Q1. Find the maximum and minimum values of the following functions.

i. $3-\sin 3x$

Sol. $f(x)=3-\sin 3x$

Here $a=3, \quad b=-1$

Maximum value of $f(x)=a+|b|=3+|(-1)|=3+1=4$

Minimum value of $f(x)=a-|b|=3-|(-1)|=3-1=2$

ii. $3+\sin 2x$

Sol. $f(x)=3+\sin 2x$

Here $a=3, \quad b=1$

Maximum value of $f(x)=a+|b|=3+|1|=3+1=4$

Minimum value of $f(x)=a-|b|=3-|1|=3-1=2$

iii. $\frac{1}{2}+\sin(5x+\pi)$

Sol. $f(x)=\frac{1}{2}+\sin(5x+\pi)$

Here $a=\frac{1}{2}, \quad b=1$

Maximum value of $f(x)=a+|b|=\frac{1}{2}+|1|=\frac{1}{2}+1=\frac{3}{2}$

Minimum value of $f(x)=a-|b|=\frac{1}{2}-|1|=\frac{1}{2}-1=-\frac{1}{2}$

iv. $\frac{3}{2}+\cos\left(x-\frac{\pi}{4}\right)$

Sol. $f(x)=\frac{3}{2}+\cos\left(x-\frac{\pi}{4}\right)$ [illegible]

Here $a=\frac{3}{2}, \quad b=1$ [illegible]

Maximum value of $f(x)=a+|b|=\frac{3}{2}+|1|=\frac{3}{2}+1=\frac{5}{2}$

Minimum value of $f(x)=a-|b|=\frac{3}{2}-|1|=\frac{3}{2}-1=\frac{1}{2}$

v. $1-3\cos 2x$

Sol. $f(x)=1-3\cos 2x$

Here $a=1, \quad b=-3$

Maximum value of $f(x)=a+|b|=1+|-3|=1+3=4$

Minimum value of $f(x)=a-|b|=1-|-3|=1-3=-2$

vi. $1+2\sin\left(x+\frac{\pi}{6}\right)$

Sol. $f(x)=1+2\sin\left(x+\frac{\pi}{6}\right)$

Here $a=1, \quad b=2$

Maximum value of $f(x)=a+|b|=1+|2|$

$=1+2=3$

Minimum value of $f(x)=a-|b|=1-|2|$

$=1-2=-1$

vii. $\frac{1}{10-2\sin 3x}$

Sol. $g(x)=\frac{1}{10-2\sin 3x}$

Here $a=10, \quad b=-2$

Maximum value of $g(x)=\frac{1}{a-|b|}=\frac{1}{10-|-2|}=\frac{1}{10-2}=\frac{1}{8}$

Minimum value of $g(x)=\frac{1}{a+|b|}=\frac{1}{10+|-2|}=\frac{1}{10+2}=\frac{1}{12}$

viii. $\dfrac{1}{7+3\cos(-2x)}$

Sol. $g(x)=\dfrac{1}{7+3\cos(-2x)}$

Here $a=7,\quad b=3$

Maximum value of $g(x)=\dfrac{1}{a-|b|}=\dfrac{1}{7-|3|}=\dfrac{1}{7-3}=\dfrac{1}{4}$

Minimum value of $g(x)=\dfrac{1}{a+|b|}=\dfrac{1}{7+|3|}=\dfrac{1}{7+3}=\dfrac{1}{10}$

ix. $\dfrac{1}{5-3\cos(3x-1)}$

Sol. $g(x)=\dfrac{1}{5-3\cos(3x-1)}$

Here $a=5,\quad b=-3$

Maximum value of $g(x)=\dfrac{1}{a-|b|}=\dfrac{1}{5-|-3|}=\dfrac{1}{5-3}=\dfrac{1}{2}$

Minimum value of $g(x)=\dfrac{1}{a+|b|}=\dfrac{1}{5+|-3|}=\dfrac{1}{5+3}=\dfrac{1}{8}$

Q2. The temperature T in degrees Celsius of a certain city varies throughout the day according to the equation $T(t)=\dfrac{13}{2}\sin\left(\dfrac{\pi}{6}t-\dfrac{\pi}{9}\right)+15$, where t is the time in hours, with t = 0 corresponding to midnight.

Sol. $T(t)=\dfrac{13}{2}\sin\left(\dfrac{\pi}{6}t-\dfrac{\pi}{9}\right)+15$

Here $a=15,\ b=\dfrac{13}{2}$

a) Find the Maximum and Minimum temperature during the day

Maximum temperature $=a+|b|=15+\left|\dfrac{13}{2}\right|$

$=15+\dfrac{13}{2}=15+6.5=21.5^{o}$

Minimum temperature $=a-|b|=15-\left|\dfrac{13}{2}\right|$

$=15-6.5=8.5^{o}$

b) Find the Temperature at t = 9hours (9:00 a.m.)

$T(9)=\dfrac{13}{2}\sin\left(\dfrac{\pi}{6}(9)-\dfrac{\pi}{9}\right)+15$

$=\dfrac{13}{2}\sin\left(\dfrac{25\pi}{18}\right)+15=8.89^{o}$

Q.3 A man on the top of a 100 m high light-house is in line with two ships on the same side of it, whose angles of depression from the man are 17^{o} and 19^{o} respectively. Find the distance between the ships.

Let d_1 be the distance from the lighthouse to the farther ship.

$\tan(17^{o})=\dfrac{100}{d_1}$

$d_1=\dfrac{100}{\tan(17^{o})}$

$d_1\approx 327.10m$

Let d_2 be the distance from the lighthouse to the closer ship.

$\tan(19^{o})=\dfrac{100}{d_2}$

$d_2=\dfrac{100}{\tan(19^{o})}$

$d_2\approx 290.42m$

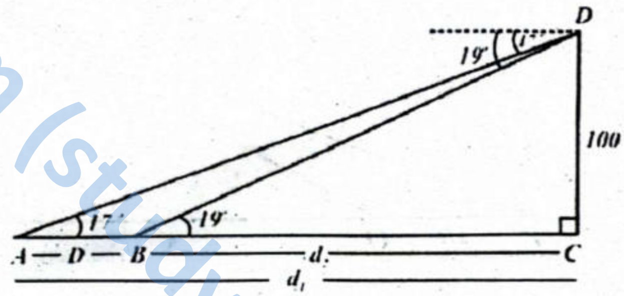
Let D be the distance between the ships.

$D=d_1-d_2$

$D=327.10-290.42$

$D\approx 36.68m$

The distance between the two ships is approximately $36.68m$.



Q.4 P and Q are two points in line with a tree. If the distance between P and Q be $30m$ and the angles of elevation of the top of the tree at P and Q be 12^{o} and 15^{o} respectively, find the height of the tree.

Let h be the height of the tree.

Let x be the distance from point Q to the base of the tree.

$\tan(15^{o})=\dfrac{h}{x}$

$\tan(12^{o})=\dfrac{h}{x+30}$

$$x=\frac{h}{\tan(15^{o})}$$

$$\tan(12^{o})=\frac{h}{x+30}=\frac{h}{\frac{h}{\tan(15^{o})}+30}$$

$$\tan(12^{o})\left(\frac{h}{\tan(15^{o})}+30\right)=h$$

$$\frac{h\tan(12^{o})}{\tan(15^{o})}+30\tan(12^{o})=h$$

$$h-\frac{h\tan(12^{o})}{\tan(15^{o})}=30\tan(12^{o})$$

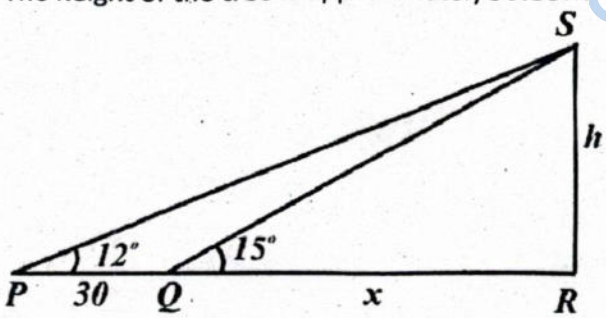
$$h\left(1-\frac{\tan(12^{o})}{\tan(15^{o})}\right)=30\tan(12^{o})$$

$$h=\frac{30\tan(12^{o})}{1-\frac{\tan(12^{o})}{\tan(15^{o})}}=\frac{30\tan(12^{o})\tan(15^{o})}{\tan(15^{o})-\tan(12^{o})}$$

$$h=\frac{30\times0.2126\times0.2679}{0.2679-0.2126}=\frac{1.705}{0.0553}$$

$h\approx30.83m$

The height of the tree is approximately $30.83m$



Q.5 **A gaint Ferris wheel has a diameter of 60feet. The lowest point of the wheel is located 6 feet above the ground. The wheel completes on full revolution every 80 seconds.**

(a) **Model an equation that represent the height h(t) of rider on the Ferris wheel at any given time t.**

(b) ***Find the maximum height of a rider***

(c) **Find the height of the rider from the ground after 35 seconds.**

$$Radius=r=\frac{60}{2}=30\,feet$$

To find the angular frequency B:

$$B=\frac{2\pi}{80}=\frac{\pi}{40}$$

To find the vertical shift D:

$D=30+6=36\,feet$

Write the equation for h(t):

$$h(t)=-30\cos\left(\frac{\pi}{40}t\right)+36$$

The maximum height is the vertical shift plus the radius:

$h_{max}=36+30=66\,feet$

Substitute $t=35$ into the equation:

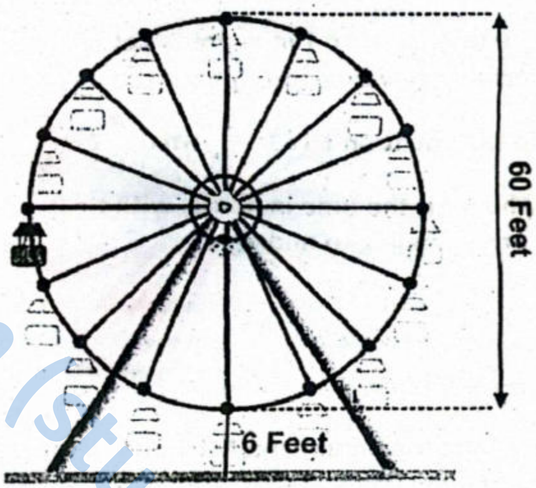
$$h(35)=-30\cos\left(\frac{\pi}{40}(35)\right)+36$$

$$h(35)=-30\cos\left(\frac{7\pi}{8}(35)\right)+36$$

$$h(35)\approx-30(-0.9239)+36$$

$$h(35)\approx27.717+36$$

$$h(35)\approx63.717\,feet$$



(a) **Model an equation that represent the height h(t) of rider on the Ferris wheel at any given time t.**

The equation for the height of the rider is

$$h(t)=-30\cos\left(\frac{\pi}{40}t\right)+36$$

(b) **Find the maximum height of a rider**

The maximum height of the rider is 66 feet.

(c) **Find the height of the *rider from the* ground after 35 seconds.**

The height of the rider after 35 seconds is approximately 63.717 feet

Q6. A child is playing on a swing in playground. The height h(t) of the swing seat above the ground (in meters) at time t (in seconds) is modeled by the function.

$$h(t)=1.5+1.2\sin(3\pi t)$$

(a). what is the maximum height reached by the swing seat?

Sol: The height function is given by $h(t)=1.5+1.2\sin(3\pi t)$. The sine function, $\sin(3\pi t)$, oscillates between -1 and 1. To find the maximum height, we need to find the maximum value of $h(t)$. This occurs when $\sin(3\pi t)$ reaches its maximum value, which is 1.

So, the maximum height is:

$h_{max}=1.5+1.2(1)=1.5+1.2=2.7$ meters.

(b). what is the minimum height reached by the swing seat?

Sol: Similarly, the minimum height occurs when $\sin(3\pi t)$ reaches its minimum value, which is -1.

So, the minimum height is:

$h_{min}=1.5+1.2(-1)=1.5-1.2=0.3$ meters.

(c). How long does it take for the swing to complete one full back-and-forth motion (period)?

Sol: The period of a sinusoidal function of the form $\sin(Bt)$ is given by $T=\frac{2\pi}{|B|}$. in our case, $B=3\pi$.

Therefore, the period fo the swing is:

$$T=\frac{2\pi}{|3\pi|}=\frac{2\pi}{3\pi}=\frac{2}{3}\text{ seconds.}$$

This means it takes $\frac{2}{3}$ seconds for the swing to complete on full back-and-forth motion.

(d). At what time (s) does the swing seat first reach a height of 2.12 meters?

Sol: We wand to find the time(s) t when h(t)=2.12 meters. So we set up the equation

$$2.12=1.5+1.2\sin(3\pi t)$$

Now, let's solve for $\sin(3\pi t)$

$$2.12-1.5=1.2\sin(3\pi t)$$

$$0.62=1.2\sin(3\pi t)$$

$$\sin(3\pi t)=\frac{0.62}{1.2}=\frac{31}{60}$$

Now we need to find the values of t for which $\sin(3\pi t)$. first, let's find the principal value:

$$3\pi t=\arcsin\left(\frac{31}{60}\right)$$

$$t=\frac{1}{3\pi}\arcsin\left(\frac{31}{60}\right)$$

Using a calculator, $\arcsin\left(\frac{31}{60}\right)\approx 0.5435$ radians.

So, the first time the swing reaches 2.12 meters is:

$$t_1=\frac{0.5435}{3\pi}\approx\frac{0.535}{9.4248}\approx 0.0577\text{ Seconds.}$$

Since the sine function is also positive in the second quadrant, there is another solution within the first period. The general solutions for $\sin(s)=k$ Are $x=\arcsin(k+2n\pi)$ and $x=\pi-\arcsin(k+2n\pi)$, where n is an integer.

For $3\pi t=\pi-arc\sin\left(\frac{31}{60}\right)$, we get:

$$t_2=\frac{1}{3\pi}\left(\pi-\arcsin\left(\frac{31}{60}\right)\right)=\frac{1}{3\pi}(\pi-0.5435)$$

$$\approx\frac{1}{9.4248}(3.1416-0.5435)$$

$$\approx\frac{2.5981}{9.4248}\approx 0.2757\text{ Seconds.}$$

The question asks for the first tie the swing seat reaches a height of 2.12 meters.

Comparing t_1 and t_2, the first time is t_1.

Therefore, the swing seat first reaches a height of approximately 0.577 seconds.

Q7. A carnival ride consist of a vertical wheel with a diameter of 40 feet. The centre of the wheel is 28 feet above the ground. The wheel rotates at a constant speed and takes 120 seconds to make one complete revolution. Model an equation that describes the height h(t) of a rider on the wheel as a function of time t. how high is the rider from the ground after 90 seconds? At what times will the rider be 36 feet above the ground?

Sol: let's model the equation: $h(t)=a\cos(bt)+d$

Where:

A=amplitude=radius=20 feet (half of diameter)

D=vertical shift =28 feet (center of wheel above ground)

Period = 120 seconds, so $b = 2\pi/120 = \pi/60$

$$h(t) = -20\cos(\pi t/60) + 28$$

Note the negative sign, as the cosine function starts at its maximum value, but we want it to start at the minimum value (lowest point).

Height after 90 seconds:

$$h(90) = -20\cos(\pi(90)/60) + 28$$

$$= -20\cos(3\pi/2) + 28 = -20(0) + 28 = 28\ feet$$

Time when rider is 36 feet above ground:

$$36 = -20\cos(\pi t/60) + 28$$

$$8 = -20\cos(\pi t/60)$$

$$\cos(\pi t/60) = -8/20$$

$$\cos(\pi t/60) = -0.4$$

$$\pi t/60 = \arccos(-0.4)$$

$$\pi t/60 \approx 1.159(or\, 113.58^\circ)$$

$$t \approx 1.159 \times 60/\pi$$

$$t \approx 22.12\, seconds$$

Since cosine is negative in the seconds quadrant:

$t \approx 22.12\, seconds$

Since cosine is negative in the second quadrant:

$$\pi t/60 = 2\pi - 1.159$$

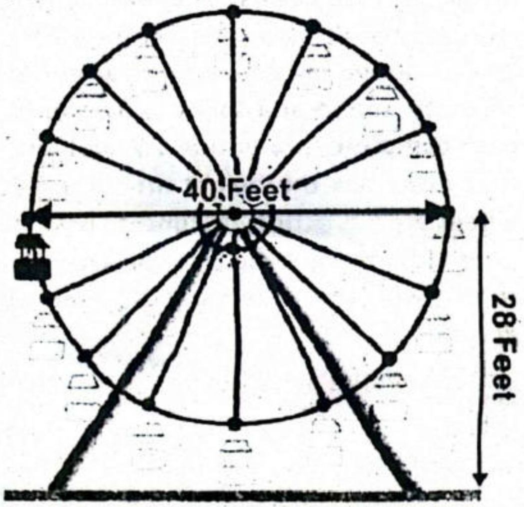
$$t \approx (2\pi - 1.159) \times 60/\pi \approx 97.88\, seconds$$

Given the periodic nature, these values repeat every 120 seconds.

The equation is $h(t) = -20\cos(\pi t/60) + 28$.

Height after 90 seconds: 28 feet

Times when rider is 36 feet above ground: Approximately 22.12 seconds and 97.88 seconds



Q8. **Suppose the temperature T in degrees Fahrenheit of Lahore city in a month of December throughout the day can be modeled by the equation:**

$$T = 64 + 8\sin\left(\frac{\pi}{12}(t-8)\right)$$ **, where t is the time in hours. The temperature oscillates 8 degrees above and below and average temperature of 64 degrees.**

(a) **Find the temperature at $t = 9$ hours?**

(b) **At what time the temperature will be maximum?**

(c) **Calculate the maximum temperature.**

(a) **Temperature at $t = 9$ hours:**

Sol. Start with the equation:

$$T = 64 + 8\sin\left(\frac{\pi}{12}(t-8)\right)$$

Substitute $t = 9$:

$$T = 64 + 8\sin\left(\frac{\pi}{12}(9-8)\right)$$

Simplify inside the sine function:

$$T = 64 + 8\sin\left(\frac{\pi}{12}(1)\right)$$

$$T = 64 + 8\sin\left(\frac{\pi}{12}\right)$$

Calculate $\sin\left(\frac{\pi}{12}\right)$:

$$\sin\left(\frac{\pi}{12}\right) \approx 0.2588$$

Multiply by 8:

$8 \times 0.2588 \approx 2.0704$

$T \approx 64 + 2.0704 \approx 66.07$

The temperature at $t = 9$ hours is approximately 66.07 degrees Fahrenheit.

(b) **Time at which the temperature will be maximum**

$\sin\left(\frac{\pi}{12}(t-8)\right)$ reaches its maximum value which is 1.

Set the sine term equal to 1.

$$\sin\left(\frac{\pi}{12}(t-8)\right) = 1$$

Find the angle for which sine is 1.

The first such angle is $\frac{\pi}{12}$

$$\frac{\pi}{12}(t-8) = \frac{\pi}{2}$$

Solve for $(t-8)$:

$$t-8=\frac{\pi}{2}\times\frac{12}{\pi}=6$$

Solve for t:

$t=6+8=14$

The temperature will be maximum at $t=14$ hours

(c) **Maximum temperature:**

The maximum temperature occurs when the sine term is 1.

Substitute the maximum value of the sine term into the temperature equation:

$T_{max}=64+8(1)$

Calculate the maximum temperature:

$T_{max}=64+8=72$

The maximum temperature is 72 degrees Fahrenheit

Q9. Suppose the population of a coastal city follows a sinusoidal pattern due to seasonal migration. The population of the city over the course of a year can be modeled by the equation:

$P(t)=70000+10000\left(\frac{\pi}{6}t-\frac{\pi}{2}\right)$, $P(t)$ is the population at time t (t is the time in months, with t = 0 corresponding to January 1st). Where t denoted the months in a year.

(a) Find the population of a city at $t=7$ months

(b) Find the maximum population

(a) **Calculate the population at $t=7$**

Substitute $t=7$ into the population model:

$$P(7)=70000+10000\cos\left(\frac{\pi}{6}(7)-\frac{\pi}{2}\right)$$

$$P(7)=70000+10000\cos\left(\frac{7\pi}{6}-\frac{3\pi}{6}\right)$$

$$P(7)=70000+10000\cos\left(\frac{4\pi}{6}\right)$$

$$P(7)=70000+10000\cos\left(\frac{2\pi}{3}\right)$$

$$P(7)=70000+10000\left(-\frac{1}{2}\right)$$

$$P(7)=70000-5000$$

$$P(7)=65000$$

(b) **Calculate the maximum population**

The maximum value of $\cos(x)$ is 1.

The maximum population is found by substituting 1 for $\cos\left(\frac{\pi}{6}t-\frac{\pi}{2}\right)$

$$P_{max}=70000+10000(1)$$

$$P_{max}=70000+10000$$

$$P_{max}=80000$$