Q1. **Find the values of $\sin 2\alpha$, $\cos 2\alpha$ and $\tan 2\alpha$, when:**

i. $\sin\alpha = \frac{3}{5};\ 0 < \alpha < \frac{\pi}{2}$

Sol: $\cos^2\alpha = 1 - \sin^2\alpha$

$$= 1 - \left(\frac{3}{5}\right)^2 = 1 - \frac{9}{25} = \frac{16}{25}$$

$$\cos\alpha = \pm\frac{4}{5} \Rightarrow \cos\alpha = \frac{4}{5}\ (\alpha\ is\ in\ I\ quad)$$

$$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{3/5}{4/5} = \frac{3}{4}$$

Now

(a) $\sin 2\alpha = 2\sin\alpha\cos\alpha$

$$= 2\left(\frac{3}{5}\right)\left(\frac{4}{5}\right) = \frac{24}{25}$$

(b) $\cos 2\alpha = 2\cos^2\alpha - 1 = 2\left(\frac{4}{5}\right)^2 - 1 = 2\left(\frac{16}{25}\right) - 1$

$$= \frac{32-25}{25} = \frac{7}{25}$$

(c) $\tan 2\alpha = \frac{\sin 2\alpha}{\cos 2\alpha} = \frac{24/25}{7/25} = \frac{24}{25} \times \frac{25}{7} = \frac{24}{7}$

ii. $\cos\alpha = \frac{4}{5};\ 0 < \alpha < \frac{\pi}{2}$

Sol: $\sin^2\alpha = 1 - \cos^2\alpha$

$$= 1 - \left(\frac{4}{5}\right)^2 = 1 - \frac{16}{25} = \frac{9}{25}$$

$$\sin\alpha = \pm\frac{3}{5} \Rightarrow \sin\alpha = \frac{3}{5}(\alpha\ is\ in\ I)$$

$$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{3/5}{4/5} = 3/4$$

(a) $\sin 2\alpha = 2\sin\alpha\cos\alpha$

$$= 2\left(\frac{3}{5}\right)\left(\frac{4}{5}\right) = \frac{24}{25}$$

(b) $\cos 2\alpha = 2\cos^2\alpha - 1$

$$= 2\left(\frac{4}{5}\right)^2 - 1 = 2\left(\frac{16}{25}\right) - 1$$

$$= \frac{32-25}{25} = \frac{7}{25}$$

(c) $\tan 2\alpha = \frac{\sin 2\alpha}{\cos 2\alpha} = \frac{24/25}{7/25} = \frac{24}{7}$

2. **Prove the following identities**

i. **$\cot\alpha - \tan\alpha = 2\cot 2\alpha$**

Sol: L.H.S = $\cot\alpha - \tan\alpha$

$$= \frac{Cos\alpha}{Sin\alpha} - \frac{Sin\alpha}{Cos\alpha} = \frac{Cos^2\alpha - Sin^2\alpha}{Sin\alpha Cos\alpha}$$

$$= \frac{2(Cos^2\alpha - Sin^2\alpha)}{2Sin\alpha Cos\alpha}$$ ('X' & ÷ by 2)

$$= \frac{2Cos2\alpha}{Sin2\alpha} = 2$$

$\text{Cot}2\alpha$ = R.H.S

ii. $\dfrac{Sin2\alpha}{1+Cos2\alpha} = \tan\alpha$

Sol. L.H.S = $\dfrac{Sin2\alpha}{1+Cos2\alpha}$

$= \dfrac{2Sin\alpha Cos\alpha}{2Cos^2\alpha} = \dfrac{Sin\alpha}{Cos\alpha} = \tan\alpha$ = R.H.S

iii. $\dfrac{1-Cos\alpha}{Sin\alpha} = \tan\dfrac{\alpha}{2}$

Sol. L.H.S = $\dfrac{1-Cos\alpha}{Sin\alpha} = \dfrac{2Sin^2\frac{\alpha}{2}}{2Sin\frac{\alpha}{2}\,Cos\frac{\alpha}{2}} = \dfrac{Sin\frac{\alpha}{2}}{Cos\frac{\alpha}{2}}$

$= \tan\dfrac{\alpha}{2}$ = R.H.S

iv. $\dfrac{Cos\alpha - Sin\alpha}{Cos\alpha + Sin\alpha} = Sec2\alpha - \tan2\alpha$

Sol. L.H.S = $\dfrac{Cos\alpha - Sin\alpha}{Cos\alpha + Sin\alpha}$

$= \dfrac{Cos\alpha - Sin\alpha}{Cos\alpha + Sin\alpha} \times \dfrac{Cos\alpha - Sin\alpha}{Cos\alpha - Sin\alpha}$

$= \dfrac{(Cos\alpha - Sin\alpha)^2}{Cos^2\alpha - Sin^2\alpha} =$

$\dfrac{Cos^2\alpha + Sin^2\alpha - 2Sin\alpha Cos\alpha}{Cos2\alpha} = \dfrac{1-Sin2\alpha}{Cos2\alpha}$

$= \dfrac{1}{Cos2\alpha} - \dfrac{Sin2\alpha}{Cos2\alpha} = Sec2\alpha - \tan2\alpha = R.H.S$

$= Sec2\alpha - \tan2\alpha = R.H.S$

v. $\sqrt{\dfrac{1+Sin\alpha}{1-Sin\alpha}} = \dfrac{Sin\alpha/2 + Cos\alpha/2}{Sin\alpha/2 - Cos\alpha/2}$

Sol. L.H.S = $\sqrt{\dfrac{1+Sin\alpha}{1-Sin\alpha}}$

$= \sqrt{\dfrac{Sin^2\alpha/2 + Cos^2\alpha/2 + 2Sin\alpha/2\,Cos\alpha/2}{Sin^2\alpha/2 + Cos^2\alpha/2 - 2Sin\alpha/2\,Cos\alpha/2}}$

$= \sqrt{\dfrac{(Sin\alpha/2 + Cos\alpha/2)^2}{(Sin\alpha/2 - Cos\alpha/2)^2}}$

$= \dfrac{Sin\alpha/2 + Cos\alpha/2}{Sin\alpha/2 - Cos\alpha/2}$ = R.H.S

vi. $\dfrac{Cosec\theta + 2Cosec2\theta}{Sec\theta} = Cot\,\theta/2$

Sol. L.H.S = $\dfrac{Cosec\theta + 2Cosec2\theta}{Sec\theta} = \dfrac{\frac{1}{Sin\theta} + \frac{2}{Sin2\theta}}{\frac{1}{Cos\theta}}$

$= \dfrac{\frac{1}{Sin\theta} + \frac{\cancel{2}}{\cancel{2}Sin\theta Cos\theta}}{\frac{1}{Cos\theta}} = \left(\dfrac{Cos\theta + 1}{Sin\theta\,\cancel{Cos\theta}}\right)\dfrac{\cancel{Cos\theta}}{1}$

$= \dfrac{\cancel{2}Cos^2\theta/2}{\cancel{2}Sin\theta/2\,Cos\theta/2} = \dfrac{Cos\theta/2}{Sin\theta/2} = Cot\,\theta/2$ = R.H.S

vii. $1 + \tan\alpha\tan2\alpha = Sec2\alpha$

Sol. L.H.S = $1 + \tan\alpha\tan2\alpha$

$= 1 + \dfrac{Sin\alpha}{Cos\alpha}\cdot\dfrac{Sin2\alpha}{Cos2\alpha} = \dfrac{Cos\alpha Cos2\alpha + Sin\alpha Sin2\alpha}{Cos\alpha Cos2\alpha}$

$= \dfrac{Cos\alpha Cos2\alpha + Sin\alpha Sin2\alpha}{Cos\alpha Cos2\alpha}$

$= \dfrac{Cos(2\alpha - \alpha)}{Cos\alpha \times Cos2\alpha} = \dfrac{Cos\alpha}{Cos\alpha \times Cos2\alpha}$

$= \dfrac{1}{Cos2\alpha} = Sec2\alpha$ = R.H.S

viii. $\dfrac{2Sin\theta\,Sin2\theta}{Cos\theta + Cos3\theta} = \tan2\theta\tan\theta$

Sol. L.H.S= $\dfrac{2Sin\theta\,Sin2\theta}{Cos\theta + Cos3\theta}$ $(Cos3\theta = 4Cos^3\theta - 3Cos\theta)$

$= \dfrac{2Sin\theta\,Sin2\theta}{Cos\theta + 4Cos^3\theta - 3Cos\theta}$

$= \dfrac{2Sin\theta\,Sin2\theta}{4Cos^3\theta - 2Cos\theta} = \dfrac{\cancel{2}Sin\theta\,Sin2\theta}{\cancel{2}Cos\theta(2Cos^2\theta - 1)}$

$= \dfrac{Sin\theta.Sin2\theta}{Cos\theta.Cos2\theta} = \tan\theta\tan2\theta$

$= \tan2\theta\,.\,\tan\theta$ = R.H.S

ix. $\dfrac{Sin3\theta}{Sin\theta} - \dfrac{Cos3\theta}{Cos\theta} = 2$

Sol. L.H.S = $\dfrac{Sin3\theta}{Sin\theta} - \dfrac{Cos3\theta}{Cos\theta}$

$= \dfrac{Sin3\theta Cos\theta - Cos3\theta Sin\theta}{Sin\theta Cos\theta} = \dfrac{Sin(3\theta - \theta)}{Sin\theta Cos\theta}$

$= \dfrac{Sin2\theta}{Sin\theta Cos\theta} = \dfrac{2\,\cancel{Sin\theta Cos\theta}}{\cancel{Sin\theta Cos\theta}} = 2$ = R.H.S

x. $\dfrac{Cos3\theta}{Cos\theta}+\dfrac{Sin3\theta}{Sin\theta}=4\,Cos2\theta$

Sol. $L.H.S=\dfrac{Cos3\theta}{Cos\theta}+\dfrac{Sin3\theta}{Sin\theta}=\dfrac{Sin\theta Cos3\theta+Cos\theta Sin3\theta}{Cos\theta Sin\theta}$

$=\dfrac{Sin(\theta+3\theta)}{Sin\theta Cos\theta}=\dfrac{2Sin4\theta}{2Sin\theta Cos\theta}=\dfrac{2Sin2(2\theta)}{Sin2\theta}$

$=\dfrac{2.2\,\cancel{Sin2\theta}\,Cos2\theta}{\cancel{Sin2\theta}}$

$=4\,Cos2\theta=R.H.S$

xi. $\dfrac{\tan\theta/2+Cot\theta/2}{Cot\theta/2-\tan\theta/2}=Sec\theta$

Sol. $L.H.S=\dfrac{\tan\theta/2+Cot\theta/2}{Cot\theta/2-\tan\theta/2}$

$=\dfrac{\dfrac{Sin\theta/2}{Cos\theta/2}+\dfrac{Cos\theta/2}{Sin\theta/2}}{\dfrac{Cos\theta/2}{Sin\theta/2}-\dfrac{Sin\theta/2}{Cos\theta/2}}$

$=\dfrac{\dfrac{Sin^2\theta/2+Cos^2\theta/2}{Sin\theta/2\,Cos\theta/2}}{\dfrac{Cos^2\theta/2-Sin^2\theta/2}{Sin\theta/2\,Cos\theta/2}}$

$=\dfrac{1}{\cancel{Cos\theta/2\,Sin\theta/2}}\times\dfrac{\cancel{Sin\theta/2\,Cos\theta/2}}{Cos^2\theta/2-Sin^2\theta/2}$

$=\dfrac{1}{Cos\theta}=Sec\theta=R.H.S$

xii. $\dfrac{Sin3\theta}{Cos\theta}+\dfrac{Cos3\theta}{Sin\theta}=2Cot2\theta$

Sol. $L.H.S=\dfrac{Sin3\theta}{Cos\theta}+\dfrac{Cos3\theta}{Sin\theta}$

$=\dfrac{Sin3\theta\,Sin\theta+Cos3\theta\,Cos\theta}{Sin\theta\,Cos\theta}$

$=\dfrac{Cos3\theta\,Cos\theta+Sin3\theta\,Sin\theta}{Sin\theta\,Cos\theta}$

$=\dfrac{2Cos(3\theta-\theta)}{2Sin\theta\,Cos\theta}=\dfrac{2Cos2\theta}{Sin2\theta}=2\,Cot\,2\theta$

xiii. $\dfrac{3+\cos4\theta}{1-\cos4\theta}=\dfrac{1}{2}\left(\tan^2\theta+\cot^2\theta\right)$

Sol: $R.H.S=\dfrac{1}{2}\left(\tan^2\theta+\cot^2\theta\right)$

$=\dfrac{1}{2}\left(\dfrac{\sin^2\theta}{\cos^2\theta}+\dfrac{\cos^2\theta}{\sin^2\theta}\right)=\dfrac{1}{2}\left(\dfrac{\sin^4\theta+\cos^4\theta}{\sin^2\theta\cos^2\theta}\right)=\dfrac{1}{2}\left(\dfrac{(\sin^2\theta)^2+(\cos^2\theta)^2}{\sin^2\theta\cos^2\theta}\right)\because\ \sin^2\theta=\dfrac{1-\cos2\theta}{2}$

$\cos^2\theta=\dfrac{1+\cos2\theta}{2}=\dfrac{1}{2}\left(\dfrac{\left(\dfrac{1-\cos2\theta}{2}\right)^2+\left(\dfrac{1+\cos2\theta}{2}\right)^2}{\sin^2\theta\cos^2\theta}\right)$

$=\dfrac{1}{2\sin^2\theta\cos^2\theta}\left(\dfrac{1+\cos^22\theta-2\cos2\theta}{4}+\dfrac{1+\cos^22\theta+2\cos2\theta}{4}\right)$

$=\dfrac{1}{2\sin^2\theta\cos^2\theta}\left(\dfrac{1+\cos^22\theta\cancel{-2\cos2\theta}+1+\cos^22\theta+\cancel{2\cos2\theta}}{4}\right)$

$=\dfrac{1}{2\sin^2\theta\cos^2\theta}\left(\dfrac{2+2\cos^22\theta}{4}\right)=\dfrac{1}{2\sin^2\theta\cos^2\theta}\left(\dfrac{\cancel{2}(1+\cos^22\theta)}{\cancel{4}_2}\right)$

$$=\frac{1}{4\sin^2\theta\cos^2\theta}\left(1+\left(\frac{1+\cos4\theta}{2}\right)\right)$$

$$=\frac{1}{4\sin^2\theta\cos^2\theta}\left(\frac{2+1+\cos4\theta}{2}\right)=\frac{1}{8\left(\frac{1-\cos2\theta}{2}\right)\left(\frac{1+\cos2\theta}{2}\right)}(3+\cos4\theta)$$

$$=\frac{3+\cos4\theta}{2(1-\cos^2 2\theta)}=\frac{3+\cos4\theta}{2\left(1-\left(\frac{1+\cos4\theta}{2}\right)\right)}=\frac{3+\cos4\theta}{\not{2}\left(\frac{2-1-\cos4\theta}{\not{2}}\right)}=\frac{3+\cos4\theta}{1-\cos4\theta}=\text{L.H.S}$$

xiv. $\mathbf{\frac{1+\sin2\theta}{1-\sin2\theta}=\tan^2\left(\frac{\pi}{4}+\theta\right)}$

Sol: $=\frac{1+\sin2\theta}{1-\sin2\theta}=\frac{1+\frac{2\tan\theta}{1+\tan^2\theta}}{1-\frac{2\tan\theta}{1+\tan^2\theta}}$ $\quad\because \sin2\theta=\frac{2\tan\theta}{1+\tan^2\theta}$

$$=\frac{\frac{1+\tan^2\theta+2\tan\theta}{1+\tan^2\theta}}{\frac{1+\tan^2\theta-2\tan\theta}{1+\tan^2\theta}}=\frac{1+\tan^2\theta+2\tan\theta}{\cancel{1+\tan^2\theta}}=\frac{\cancel{1+\tan^2\theta}}{1+\tan^2\theta-2\tan\theta}$$

$$=\frac{1+\tan^2\theta+2\tan\theta}{1+\tan^2\theta-2\tan\theta}=\frac{(1+\tan\theta)^2}{(1-\tan\theta)^2}=\left(\frac{1+\tan\theta}{1-\tan\theta}\right)^2=\left(\frac{\tan\frac{\pi}{4}+\tan\theta}{1-\tan\frac{\pi}{4}\tan\theta}\right)^2=\left(\tan\left(\frac{\pi}{4}+\theta\right)\right)^2$$

$$=\tan^2\left(\frac{\pi}{4}+\theta\right)=R.H.S$$

xv. $\mathbf{\cos^2\frac{\pi}{8}+\cos^2\frac{3\pi}{8}+\cos^2\frac{5\pi}{8}+\cos^2\frac{7\pi}{8}=2}$

Sol:

L.H.S $=\cos^2\frac{\pi}{8}+\cos^2\frac{3\pi}{8}+\cos^2\frac{5\pi}{8}+\cos^2\frac{7\pi}{8}$

$$=\left(\frac{1+\cos2\left(\frac{\pi}{8}\right)}{2}\right)+\left(\frac{1+\cos2\left(\frac{3\pi}{8}\right)}{2}\right)+\left(\frac{1+\cos2\left(\frac{5\pi}{8}\right)}{2}\right)+\left(\frac{1+\cos2\left(\frac{7\pi}{8}\right)}{2}\right)$$

$$=\frac{1+\cos\frac{\pi}{4}}{2}+\frac{1+\cos\frac{3\pi}{4}}{2}+\frac{1+\cos\frac{5\pi}{4}}{2}+\frac{1+\cos\frac{7\pi}{4}}{2}=\frac{1+\cos\frac{\pi}{4}+1+\cos\frac{3\pi}{4}+1+\cos\frac{5\pi}{4}+1+\cos\frac{7\pi}{4}}{2}$$

$$=\frac{1}{2}\left(1+\cancel{\frac{1}{\sqrt2}}+1-\cancel{\frac{1}{\sqrt2}}+1-\cancel{\frac{1}{\sqrt2}}+1+\cancel{\frac{1}{\sqrt2}}\right)$$

$\because \cos\frac{\pi}{4}=\cos45^\circ=\frac{1}{\sqrt2}$ $\quad\because \cos\frac{3\pi}{4}=\cos135^\circ=-\frac{1}{\sqrt2}$

$$=\frac{1}{2}(4)=2=R.H.S$$

$\because \cos\frac{5\pi}{4}=\cos225^\circ=-\frac{1}{\sqrt2}$ $\quad\because \cos\frac{7\pi}{4}=\cos315^\circ=\frac{1}{\sqrt2}$

Q3. Show that: $2\cos\theta = \sqrt{2+\sqrt{2+2\cos 4\theta}}$

Sol: $2\cos\theta = \sqrt{2+\sqrt{2+2\cos 4\theta}}$

R.H.S $= \sqrt{2+\sqrt{2+2\cos 4\theta}}$

$= \sqrt{2+\sqrt{2(1+\cos 4\theta)}} = \sqrt{2+\sqrt{2(2\cos^2 2\theta)}} = \sqrt{2+\sqrt{4\cos^2 2\theta}} = \sqrt{2+2\cos 2\theta} = \sqrt{2(1+\cos 2\theta)}$

$= \sqrt{2(2\cos^2\theta)} = \sqrt{4\cos^2\theta} = 2\cos\theta = L.H.S$

Q4. Reduce $\sin^4\theta$ to an expression involving only function of multiples of θ, raised to the first power.

Sol. $\sin^4\theta = (\sin^2\theta)^2 = \left(\frac{1-\cos 2\theta}{2}\right)^2$

$\frac{1-2\cos 2\theta+\cos^2 2\theta}{4} = \frac{1}{4}\left[1-2\cos 2\theta+\frac{1+\cos 4\theta}{2}\right]$ $\boxed{\sin^2\theta = \frac{1-\cos 2\theta}{2}}$

$=\frac{1}{4}\left[\frac{2-4\cos 2\theta+1+\cos 4\theta}{2}\right] = \frac{1}{8}\,[3-4\cos 2\theta+\cos 4\theta]$ $\boxed{\cos^2 2\theta = \frac{1+\cos 4\theta}{2}}$

$= \frac{3-4\cos 2\theta+\cos 4\theta}{8}$

Q5. Find the values of $\sin\theta$ and $\cos\theta$ without using table or calculator when θ is.

(i) 18° (ii) 36° (iii) 54° (iv) 72°

Hence prove that

$\cos 36^\circ \cos 72^\circ \cos 108^\circ \cos 144^\circ = \frac{1}{16}$

When $\theta = 18^\circ$ Multiply by 5

Sol. then $5\theta = 90^\circ \Rightarrow 2\theta + 3\theta = 90^\circ \Rightarrow 2\theta = 90^\circ - 3\theta$

$\sin(2\theta) = \sin(90^\circ - 3\theta) \Rightarrow 2\sin\theta\cos\theta = \cos 3\theta$

$2\sin\theta \cancel{\cos\theta} = 4\cos^3\theta - 3\cos\theta = \cancel{\cos\theta}\,(4\cos^2\theta - 3)$

$2\sin\theta = 4(1-\sin^2\theta) - 3 \Rightarrow 2\sin\theta = 4 - 4\sin^2\theta - 3$

$4\sin^2\theta + 2\sin\theta - 1 = 0$ $\quad a = 4, b = 2, c = -1$

$\sin\theta = \frac{-b\pm\sqrt{b^2-4ac}}{2a} = \frac{-2\pm\sqrt{(2)^2-4(4)(-1)}}{2(4)}$

$\sin\theta = \frac{-2\pm\sqrt{4+16}}{8} = \frac{-2\pm\sqrt{20}}{8} = \frac{-2\pm2\sqrt{5}}{8} = \frac{\cancel{2}(-1\pm\sqrt{5})}{\cancel{8}_4}$

$\sin\theta = \frac{-1\pm\sqrt{5}}{4}$ Put $\theta = 18^\circ$ then $\boxed{\sin 18^\circ = \frac{-1+\sqrt{5}}{4}}$ because 18° is in I quadrant

$\cos^2\theta = 1-\sin^2\theta = 1-\left(\frac{-1+\sqrt{5}}{4}\right)^2 = 1-\left(\frac{\sqrt{5}-1}{4}\right)^2 = 1-\left(\frac{5+1-2\sqrt{5}}{16}\right) = \frac{16-6+2\sqrt{5}}{16} = \frac{10+2\sqrt{5}}{16}$

$\cos^2\theta = \frac{10+2\sqrt{5}}{16} \Rightarrow \cos\theta = \pm\frac{\sqrt{10+2\sqrt{5}}}{4} \Rightarrow \boxed{\cos 18^\circ = \pm\frac{\sqrt{10+2\sqrt{5}}}{4}}$

$\cos 18^\circ = \frac{\sqrt{10+2\sqrt{5}}}{4}$ (Because 18° in I quad)

ii. **When θ = 36°**

Sol. then $\cos 2\theta = 2\cos^2\theta - 1$

Put $\theta = 18° \Rightarrow \cos 2(18°) = 2\cos^2(18°) - 1$

$$\cos 36° = 2\left(\frac{\sqrt{10+2\sqrt{5}}}{4}\right)^2 - 1 = 2\left(\frac{10+2\sqrt{5}}{16}\right) - 1 = \frac{10+2\sqrt{5}-8}{8} = \frac{2+2\sqrt{5}}{8} = \frac{2(1+\sqrt{5})}{8}$$

$$\boxed{\cos 36° = \frac{1+\sqrt{5}}{4}}, \quad \sin^2\theta = 1 - \cos^2\theta$$

$$\sin^2 36° = 1 - \cos^2 36°$$

$$\sin^2 36° = 1 - \left(\frac{1+\sqrt{5}}{4}\right)^2 = 1 - \left(\frac{1+5+2\sqrt{5}}{16}\right) = \frac{16-6-2\sqrt{5}}{16}$$

$$= \frac{10-2\sqrt{5}}{16} \Rightarrow \boxed{\sin 36° = \frac{\sqrt{10-2\sqrt{5}}}{4}}$$

iii. **When θ = 54°**

Sol. $\cos 54° = \sin(90° - 54°) = \sin 36° = \frac{10-2\sqrt{5}}{4}$

$\sin 54° = \cos(90° - 54°) = \cos 36° = \frac{1+\sqrt{5}}{4}$

iv. **When θ = 72°**

Sol. $\sin 72° = \sin(90° - 18°) = \cos 18° = \frac{\sqrt{10+2\sqrt{2}}}{4}$

$\cos 72° = \cos(90° - 18°) = \sin 18° = \frac{-1+\sqrt{5}}{4}$

ALSO $\cos 36° \cos 72° \cos 108° \cos 144° = \frac{1}{16}$

Sol. L.H.S = $\cos 36° \cos 72° \cos 108° \cos 144°$

$= \cos 36° \cos 72° \cos(180° - 72°)\cos(180° - 36°)$

$= \cos 36° \cos 72° (-\cos 72°)(-\cos 36°)$

$= \cos^2 36° \cos^2 72°$

$$= \left(\frac{1+\sqrt{5}}{4}\right)^2 \left(\frac{\sqrt{5}-1}{4}\right)^2$$

$$= \left(\frac{1+5+2\sqrt{5}}{16}\right)\left(\frac{5+1-2\sqrt{5}}{16}\right)$$

$$= \frac{(6+2\sqrt{5})(6-2\sqrt{5})}{16 \times 16} = \frac{(6)^2 - (2\sqrt{5})^2}{16 \times 16}$$

$$= \frac{36-(4\times 5)}{16\times 16} = \frac{36-20}{16\times 16} = \frac{16}{16\times 16} = \frac{1}{16} = \text{R.H.S}$$