Exercise 10.1

Q1. Without using the tables, find the values of

i. $\cos(-1230^\circ)$

Sol: $\cos(-1230^\circ)=\cos(1230^\circ)=\cos(13\times 90+60)^\circ=-\sin 60^\circ=-\frac{\sqrt{3}}{2}$

ii. $\tan(-1035^\circ)$

Sol: $\tan(-1035^\circ)=-\tan(1035^\circ)=-\tan(11\times 90+45)^\circ=-(-\cot 45^\circ)=\cot 45^\circ=\frac{1}{\tan 45^\circ}=\frac{1}{1}=1$

iii. $\sec(1140^\circ)$

Sol: $\sec(1140^\circ)=\sec(12\times 90+60)^\circ=\sec(60^\circ)=\frac{1}{\cos 60^\circ}=\frac{1}{1/2}=2$

iv. $cosec(-690^\circ)$

Sol: $\cos ec(-690^\circ)=-\cos ec(690^\circ)=-\cos ec(7\times 90^\circ+60^\circ)=-(-\sec 60^\circ)=\sec 60^\circ=\frac{1}{\cos 60^\circ}\frac{1}{1/2}=2$

v. $\cot(1320^\circ)$

Sol: $\cot(1320^\circ)=\cot(14\times 90^\circ+60^\circ)=\cot 60^\circ=\frac{1}{\tan 60^\circ}=\frac{1}{\sqrt{3}}$

vi. $\cos(-240^\circ)$

Sol: $\cos(-240^\circ)=\cos(240^\circ)=\cos(2\times 90^\circ+60^\circ)=-\cos(60^\circ)=-\frac{1}{2}$

Q2. Express each of the following as a trigonometric functions of an angle of positive degree measure of less then 45°

i. $\cos(168^\circ)$

Sol: $\cos(168^\circ)=\cos(180-12)^\circ=\cos(2\times 90-12)^\circ=-\cos(12^\circ)$

ii. $\sin(192^\circ)$

Sol: $\sin(192^\circ)=\sin(180+12)^\circ=\sin(2\times 90+12)^\circ=-\sin(12^\circ)$

iii. $\cos(333^\circ)$

Sol: $\cos(333^\circ)=\cos(360-27)^\circ=\cos(4\times 90-27)^\circ=\cos(27^\circ)$

iv. $\tan(213^\circ)$

Sol: $\tan(213^\circ)=\tan(180+33)^\circ=\tan(2\times 90+33)^\circ=\tan(33^\circ)$

v. $\cos(-435^\circ)$

Sol: $\cos(-435^\circ)=\cos(435^\circ)=\cos(5\times 90-15)^\circ=\sin(15^\circ)$

vi. $\sin(219^\circ)$

Sol: $\sin(219^\circ)=\sin(180+39)^\circ=\sin(2\times 90+39)^\circ=-\sin(39^\circ)$

vii. $\tan(-597^\circ)$

Sol: $\tan(-597^\circ) = -\tan(-597^\circ) = -\tan(7\times 90 - 33)^\circ = -\cot(33^\circ)$

viii. $\cos(-111^\circ)$

Sol: $\cos(-111^\circ) = \cos(111^\circ) = \cos(90^\circ + 21^\circ) = -\sin(21^\circ)$

ix. $\sin(-390^\circ)$

Sol: $\sin(-390^\circ) = -\sin(390^\circ) = -\sin(4\times 90^\circ + 30^\circ) = -\sin 30^\circ$

Q3. Prove the following

i. **$\text{Sin}(180^\circ + \alpha)\,\text{Sin}(90^\circ - \alpha) = -\text{Sin}\,\alpha\,\text{Cos}\,\alpha$**

Sol. L.H.S = $\text{Sin}(180^\circ + \alpha)\,\text{Sin}\,90^\circ - \alpha) = \text{Sin}(2\times 90^\circ + \alpha)\,\text{Sin}(1\times 90^\circ - \alpha) = (-\sin\alpha)(\cos\alpha)$
$= -\text{Sin}\,\alpha\ \text{Cos}\,\alpha$ = R.H.S

ii. **$Sin\,810^\circ \sin 630^\circ + \cos 135^\circ \sin 225^\circ = -1/2$**

Sol: L.H.S

$$= \sin 810^\circ \sin 630^\circ + \cos 135^\circ \sin 225^\circ$$
$$= \sin(2\times 360^\circ + 90)\sin(2\times 360^\circ - 90^\circ) + \cos(90^\circ + 45^\circ)\sin(180^\circ + 45^\circ)$$
$$= \sin(90^\circ)[-\sin(90^\circ)] + (-\sin 45^\circ)(-\sin 45^\circ)$$
$$= (1)(-1) + \left(-\frac{1}{\sqrt{2}}\right)\left(-\frac{1}{\sqrt{2}}\right) = -1 + \frac{1}{2} = -1/2 = \text{R.H.S}$$

iii. **$\tan 150^\circ \cot 330^\circ - 2\sec 135^\circ\, cosec 225^\circ = -3$**

Sol: L.H.S $= \tan 150^\circ \cot 330^\circ - 2\sec 135^\circ \operatorname{cos}ec 225^\circ$

$$= \tan(180-30)^\circ \cot(360-30)^\circ - 2\sec(180-45)^\circ\, cosec(180+45)^\circ$$
$$= \tan(2\times 90 - 30)^\circ \cot(4\times 90 - 30)^\circ - 2\sec(2\times 90 - 45)^\circ\, cosec(2\times 90 + 45)^\circ$$
$$= -\tan(30^\circ)(-\cot 30^\circ) - 2(-\sec 45^\circ)(-\operatorname{cos}ec 45^\circ)$$
$$= \left(-\frac{1}{\sqrt{3}}\right)(-\sqrt{3}) - 2(\sqrt{2})(\sqrt{2}) = 1 - 4 = -3$$

iv. **$\sin 210^\circ + \cos 240^\circ + \tan 225^\circ + \cot 225^\circ = 1$**

Sol: L.H.S $= \sin 210^\circ + \cos 240^\circ + \tan 225^\circ + \cot 225^\circ$

$$= \sin(180+30)^\circ + \cos(180+60)^\circ + \tan(180+45)^\circ + \cot(180+45)^\circ$$
$$= \sin(2\times 90 + 30)^\circ + \cos(2\times 90 + 60)^\circ + \tan(2\times 90 + 45)^\circ + \cot(2\times 90 + 45)^\circ$$
$$= -\sin 30^\circ + (-\cos 60^\circ) + \tan 45^\circ + \cot 45^\circ$$
$$= -\frac{1}{2} - \frac{1}{2} + 1 + 1 = \cancel{-1}\ \cancel{+1} + 1 = 1 = \text{R.H.S}$$

Q4. Prove that

i. $$\frac{\tan(180^\circ + \alpha)\cot(90^\circ - \alpha)}{\sin(360^\circ - \alpha)\cos(270^\circ + \alpha)} = -\sec^2\alpha$$

Sol: $$\text{L.H.S} = \frac{\tan(180^\circ + \alpha)\cot(90^\circ - \alpha)}{\sin(360^\circ - \alpha)\ \cos(270^\circ + \alpha)} = \frac{\tan(2\times 90^\circ + \alpha)\cot(90^\circ - \alpha)}{\sin(4\times 90^\circ - \alpha)\ \cos(3\times 90^\circ + \alpha)}$$

$$=\frac{\tan\alpha.\tan\alpha}{(-\sin\alpha).\sin\alpha}=\frac{\tan^2\alpha}{-\sin^2\alpha}=\frac{\sin^2\alpha}{\cos^2\alpha}\times\frac{1}{-\sin^2\alpha}=-\frac{1}{\cos^2\alpha}=-\sec^2\alpha=\text{R.H.S}$$

ii. $$\frac{Sin^2(\pi+\theta)\tan(\frac{3\pi}{2}+\theta)}{Cot^2(\frac{3\pi}{2}-\theta)Cos^2(\pi-\theta)Co\sec(2\pi-\theta)}=Cos\theta$$

Sol: $$\text{L.H.S}=\frac{Sin^2(\pi+\theta)\tan(\frac{3\pi}{2}+\theta)}{Cot^2(\frac{3\pi}{2}-\theta)Cos^2(\pi-\theta)Co\sec(2\pi-\theta)}=\frac{(-Sin\theta)^2(-Cot\theta)}{\tan^2\theta(-Cos)^2(-Co\sec\theta)}$$

$$=\frac{Sin^2\theta Cot\theta}{\frac{Sin^2\theta}{Cos^2\theta}Cos^2\theta\frac{1}{Sin\theta}}=Sin^2\theta\times\frac{Cos\theta}{Sin\theta}\times\frac{1}{Sin^2\theta}\times Sin\theta=Cos\theta=R.H.S$$

iii. $$\frac{Cos(90^o+\theta)Sec(-\theta)\tan(180^o-\theta)}{Sec(360^o-\theta)\,Sin\left(180^o+\theta\right)Cot(90^o-\theta)}=-1$$

Sol. $$\text{L.H.S}=\frac{Cos(90^o+\theta)Sec(-\theta)\tan(180^o-\theta)}{Sec(360^o-\theta)\,Sin(180^o+\theta)Cot(90^o-\theta)}=\frac{-Sin\theta\,Sec\theta(-\tan\theta)}{Sec\theta(-Sin\theta)\tan\theta}=\frac{Sin\theta\;Sec\theta\;\tan\theta}{-Sin\theta\;Sec\theta\;\tan\theta}$$

$=-1=\text{R.H.S}$

Q5. **Show that** $\sec\left(\frac{3\pi}{2}-\theta\right)\sec\left(\frac{5\pi}{2}-\theta\right)-\tan\left(\frac{3\pi}{2}-\theta\right)\tan\left(\frac{5\pi}{2}+\theta\right)=-1$

Sol: $$\text{L.H.S}=\sec\left(\frac{3\pi}{2}-\theta\right)\sec\left(\frac{5\pi}{2}-\theta\right)-\tan\left(\frac{3\pi}{2}-\theta\right)\tan\left(\frac{5\pi}{2}+\theta\right)$$

$$=(-cosec\,\theta)(cosec\,\theta)-(-\cot\theta)(\cot\theta)$$

$$=-\left(cosec^2\theta-\cot^2\theta\right)=-\left(\frac{1}{\sin^2\theta}-\frac{\cos^2\theta}{\sin^2\theta}\right)=-\left(\frac{1-\cos^2\theta}{\sin^2\theta}\right)=-\left(\frac{\sin^2\theta}{\sin^2\theta}\right)=-1$$

Q6. **If α,β,γ are angle of Triangle ABC, then prove that**

i. **$\text{Sin}(\alpha+\beta)=\text{Sin}\,\gamma$**

Sol. As $\alpha+\beta+\gamma=180^0$ (sum of angles of triangle=180^0)

$\alpha+\beta=180^0-\gamma$

$\text{Sin}(\alpha+\beta)=\sin(2\times90^0-\gamma)$

$\text{Sin}(\alpha+\beta)=\text{Sin}\,\gamma$ Hence proved.

ii. $$\sec\left(\frac{\alpha+\beta}{2}\right)=cosec\frac{\gamma}{2}$$

Sol: $\alpha+\beta+\gamma=180^\circ$

$$\alpha+\beta=180^\circ-\gamma\Rightarrow\frac{\alpha+\beta}{2}=\frac{180^\circ-\gamma}{2}\Rightarrow\frac{\alpha+\beta}{2}=90^\circ-\frac{\gamma}{2}$$

$$\Rightarrow\sec\left(\frac{\alpha+\beta}{2}\right)=\sec\left(90^\circ-\frac{\gamma}{2}\right)$$

$$\sec\left(\frac{\alpha+\beta}{2}\right)=cosec\frac{\gamma}{2}$$

iii. $cosec\alpha = \frac{1}{\sin(\beta+\gamma)}$

Sol: $\alpha+\beta+\gamma=180^\circ$

$\beta+\gamma=180^\circ-\alpha$

$\sin(\beta+\gamma)=\sin(180^\circ-\alpha)=\sin(2\times 90^\circ-\alpha)$

$\sin(\beta+\gamma)=\sin\alpha$

$\sin(\beta+\gamma)=\frac{1}{cosec\alpha}$ or $cosec\alpha=\frac{1}{\sin(\beta+\gamma)}$

iv. **$\tan(\alpha+\beta)+\tan\gamma=0$**

Sol. let $\alpha+\beta+\gamma=180^\circ$

$\alpha+\beta=180^\circ-\gamma$

$\tan(\alpha+\beta)=\tan(2\times 90^\circ-\gamma)$

$\tan(\alpha+\beta)=-\tan\gamma \Rightarrow \tan(\alpha+\beta)+\tan\gamma=0$ Hence Proved