Exercise 12.2

Q1. Determine the left hand limit and the right hand limit and then, find limit of the following functions when $x \to c$.

(i) $f(x) = 2x^2 + x - 5$, $c = 1$

L.H.L $= \lim_{x\to 1^-} f(x) = \lim_{x\to 1} (2x^2+x-5)$

$= 2(1)^2 + 1 - 5 = 2+1-5 = -2$

R.H.L $= \lim_{x\to 1^+} f(x) = \lim_{x\to 1} (2x^2+x-5)$

$= 2(1)^2+1-5 = 2+1-5 = -2$

L.H.L = R.H.L $= -2$ so $\lim_{x\to 1} f(x) = -2$

(ii) $f(x) = \dfrac{x^2-9}{x-3}, c = -3$

L.H.L $= \lim_{x\to -3^-} f(x) = \lim_{x\to -3} \dfrac{x^2-9}{x-3} = \dfrac{(-3)^2-9}{-3-3} = \dfrac{9-9}{-6} = 0$

R.H.L $= \lim_{x\to -3^+} f(x) = \lim_{x\to -3} \dfrac{x^2-9}{x-3}$

$= \dfrac{(-3)^2-9}{-3-3} = \dfrac{9-9}{-6} = 0$

L.H.L = R.H.L = 0 So, $\lim_{x\to 1} f(x) = 0$

(iii) $f(x) = |x-5|$, $c = 5$

L.H.L $= \lim_{x\to -5^-} f(x) = \lim_{x\to -5} -(x-5) = -(5-5) = 0$

R.H.L $= \lim_{x\to 5^+} f(x) = \lim_{x\to 5} (x-5) = 5-5 = 0$

L.H.L = R.H.L = 0 So, $\lim_{x\to 5} f(x) = 0$

Q2. Discuss the continuity of f(x) at x = c

(i) $f(x) = \begin{cases} 2x+5 & \text{if } x \le 2 \\ 4x+1 & \text{if } x > 2 \end{cases}, \ c = 2$

L.H.L $= \lim_{x\to 2^-} f(x) = \lim_{x\to 2} (2x+5)$

$= 2(2) + 5 = 4 + 5 = 9$

R.H.L $= \lim_{x\to 2^+} f(x) = \lim_{x\to 2} (4x+1) = 4(2) + 1 = 9$

$f(2) = 2(2) + 5 = 4+5 = 9$

L.H.L = R.H.L = f(2)

so f(x) is Continuous at $x = 2$

(ii) $f(x) = \begin{cases} 3x-1 & \text{if } x < 1 \\ 4 & \text{if } x = 1 \\ 2x & \text{if } x > 1 \end{cases}, c = 1$

L.H.L $= \lim_{x\to 1^-} f(x) = \lim_{x\to 1} (3x-1)$

$= 3(1) - 1 = 3 - 1 = 2$

R.H.L $= \lim_{x\to 1^+} f(x) = \lim_{x\to 1}(2x) = 2(1) = 2$

f(1) = 4

f(1) $\neq$ L.H.L = R. H. L

so f(x) is Discontinuous at $x = 1$

Q3. If f(x) = $\begin{cases} 3x & if \quad x \le -2 \\ x^2 - 1 & if \quad -2 < x < 2 \\ 3 & if \quad x \ge 2 \end{cases}$,

Discuss continuity at $x = 2$ *and* $x = -2$

Case I for $x = 2$

L.H.L= $\lim_{x\to 2^-} f(x) = \lim_{x\to 2} (x^2-1) = (2)^2-1 = 4-1 = 3$

R.H.L= $\lim_{x\to 2^+}$ f(x)= $\lim_{x\to 2}$ 3=3 f(2) = 3

L.H.L = R.H.L = f(2)

f(x) is Continuous at c = 2

Case II For $x = -2$

L.H.L = $\lim_{x\to -2^-}$ f(x) = $\lim_{x\to -2}$ (3x) = 3 (–2) = –6

R.H.L = $\lim_{x\to -2^+} f(x) = \lim_{x\to -2^+} (x^2 - 1)$

$= (-2)^2 - 1 = 4 - 1 = 3$

f(–2) = 3 (–2) = –6

L.H.L= f(–2) $\neq$ R.H.L

f(x) is Discontinuous at $x = -2$

Q4. f(x)= $\begin{cases} x+2 \ , \ x \le -1 \\ c+2 \ , x > -1 \end{cases}$ **find c so that $\lim_{x\to -1}$ f(x) exist**

L.H.L $= \lim_{x\to -1^-} f(x) = \lim_{x\to -1}(x+2) = -1+2 = 1$

R.H.L $= \lim_{x\to -1^+} f(x) = \lim_{x\to -1}(c+2) = c+2$

Given limit exist mean

L.H.L = R.H.L

1 = c + 2 ⇒ $\boxed{c = -1}$

Q5. **Find the values m and n, so that given function f is continuous at $x = 3$**

(i) f(x) = $\begin{cases} mx & if \ x<3 \\ n & if \ x = 3 \\ -2x+9 & if \ x > 3 \end{cases}$

L.H.L = $\lim_{x\to 3^-}$ f(x) = $\lim_{x\to 3}$ (mx) = 3m

R.H.L $= \lim_{x\to 3^+} f(x) = \lim_{x\to 3}(-2x+9)$

= –2(3) + 9 = –6 +9 = 3 and f(3) = n

Given f(x) is Continuous

So L.H.L = R.H.L = f(3) ⇒ 3m = 3 = n

⇒3m = 3 and n = 3

⇒$\boxed{m = 1}$ and $\boxed{n = 3}$

(ii) f(x) = $\begin{cases} mx & if \ x < 3 \\ x^2 & if \ x \ge 3 \end{cases}$

L.H.L = $\lim_{x\to 3^-}$ f(x) = $\lim_{x\to 3}$ (mx) = 3m

R.H.L $= \lim_{x\to 3^+} f(x) = \lim_{x\to 3} x^2 = (3)^2 = 9$

f(3) = $(3)^2$ = 9

Given f(x) continuous mean

L.H.L = R.H.L = f(3)

⇒ 3m = 9 = 9 ⇒$\boxed{m = 3}$

Q6. f(x) = $\begin{cases} \dfrac{\sqrt{2x+5}-\sqrt{x+7}}{x-2}, & x \ne 2 \\ k & x = 2 \end{cases}$

Find value of K so that f is continuous at x =2

$$\lim_{x\to 2} f(x) = \lim_{x\to 2} \frac{\sqrt{2x+5}-\sqrt{x+7}}{x-2} \times \frac{\sqrt{2x+5}+\sqrt{x+7}}{\sqrt{2x+5}+\sqrt{x+7}}$$

$$= \lim_{x\to 2} \frac{\left(\sqrt{2x+5}\right)^2 - \left(\sqrt{x+7}\right)^2}{(x-2)\left(\sqrt{2x+5}+\sqrt{x+7}\right)}$$

$$= \lim_{x\to 2} \frac{2x+5-x-7}{(x-2)(\sqrt{2x+5}+\sqrt{x+7})}$$

$$= \lim_{x\to 2} \frac{(x-2)}{(x-2)\left(\sqrt{2x+5}+\sqrt{x+7}\right)} = \lim_{x\to 2} \frac{1}{\left(\sqrt{2x+5}+\sqrt{x+7}\right)}$$

$$= \frac{1}{\sqrt{2(2)+5}+\sqrt{2+7}} = \frac{1}{\sqrt{9}+\sqrt{9}} = \frac{1}{3+3} = \frac{1}{6}$$

Given f(x) is continuous so

$\lim_{x\to 2}$ f(x)=f(2) $\Rightarrow \frac{1}{6} = k$ *or* $\boxed{k = \frac{1}{6}}$

Q.7 **Given the function $f(x) = \begin{cases} 2x+3 \ , & x \le 1 \\ -x+4 \ , & x > 1 \end{cases}$**

Discuss the limit and continuity at $x = 1$

Sol: $f(1) = 2(1)+3 = 2+3 = 5$

L.H.L$= \lim_{x\to 1^-} f(x) = \lim_{x\to 1^-}(2x+3) = 2(1)+3 = 2+3 = 5$

R.H.L $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+}(-x+4) = -1+4 = 3$

Since $\lim_{x\to 1} f(x) \ne \lim_{x\to 1} f(1)$

Limit does not exist at $x = 1$

Also f(x) is not continuous at x = 1