Exercise 12.3

Q1. A substance decays exponentially following the formula $A(t) = A_0 e^{-0.1t}$, where A_0 is the initial amount. Find the limit of $A(t)$ as $t \to \infty$.

Sol: $A(t) = A_0 e^{-0.1t} = A_0 \lim_{t\to\infty}\left(\frac{1}{e^{0.1t}}\right)$

$$= A_0 \cdot \frac{1}{e^{\infty}} = \frac{1}{\infty} = A_0(0) = 0$$

Q2. A town's population is modeled by $P(t) = \frac{100000}{1+9e^{-0.5t}}$. what is the long-term population as $t \to \infty$.

Sol: $P(t) = \frac{100000}{1+9e^{-0.5t}}$

$$\lim_{t\to\infty} P(t) = \lim_{t\to\infty} \frac{100000}{1+9e^{-0.5t}}$$

$$= \frac{100000}{1+\frac{9}{e^{\infty}}} = \frac{100000}{1+0} = 100000$$

Q3. A company's weekly sales (in thousands) follow the functions $S(t) = \frac{500t}{t+10}$. what is the limit of $S(t)$ as $t \to \infty$ and what does it represent?

Sol: $S(t) = \frac{500t}{t+10}$

$$\lim_{t\to\infty} S(t) = \lim_{t\to\infty} \frac{500t}{t+10}$$

$$= \lim_{t\to\infty} \frac{500\not{t}}{\not{t}\left(1+\frac{10}{t}\right)} = \left(\frac{500}{1+\frac{10}{\infty}}\right) = \frac{500}{1+0} = 500$$

Q4. Signal strength $S(d)$ at a distance d from a tower modeled as $S(d) = \frac{1000}{d^2}$.

i. what is the signal at $d = 10$?

Sol: $S(10) = \frac{1000}{(10)^2} = \frac{1000}{100} = 10$

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ii. **What happens to signal strength as $d \to \infty$?**

Sol: $\lim_{d \to \infty} \frac{1000}{d^2} = \frac{1000}{\infty^2} = \frac{1000}{\infty} = 0$

Q5. **A stock price grows according to the function $P(t) = 50e^{0.05t}$**

i. **Find the limit of $P(t)$ as $t \to \infty$**

Sol: $\lim_{t \to \infty} P(t) = \lim_{t \to \infty} 50e^{0.05t} = 50e^{\infty} = \infty$

ii. **Calculate the price of ten 10 years.**
Put t = 10

Sol: $P(10) = 50e^{0.05(10)} = 82.44$

Q6. **The factory's cost function is given as:**

$$C(x) = \begin{cases} 10x + 500 & if \quad x \le 100 \\ 12x + 300 & if \quad x > 100 \end{cases}$$

In the cost function continuous at $x = 100$?

Sol: $C(x) = \begin{cases} 10x + 500 & if \quad x \le 100 \\ 12x + 300 & if \quad x > 100 \end{cases}$

$C(100) = 10(100) + 500 = (1000 + 500) = 1500$

L.H.L $= \lim_{x \to 100^-} f(x)$
$= \lim_{x \to 100^-} (10x + 500)$
$= 10(100) + 500 = 1000 + 500 = 1500$

R.H.L $= \lim_{x \to 100^+} C(x) = \lim_{x \to 100^+} (12x + 300)$
$= 12(100) + 300 = 1200 + 300 = 1500$

$\Rightarrow \lim_{x \to 100} C(x) = C(100)$

$\Rightarrow C(x)$ is continuous at $x = 100$

Q7. **Inflation is modeled by $I(t) = I_0 e^{0.03t}$, where I_0 is the initial price index and t is the number of years**

i. **Find the inflation rate after 5 years if $I_0 = 100$**

Sol: Inflation rate after 5 years if $I_0 = 100$

$I_0 = 100 \quad , \quad t = 5$

$I(5) = 100e^{0.03(5)} = (100)(1.11618) = 116.18$

Inflation Rate $= \left(\frac{I - I_0}{I_0}\right) \times 100\%$

$= \left(\frac{116.18 - 100}{100}\right) \times 100\% = 16.15\%$

ii. **What is the expected price index after 10 years**

Sol: Expected Price index after 10 years

$I(10) = 100e^{0.03(10)} = 134.99$

Q8. **The cost to produce x units is:**

$$C(x) = \begin{cases} 5x + 20 & if \quad x \le 10 \\ 6x + 10 & if \quad x > 10 \end{cases}$$

If the cost function at $x = 10$?

Sol: $C(x) = \begin{cases} 5x + 20 & if \quad x \le 10 \\ 6x + 10 & if \quad x > 10 \end{cases}$

$C(10) = 5(10) + 20 = 50 + 20 = 70$

L.H.L $= \lim_{x \to 10^-} C(x) = \lim_{x \to 10^-} (5x + 20)$
$= 5(10) + 20 = 50 + 20 = 70$

R.H.L $= \lim_{x \to 10^+} C(x) = \lim_{x \to 10^+} (6x + 10)$
$= 6(10) + 10 = 60 + 10 = 70$

$\Rightarrow \lim_{x \to 10} C(x) = C(10)$

$\Rightarrow \lim_{x \to 10} C(x)$ is continuous at $x = 10$

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