



Exercise 10.4



Q1. Express the following products as sum or differences.

i. $2\sin 3\theta \cos \theta$

Sol. $= \sin(3\theta + \theta) + \sin(3\theta - \theta)$
 $= \sin 4\theta + \sin 2\theta$

iii. $\sin 5\theta \cos 2\theta$

Sol. $= \frac{1}{2}(2\sin 5\theta \cos 2\theta)$
 $= \frac{1}{2}[\sin(5\theta + 2\theta) + \sin(5\theta - 2\theta)]$
 $= \frac{1}{2}(\sin 7\theta + \sin 3\theta)$

ii. $2\cos 5\theta \sin \theta$

Sol. $= \sin(5\theta + \theta) - \sin(5\theta - \theta)$
 $= \sin 6\theta - \sin 4\theta$

iv. $2\sin 7\theta \sin 2\theta$

Sol. $= -(-2\sin 7\theta \sin 2\theta)$
 $= -[\cos(7\theta + 2\theta) - \cos(7\theta - 2\theta)]$
 $= -(\cos 9\theta - \cos 5\theta)$
 $= \cos 5\theta - \cos 9\theta$

v. $\cos(x + y) \sin(x - y)$

Sol. $= \frac{1}{2}(2\cos(x + y) \sin(x - y))$
 $= \frac{1}{2}(\sin((x + y) + (x - y)) - \sin(x + y) - (x - y))$
 $= \frac{1}{2}(\sin(x + \cancel{y} + x - \cancel{y}) - \sin(\cancel{x} + y - \cancel{x} + y))$
 $= \frac{1}{2}(\sin 2x - \sin 2y)$



vi. $\text{Cos}(2x + 30^\circ) \text{Cos}(2x - 30^\circ)$

Sol. $= \frac{1}{2} (2 \text{Cos}(2x + 30^\circ) \text{Cos}(2x - 30^\circ))$
 $= \frac{1}{2} [\text{Cos}(2x + 30^\circ + 2x - 30^\circ) + \text{Cos}((2x + 30^\circ) - (2x - 30^\circ))]$
 $= \frac{1}{2} [\text{Cos}(2x + 30^\circ + 2x - 30^\circ) + \text{Cos}(2x + 30^\circ - 2x + 30^\circ)]$
 $= \frac{1}{2} (\text{Cos}4x + \text{Cos}60^\circ)$

vii. $\text{Sin}12^\circ \text{Sin}46^\circ$

Sol. $= \frac{-1}{2} (-2 \text{Sin}12^\circ \text{Sin}46^\circ) = -\frac{1}{2} (\text{Cos}(12^\circ + 46^\circ) - \text{Cos}(12^\circ - 46^\circ))$
 $= -\frac{1}{2} (\text{Cos}58^\circ - \text{Cos}(-34^\circ)) = -\frac{1}{2} (\text{Cos}58^\circ - \text{Cos}34^\circ)$

viii. $\text{Sin}(x + 45^\circ) \text{Sin}(x - 45^\circ)$

Sol. $= -\frac{1}{2} (-2 \text{Sin}(x + 45^\circ) \text{Sin}(x - 45^\circ))$
 $= -\frac{1}{2} [\text{Cos}[(x + 45^\circ) + (x - 45^\circ)] - \text{Cos}[(x + 45^\circ) - (x - 45^\circ)]]$
 $= -\frac{1}{2} [\text{Cos}(x + 45^\circ + x - 45^\circ) - \text{Cos}(x + 45^\circ - x + 45^\circ)]$
 $= -\frac{1}{2} [\text{Cos}2x - \text{Cos}90^\circ] = \frac{1}{2} (-\text{Cos}2x + \text{Cos}90^\circ)$
 $= \frac{1}{2} (\text{Cos}90^\circ - \text{Cos}2x)$

Q2. Express the following sums or differences as products

i. $\text{Sin}5\theta + \text{Sin}3\theta$

Sol. $= 2 \text{Sin} \frac{5\theta + 3\theta}{2} \text{Cos} \frac{5\theta - 3\theta}{2} = 2 \text{Sin} 4\theta \text{Cos} \theta$

ii. $\text{Sin}8\theta - \text{Sin}4\theta$

Sol. $= 2 \text{Cos} \frac{8\theta + 4\theta}{2} \text{Sin} \frac{8\theta - 4\theta}{2} = 2 \text{Cos} 6\theta \text{Sin} 2\theta$

iii. $\text{Cos}6\theta + \text{Cos}3\theta$

Sol. $= 2 \text{Cos} \frac{6\theta + 3\theta}{2} \text{Cos} \frac{6\theta - 3\theta}{2} = 2 \text{Cos} \frac{9\theta}{2} \text{Cos} \frac{3\theta}{2}$

iv. $\text{Cos}7\theta - \text{Cos}\theta$

Sol. $= -2 \text{Sin} \frac{7\theta + \theta}{2} \text{Sin} \frac{7\theta - \theta}{2} = -2 \text{Sin} 4\theta \text{Sin} 3\theta$

v. $\text{Cos}12^\circ + \text{Cos}48^\circ$

Sol. $= 2 \text{Cos} \frac{12^\circ + 48^\circ}{2} \text{Cos} \frac{12^\circ - 48^\circ}{2}$

$= 2 \text{Cos} \frac{60^\circ}{2} \text{Cos} \frac{(-36^\circ)}{2}$ $\text{Cos}(-\theta) = \text{Cos}\theta$

$= 2 \text{Cos} 30^\circ \text{Cos}(18^\circ)$

vi. $\sin(x + 30^\circ) + \sin(x - 30^\circ)$

Sol. $= 2\sin\left(\frac{x + 30^\circ + x - 30^\circ}{2}\right) \cos\frac{(x + 30^\circ) - (x - 30^\circ)}{2}$
 $= 2\sin\left(\frac{2x}{2}\right) \cos\left(\frac{x + 30^\circ - x + 30^\circ}{2}\right) = 2\sin x \cos 30^\circ$

Q3. Prove the following identities

i. $\frac{\sin 3x - \sin x}{\cos x - \cos 3x} = \cot 2x$

Sol. L.H.S. $= \frac{\sin 3x - \sin x}{\cos x - \cos 3x}$
 $= \frac{2\cos\frac{3x+x}{2} \sin\frac{3x-x}{2}}{-2\sin\frac{3x+x}{2} \sin\frac{x-3x}{2}} = \frac{\cos 2x \sin x}{-\sin 2x \sin(-x)} = \frac{\cos 2x \sin x}{-\sin 2x (-\sin x)} = \frac{\cos 2x \sin x}{\sin 2x \sin x}$
 $= \cot 2x = \text{R.H.S}$

ii. $\frac{\sin 8x + \sin 2x}{\cos 8x + \cos 2x} = \tan 5x$

Sol. L.H.S. $= \frac{\sin 8x + \sin 2x}{\cos 8x + \cos 2x} = \frac{2\sin\frac{8x+2x}{2} \cos\frac{8x-2x}{2}}{2\cos\frac{8x+2x}{2} \cos\frac{8x-2x}{2}}$
 $= \frac{\sin 5x \cos 3x}{\cos 5x \cos 3x} = \tan 5x = \text{R.H.S}$

iii. $\frac{\sin A - \sin B}{\sin A + \sin B} = \tan \frac{A - B}{2} \cot \frac{A + B}{2}$

Sol. L.H.S. $= \frac{\sin A - \sin B}{\sin A + \sin B}$
 $= \frac{\cancel{2} \cos\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right)}{\cancel{2} \sin\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)} = \frac{\cos\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right)}{\sin\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)}$
 $= \cot\left(\frac{A+B}{2}\right) \tan\left(\frac{A-B}{2}\right) = \tan\left(\frac{A-B}{2}\right) \cot\left(\frac{A+B}{2}\right)$

iv. $\frac{\sin 80^\circ + \sin 40^\circ}{\cos 80^\circ + \cos 40^\circ} = \sqrt{3}$

Sol: L.H.S. $= \frac{\sin 80^\circ + \sin 40^\circ}{\cos 80^\circ + \cos 40^\circ}$
 $= \frac{\cancel{2} \sin\left(\frac{80^\circ + 40^\circ}{2}\right) \cos\left(\frac{80^\circ - 40^\circ}{2}\right)}{\cancel{2} \cos\left(\frac{80^\circ + 40^\circ}{2}\right) \cos\left(\frac{80^\circ - 40^\circ}{2}\right)} = \frac{\sin 60^\circ \cancel{\cos 20^\circ}}{\cos 60^\circ \cancel{\cos 20^\circ}} = \frac{\sin 60^\circ}{\cos 60^\circ} = \tan 60^\circ = \sqrt{3} = \text{R.H.S}$

Q4. Prove that:

i. $\cos 15^\circ + \cos 105^\circ + \cos 195^\circ + \cos 285^\circ = 0$

Sol:

L.H.S

$$\begin{aligned} & \cos 15^\circ + \cos 105^\circ + \cos 195^\circ + \cos 285^\circ \\ &= \cos 15^\circ + \cos(90^\circ + 15^\circ) + \cos(180^\circ + 15^\circ) + \cos(270^\circ + 15^\circ) = \cancel{\cos 15^\circ} - \cancel{\sin 15^\circ} - \cancel{\cos 15^\circ} + \cancel{\sin 15^\circ} \\ &= 0 = R.H.S \end{aligned}$$

ii.

$$\frac{\sin 2\theta + \sin 4\theta + \sin 6\theta + \sin 8\theta}{\cos 2\theta + \cos 4\theta + \cos 6\theta + \cos 8\theta} = \tan 5\theta$$

Sol:

$$\begin{aligned} \text{L.H.S} &= \frac{\sin 2\theta + \sin 4\theta + \sin 6\theta + \sin 8\theta}{\cos 2\theta + \cos 4\theta + \cos 6\theta + \cos 8\theta} \\ &= \frac{2 \sin \left(\frac{2\theta + 4\theta}{2} \right) \cos \left(\frac{2\theta - 4\theta}{2} \right) + 2 \sin \left(\frac{6\theta + 8\theta}{2} \right) \cos \left(\frac{6\theta - 8\theta}{2} \right)}{2 \cos \left(\frac{2\theta + 4\theta}{2} \right) \cos \left(\frac{2\theta - 4\theta}{2} \right) + 2 \cos \left(\frac{6\theta + 8\theta}{2} \right) \cos \left(\frac{6\theta - 8\theta}{2} \right)} \\ &= \frac{2 \sin \left(\frac{6\theta}{2} \right) \cos \left(\frac{-2\theta}{2} \right) + 2 \sin \left(\frac{14\theta}{2} \right) \cos \left(\frac{-2\theta}{2} \right)}{2 \cos \left(\frac{6\theta}{2} \right) \cos \left(\frac{-2\theta}{2} \right) + 2 \cos \left(\frac{14\theta}{2} \right) \cos \left(\frac{-2\theta}{2} \right)} = \frac{2 \sin 3\theta \cos \theta + 2 \sin 7\theta \cos \theta}{2 \cos 3\theta \cos \theta + 2 \cos 7\theta \cos \theta} \\ &= \frac{\cancel{2} \cos \theta (\sin 3\theta + \sin 7\theta)}{\cancel{2} \cos \theta [\cos 3\theta + \cos 7\theta]} = \frac{\cancel{2} \sin 5\theta \cos(-2\theta)}{\cancel{2} \cos 5\theta \cos(-2\theta)} \\ &= \frac{\sin 5\theta}{\cos 5\theta} = \tan 5\theta = R.H.S \end{aligned}$$

iii.

$$\cos^2 \left(\frac{\pi}{4} - \frac{\alpha}{2} \right) - \cos^2 \left(\frac{\pi}{4} + \frac{\alpha}{2} \right) = \sin \alpha$$

Sol:

$$\begin{aligned} \text{L.H.S} &= \cos^2 \left(\frac{\pi}{4} - \frac{\alpha}{2} \right) - \cos^2 \left(\frac{\pi}{4} + \frac{\alpha}{2} \right) \\ &= \left[\cos \left(\frac{\pi}{4} - \frac{\alpha}{2} \right) - \cos \left(\frac{\pi}{4} + \frac{\alpha}{2} \right) \right] \left[\cos \left(\frac{\pi}{4} - \frac{\alpha}{2} \right) + \cos \left(\frac{\pi}{4} + \frac{\alpha}{2} \right) \right] \\ &= \left[-2 \sin \left(\frac{\frac{\pi}{4} - \frac{\alpha}{2} + \frac{\pi}{4} + \frac{\alpha}{2}}{2} \right) \sin \left(\frac{\frac{\pi}{4} - \frac{\alpha}{2} - \frac{\pi}{4} - \frac{\alpha}{2}}{2} \right) \right] \left[2 \cos \left(\frac{\frac{\pi}{4} - \frac{\alpha}{2} + \frac{\pi}{4} + \frac{\alpha}{2}}{2} \right) \cos \left(\frac{\frac{\pi}{4} - \frac{\alpha}{2} - \frac{\pi}{4} - \frac{\alpha}{2}}{2} \right) \right] \\ &= \left[-2 \sin \left(\frac{2\pi}{4} \right) \sin \left(\frac{-2\alpha}{2} \right) \right] \left[2 \cos \left(\frac{2\pi}{4} \right) \cos \left(\frac{-2\alpha}{2} \right) \right] = \left[-2 \sin \left(\frac{\pi}{4} \right) \sin \left(\frac{-\alpha}{2} \right) \right] \left[2 \cos \left(\frac{\pi}{4} \right) \cos \left(-\frac{\alpha}{2} \right) \right] \\ &= \left[2 \cdot \frac{1}{\sqrt{2}} \cdot \sin \frac{\alpha}{2} \right] \left[2 \cdot \frac{1}{\sqrt{2}} \cdot \cos \frac{\alpha}{2} \right] = \frac{4}{2} \sin \frac{\alpha}{2} \cos \frac{\alpha}{2} \\ &= 2 \sin \frac{\alpha}{2} \cos \frac{\alpha}{2} = \sin \alpha = R.H.S \end{aligned}$$

Note:

Wrong statement is given. It cannot be solved with $\cos 160^\circ$

$$\text{iv. } \sin\left(\frac{\pi}{4} - \theta\right) \sin\left(\frac{\pi}{4} + \theta\right) = \frac{1}{2} \cos 2\theta$$

$$\begin{aligned} \text{Sol. L.H.S} &= \sin\left(\frac{\pi}{4} - \theta\right) \sin\left(\frac{\pi}{4} + \theta\right) \\ &= -\frac{1}{2} (-2\sin\left(\frac{\pi}{4} - \theta\right) \sin\left(\frac{\pi}{4} + \theta\right)) \\ &= -\frac{1}{2} [\cos\left(\frac{\pi}{4} - \theta + \frac{\pi}{4} + \theta\right) - \cos\left[\left(\frac{\pi}{4} - \theta\right) - \left(\frac{\pi}{4} + \theta\right)\right]] \\ &= -\frac{1}{2} [\cos\left(2 \times \frac{\pi}{4}\right) - \cos\left(\frac{\pi}{4} - \theta - \frac{\pi}{4} - \theta\right)] \\ &= -\frac{1}{2} [\cos \frac{\pi}{2} - \cos(-2\theta)] \\ &= -\frac{1}{2} [0 - \cos 2\theta] = \frac{1}{2} \cos 2\theta = \text{R.H.S} \end{aligned}$$

$$\text{v. } \frac{\sin\theta + \sin 3\theta + \sin 5\theta + \sin 7\theta}{\cos\theta + \cos 3\theta + \cos 5\theta + \cos 7\theta} = \tan 4\theta$$

$$\begin{aligned} \text{Sol. L.H.S} &= \frac{\sin\theta + \sin 3\theta + \sin 5\theta + \sin 7\theta}{\cos\theta + \cos 3\theta + \cos 5\theta + \cos 7\theta} = \frac{\sin 7\theta + \sin\theta + \sin 5\theta + \sin 3\theta}{\cos 7\theta + \cos\theta + \cos 5\theta + \cos 3\theta} \\ &= \frac{2\sin \frac{7\theta + \theta}{2} \cos \frac{7\theta - \theta}{2} + 2\sin \frac{5\theta + 3\theta}{2} \cos \frac{5\theta - 3\theta}{2}}{2\cos \frac{7\theta + \theta}{2} \cos \frac{7\theta - \theta}{2} + 2\cos \frac{5\theta + 3\theta}{2} \cos \frac{5\theta - 3\theta}{2}} \\ &= \frac{2\sin 4\theta \cdot \cos 3\theta + 2\sin 4\theta \cdot \cos\theta}{2\cos 4\theta \cdot \cos 3\theta + 2\cos 4\theta \cdot \cos\theta} = \frac{\cancel{2}\sin 4\theta [\cancel{\cos 3\theta} + \cos\theta]}{\cancel{2}\cos 4\theta [\cancel{\cos 3\theta} + \cos\theta]} = \frac{\sin 4\theta}{\cos 4\theta} \\ &= \tan 4\theta = \text{R.H.S} \end{aligned}$$

5. Prove that

$$\text{i } \cos 20^\circ \cos 40^\circ \cos 60^\circ \cos 80^\circ = \frac{1}{16}$$

$$\begin{aligned} \text{Sol. L.H.S} &= \cos 20^\circ \cos 40^\circ \cos 60^\circ \cos 80^\circ \\ &= \cos 20^\circ \cos 40^\circ \left(\frac{1}{2}\right) \cos 80^\circ \\ &= \frac{1}{2} (\cos 20^\circ \cos 40^\circ) \cos 80^\circ \\ &= \frac{1}{4} (2\cos 20^\circ \cos 40^\circ) \cos 80^\circ \quad ('x' \& ' \div ' \text{ by } 2) \\ &= \frac{1}{4} (\cos(20^\circ + 40^\circ) + \cos(20^\circ - 40^\circ)) \cos 80^\circ \\ &= \frac{1}{4} [(\cos(20^\circ + 40^\circ)) \cos 80^\circ + (\cos(20^\circ - 40^\circ)) \cos 80^\circ] \\ &= \frac{1}{4} (\cos 60^\circ \cos 80^\circ + \cos 20^\circ \cos 80^\circ) \quad (\cos(-20^\circ) = \cos 20^\circ) \end{aligned}$$

$$\begin{aligned}
&= \frac{1}{4} \left(\frac{1}{2} \cos 80^\circ + \cos 20^\circ \cos 80^\circ \right) = \frac{1}{4} \left(\frac{\cos 80^\circ + 2 \cos 20^\circ \cos 80^\circ}{2} \right) \\
&= \frac{1}{8} (\cos 80^\circ + \cos (20^\circ + 80^\circ) + \cos (20^\circ - 80^\circ)) \\
&= \frac{1}{8} (\cos (180^\circ - 100^\circ) + \cos (100^\circ) + \cos (-60^\circ)) \\
&= \frac{1}{8} [\cos 180^\circ \cos 100^\circ + \sin 180^\circ \sin 100^\circ + \cos 100^\circ + \cos 60^\circ] \\
&= \frac{1}{8} [(-1) \cos 100^\circ + (0) \sin 100^\circ + \cos 100^\circ + \frac{1}{2}] \\
&= \frac{1}{8} \left[-\cancel{\cos 100^\circ} + \cancel{\cos 100^\circ} + \frac{1}{2} \right] \\
&= \frac{1}{8} \left(\frac{1}{2} \right) = \frac{1}{16} = R.H.S
\end{aligned}$$

ii. $\sin \frac{\pi}{9} \sin \frac{2\pi}{9} \sin \frac{\pi}{3} \sin \frac{4\pi}{9} = \frac{3}{16}$

Sol. L.H.S = $\sin \frac{\pi}{9} \sin \frac{2\pi}{9} \sin \frac{\pi}{3} \sin \frac{4\pi}{9}$

$$\begin{aligned}
&= \sin 20^\circ \sin 40^\circ \sin 60^\circ \sin 80^\circ \\
&= \sin 20^\circ \sin 40^\circ \frac{\sqrt{3}}{2} \sin 80^\circ \\
&= \frac{\sqrt{3}}{2} (\sin 20^\circ \sin 40^\circ) \sin 80^\circ = -\frac{\sqrt{3}}{4} (-2 \sin 20^\circ \sin 40^\circ) \sin 80^\circ \\
&= -\frac{\sqrt{3}}{4} [\cos (20^\circ + 40^\circ) - \cos (20^\circ - 40^\circ)] \sin 80^\circ \\
&= \frac{-\sqrt{3}}{4} [\cos 60^\circ - \cos (-20^\circ)] \sin 80^\circ = \frac{-\sqrt{3}}{4} \left(\frac{1}{2} - \cos 20^\circ \right) \sin 80^\circ \\
&= -\frac{\sqrt{3}}{4} \left[\frac{1}{2} \sin 80^\circ - \cos 20^\circ \sin 80^\circ \right] \\
&= -\frac{\sqrt{3}}{4} \left[\frac{\sin 80^\circ - 2 \cos 20^\circ \sin 80^\circ}{2} \right] \\
&= -\frac{\sqrt{3}}{8} \{ \sin (180^\circ - 100^\circ) - [\sin (20^\circ + 80^\circ) - \sin (20^\circ - 80^\circ)] \} \\
&= -\frac{\sqrt{3}}{8} [\cancel{\sin 100^\circ} - \cancel{\sin 100^\circ} + \sin (-60^\circ)] \\
&= -\frac{\sqrt{3}}{8} (-\sin 60^\circ) = -\frac{\sqrt{3}}{8} \left(-\frac{\sqrt{3}}{2} \right) \\
&= \frac{3}{16} = R.H.S
\end{aligned}$$

$$\text{iii. } \sin 10^\circ \sin 30^\circ \sin 50^\circ \sin 70^\circ = \frac{1}{16}$$

$$\begin{aligned} \text{Sol. L.H.S} &= \sin 10^\circ \sin 30^\circ \sin 50^\circ \sin 70^\circ \\ &= \sin 10^\circ \left(\frac{1}{2}\right) \sin 50^\circ \sin 70^\circ = \frac{1}{2} (\sin 10^\circ \sin 50^\circ) \sin 70^\circ \\ &= -\frac{1}{4} (-2 \sin 10^\circ \sin 50^\circ) \sin 70^\circ = \frac{1}{4} [\cos (10^\circ + 50^\circ) - \cos (10^\circ - 50^\circ)] \sin 70^\circ \\ &= -\frac{1}{4} [\cos 60^\circ - \cos(-40^\circ)] \sin 70^\circ = -\frac{1}{4} [\cos 60^\circ \sin 70^\circ - \cos 40^\circ \sin 70^\circ] \\ &= -\frac{1}{4} \left[\frac{1}{2} \sin 70^\circ - \cos 40^\circ \sin 70^\circ \right] = -\frac{1}{4} \left(\frac{\sin 70^\circ - 2 \cos 40^\circ \sin 70^\circ}{2} \right) \\ &= -\frac{1}{8} [\sin (70^\circ) - \{(\sin (40^\circ + 70^\circ) - \sin (40^\circ - 70^\circ))\}] = -\frac{1}{8} [\sin (180^\circ - 110^\circ) - \sin 110^\circ + \sin (-30^\circ)] \\ &= -\frac{1}{8} [\cancel{\sin 110^\circ} - \cancel{\sin 110^\circ} - \sin 30^\circ] = -\frac{1}{8} \left[-\frac{1}{2} \right] = \frac{1}{16} = R.H.S \end{aligned}$$

Q6. Prove that $\frac{\sin 3\theta}{1 + 2 \cos 2\theta} = \sin \theta$, and deduce the value of $\sin 15^\circ$.

Sol:

$$\begin{aligned} \text{L.H.S} &= \frac{\sin 3\theta}{1 + 2 \cos 2\theta} = \frac{3 \sin \theta - 4 \sin^3 \theta}{1 + 2(1 - 2 \sin^2 \theta)} = \frac{\sin \theta (3 - 4 \sin^2 \theta)}{1 + 2 - 4 \sin^2 \theta} \\ &= \frac{\sin \theta (3 - 4 \sin^2 \theta)}{(3 - 4 \sin^2 \theta)} \sin \theta = R.H.S \end{aligned}$$

To deduce $\sin 15^\circ$

$$\begin{aligned} \sin 15^\circ &= \frac{\sin (3 \times 15^\circ)}{1 + 2 \cos (2 \times 15^\circ)} \quad (\text{put } \theta = 15^\circ \text{ in given question}) \\ &= \frac{\sin (45^\circ)}{1 + 2 \cos 30^\circ} = \frac{\frac{1}{\sqrt{2}}}{1 + 2 \cdot \frac{\sqrt{3}}{2}} = \frac{\frac{1}{\sqrt{2}}}{1 + \sqrt{3}} = \frac{1}{\sqrt{2} + \sqrt{6}} = \frac{\sqrt{2} - \sqrt{6}}{(\sqrt{2} + \sqrt{6})(\sqrt{2} - \sqrt{6})} \\ &= \frac{\sqrt{2} - \sqrt{6}}{(\sqrt{2})^2 - (\sqrt{6})^2} = \frac{\sqrt{2} - \sqrt{6}}{2 - 6} = \frac{\sqrt{2} - \sqrt{6}}{-4} = \frac{\sqrt{6} - \sqrt{2}}{4} \end{aligned}$$

Q7. Prove that $\tan 75^\circ - \tan 15^\circ = 2\sqrt{3}$

$$\begin{aligned} \text{Sol: L.H.S} &= \tan 75^\circ - \tan 15^\circ \\ &= \tan (45^\circ + 30^\circ) - \tan (45^\circ - 30^\circ) \\ &= \frac{\tan 45^\circ + \tan 30^\circ}{1 - \tan 45^\circ \tan 30^\circ} - \frac{\tan 45^\circ - \tan 30^\circ}{1 + \tan 45^\circ \tan 30^\circ} \\ &= \frac{1 + \tan 30^\circ}{1 - \tan 30^\circ} - \frac{1 - \tan 30^\circ}{1 + \tan 30^\circ} \end{aligned}$$

$$\begin{aligned}
&= \frac{1 + \frac{1}{\sqrt{3}}}{1 - \frac{1}{\sqrt{3}}} - \frac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}} = \frac{\frac{\sqrt{3}+1}{\sqrt{3}} - \frac{\sqrt{3}-1}{\sqrt{3}}}{\frac{\sqrt{3}-1}{\sqrt{3}} + \frac{\sqrt{3}+1}{\sqrt{3}}} = \frac{\sqrt{3}+1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}-1} - \frac{\sqrt{3}-1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}+1} \\
&= \frac{\sqrt{3}+1}{\sqrt{3}-1} - \frac{\sqrt{3}-1}{\sqrt{3}+1} = \frac{(\sqrt{3}+1)^2 - (\sqrt{3}-1)^2}{(\sqrt{3}-1)(\sqrt{3}+1)} = \frac{(3+1+2\sqrt{3}) - (3+1-2\sqrt{3})}{(\sqrt{3})^2 - (1)^2} \\
&= \frac{4 + 2\sqrt{3} - 4 + 2\sqrt{3}}{3-1} = \frac{4\sqrt{3}}{2} = 2\sqrt{3} = R.H.S
\end{aligned}$$

Q8. Prove that $\cos 15^\circ - \sin 15^\circ = \frac{1}{\sqrt{2}}$

Sol: L.H.S = $\cos 15^\circ - \sin 15^\circ$
 $= \cos(45^\circ - 30^\circ) - \sin(45^\circ - 30^\circ)$
 $= [\cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ] - [\sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ]$
 $= \left[\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \cdot \frac{1}{2} \right] - \left[\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \cdot \frac{1}{2} \right] = \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} - \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}} = R.H.S$

Q9. Prove that $\frac{\sin^2 \alpha - \sin^2 \beta}{\sin \alpha \cos \alpha - \sin \beta \cos \beta} = \tan(\alpha + \beta)$

Sol: R.H.S = $\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta} = \frac{\frac{\sin \alpha}{\cos \alpha} + \frac{\sin \beta}{\cos \beta}}{1 - \frac{\sin \alpha \sin \beta}{\cos \alpha \cos \beta}}$
 $= \frac{\frac{\sin \alpha \cos \beta + \cos \alpha \sin \beta}{\cos \alpha \cos \beta}}{\frac{\cos \alpha \cos \beta - \sin \alpha \sin \beta}{\cos \alpha \cos \beta}} = \frac{\sin \alpha \cos \beta + \cos \alpha \sin \beta}{\cos \alpha \cos \beta - \sin \alpha \sin \beta} \times \frac{\sin \alpha \cos \beta - \cos \alpha \sin \beta}{\sin \alpha \cos \beta - \cos \alpha \sin \beta}$
 $= \frac{\sin^2 \alpha \cos^2 \beta - \cos^2 \alpha \sin^2 \beta}{\cos \alpha \cos \beta \sin \alpha \cos \beta - \cos \alpha \cos \beta \cos \alpha \sin \beta - \sin \alpha \sin \beta \sin \alpha \cos \beta + \sin \alpha \sin \beta \cos \alpha \sin \beta}$
 $= \frac{\sin^2 \alpha (1 - \sin^2 \beta) - (1 - \sin^2 \alpha) \sin^2 \beta}{\sin \alpha \cos \alpha \cos^2 \beta - \cos^2 \alpha \cos \beta \sin \beta - \sin^2 \alpha \sin \beta \cos \beta + \sin \alpha \cos \alpha \sin^2 \beta}$
 $= \frac{\sin^2 \alpha - \sin^2 \alpha \sin^2 \beta - \sin^2 \beta + \sin^2 \alpha \sin^2 \beta}{\sin \alpha \cos \alpha (1 - \sin^2 \beta) - (1 - \sin^2 \alpha) \cos \beta \sin \beta - \sin^2 \alpha \sin \beta \cos \beta + \sin \alpha \cos \alpha \sin^2 \beta}$
 $= \frac{\sin^2 \alpha - \sin^2 \beta}{\sin \alpha \cos \alpha - \sin \alpha \cos \alpha \sin^2 \beta - \cos \beta \sin \beta + \sin^2 \alpha \sin \beta \cos \beta - \sin^2 \alpha \sin \beta \cos \beta + \sin^2 \beta \sin \alpha \cos \alpha}$
 $= \frac{\sin^2 \alpha - \sin^2 \beta}{\sin \alpha \cos \alpha - \sin \beta \cos \beta} = L.H.S$

Q10. Prove that $\sin \alpha + \sin \beta + \sin \gamma - \sin(\alpha + \beta + \gamma) = 4 \sin\left(\frac{\alpha + \beta}{2}\right) \sin\left(\frac{\beta + \gamma}{2}\right) \sin\left(\frac{\gamma + \alpha}{2}\right)$

Sol: L.H.S = $\sin \alpha + \sin \beta + \sin \gamma - \sin(\alpha + \beta + \gamma)$

$$= (\sin \alpha + \sin \beta) + (\sin \gamma - \sin(\alpha + \beta + \gamma))$$

$$= 2 \sin\left(\frac{\alpha + \beta}{2}\right) \cos\left(\frac{\alpha - \beta}{2}\right) + 2 \cos\left(\frac{\gamma + \alpha + \beta + \gamma}{2}\right) \sin\left(\frac{\gamma - \alpha - \beta - \gamma}{2}\right)$$

$$= 2 \sin\left(\frac{\alpha + \beta}{2}\right) \cos\left(\frac{\alpha - \beta}{2}\right) + 2 \cos\left(\frac{\alpha + \beta + 2\gamma}{2}\right) \left(-\sin\left(\frac{\alpha + \beta}{2}\right)\right)$$

$$= 2 \sin\left(\frac{\alpha + \beta}{2}\right) \left[\cos\left(\frac{\alpha - \beta}{2}\right) - \cos\left(\frac{\alpha + \beta + 2\gamma}{2}\right) \right]$$

$$= 2 \sin\left(\frac{\alpha + \beta}{2}\right) \left\{ -2 \sin\left(\frac{\alpha - \beta}{2} + \frac{\alpha + \beta + 2\gamma}{2}\right) \sin\left(\frac{\alpha - \beta}{2} - \frac{\alpha + \beta + 2\gamma}{2}\right) \right\}$$

$$= -4 \sin\left(\frac{\alpha + \beta}{2}\right) \sin\left(\frac{\alpha - \beta + \alpha + \beta + 2\gamma}{2 \times 2}\right) \sin\left(\frac{\alpha - \beta - \alpha - \beta - 2\gamma}{2 \times 2}\right)$$

$$= -4 \sin\left(\frac{\alpha + \beta}{2}\right) \sin\left(\frac{2\alpha + 2\gamma}{2 \times 2}\right) \sin\left(\frac{-2\beta - 2\gamma}{2 \times 2}\right)$$

$$= -4 \sin\left(\frac{\alpha + \beta}{2}\right) \sin\left(\frac{\alpha + \gamma}{2}\right) \sin\left(\frac{-(\beta + \gamma)}{2}\right)$$

$$= 4 \sin\left(\frac{\alpha + \beta}{2}\right) \sin\left(\frac{\beta + \gamma}{2}\right) \sin\left(\frac{\gamma + \alpha}{2}\right)$$