^2+3z+2=0$

Factoring the quadratic:

$(z+1)(z+2)=0$

Setting each factor to zero to find the roots (Zeros):

$z+1=0 \Rightarrow z=-1$

$z+2=0 \Rightarrow z=-2$

Therefore, the zeros of the transfer function are $z=-1, -2$

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5. A signal process system has a transfer function $H(z)=\dfrac{z^2-0.5z-0.5}{z^3+1}$. Find the zero(s) of the transfer function by using factor theorem.

Sol: To find the zeros of $H(z)=\dfrac{z^2-0.5z-0.5}{z^3+1}$, we need to find the roots of the numerator:

$z^2-0.5z-0.5=0$

Using the quadratic formula, $z=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

Where $a=1, b=-0.5$, $and\ c=-0.5$:

$$z=\frac{-(-0.5)\pm\sqrt{(-0.5)^2-4(1)(-0.5)}}{2(1)}=\frac{0.5\pm\sqrt{0.25+2}}{2}=\frac{0.5\pm\sqrt{2.25}}{2}=\frac{0.5\pm1.5}{2}$$

$$z=\frac{0.5+1.5}{2}=\frac{2}{2}=1 \quad \text{and} \quad z=\frac{0.5-1.5}{2}=\frac{-1}{2}=-0.5$$

Thus, the zeros of the transfer function are 1 and -0.5

6. **A signal processing system has a transfer function with a denominator $A(z) = z^2 - 0.3z - 0.4$. Use factor theorem to find the poles of the system and determine if the system is stable.**

Sol: Given the denominator $A(z) = z^2 - 0.3z - 0.4$

To find the poles, we set $A(z) = 0$:

$$z^2 - 0.3z - 0.4 = 0 \Rightarrow z^2 - 0.8z + 0.5z - 0.4 = 0$$
$$z(z-0.8) + 0.5(z-0.8) = 0$$
$$(z+0.5)(z-0.8) = 0$$

Setting each factor to zero to find the poles:

$z - 0.8 = 0 \quad \Rightarrow \quad z = 0.8$

$z + 0.5 = 0 \quad \Rightarrow \quad z = -0.5$

The poles of the system are located at $z = 0.8$ and $z = -0.5$

Stability Assessment:

For stability, the magnitude of each pole must be less than 1:

$$|0.8| = 0.8 < 1$$
$$|-0.5| = 0.5 < 1$$

Since both poles have a magnitude less than 1, the system is stable

7. **The denominator of signal processing system's transfer function equal to $A(z) = z^2 + 1.2z + 0.35$. Use factor theorem to determine the location of the corresponding poles and assess the stability of the system.**

Sol: Given the denominator $A(z) = z^2 + 1.2z + 0.35$

To find the poles, we set $A(z) = 0$:

$$z^2 + 1.2z + 0.35 = 0$$
$$\Rightarrow z^2 + 0.7z + 0.5z + 0.35 = 0$$
$$\Rightarrow z(z+0.7) + 0.5(z+0.7) = 0$$

Factoring the quadratic:

$$\Rightarrow (z+0.5)(z+0.7) = 0$$

Setting each factor to zero to find the poles:

$z + 0.5 = 0 \quad \Rightarrow \quad z = -0.5$

$z + 0.7 = 0 \quad \Rightarrow \quad z = -0.7$

The poles of the system are located at $z = -0.5$ and $z = -0.7$

Stability Assessment:

For stability, the magnitude of each pole must be less than 1:

$$|-0.5| = 0.5 < 1$$
$$|-0.7| = 0.7 < 1$$

Since both poles have a magnitude less than 1, the system is stable.

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