



Exercise 13.1



Q1. Find by definition, the derivatives w.r.t 'x' of the following function defined as.

(i) $2x^2 + 1$

Sol: Let $y = 2x^2 + 1$

$$\text{Then } y + \delta y = 2(x + \delta x)^2 + 1$$

$$\delta y = 2(x^2 + 2x\delta x + (\delta x)^2) + 1 - y$$

$$\delta y = 2x^2 + 4x\delta x + 2(\delta x)^2 + 1 - (2x^2 + 1)$$

$$\delta y = \cancel{2x^2} + 4x\delta x + 2(\delta x)^2 + \cancel{1} - \cancel{2x^2} - \cancel{1}$$

$$\delta y = 4x\delta x + 2(\delta x)^2$$

$$\delta y = \delta x(4x + 2\delta x)$$

Divide both side by δy and take limit $\delta x \rightarrow 0$

$$\lim_{\delta x \rightarrow 0} \frac{\delta y}{\delta x} = \lim_{\delta x \rightarrow 0} \frac{\delta x(4x + 2\delta x)}{\delta x} \Rightarrow \frac{dy}{dx} = 4x$$

(ii) $2 - \sqrt{x}$

Sol: Let $y = 2 - \sqrt{x}$

$$\text{Then } y + \delta y = 2 - \sqrt{x + \delta x}$$

$$\delta y = 2 - \sqrt{x + \delta x} - y = \cancel{2} - \sqrt{x + \delta x} - \cancel{2} + \sqrt{x} = \sqrt{x} - \sqrt{x + \delta x}$$

$$\delta y = (\sqrt{x} - \sqrt{x + \delta x}) \times \frac{\sqrt{x} + \sqrt{x + \delta x}}{\sqrt{x} + \sqrt{x + \delta x}} \quad \left(\text{multiply and divide by } \frac{1}{\sqrt{x} + \sqrt{x + \delta x}} \right)$$

$$\delta y = \frac{(\sqrt{x})^2 - (\sqrt{x + \delta x})^2}{\sqrt{x} + \sqrt{x + \delta x}} = \frac{\cancel{x} - \cancel{x} - \delta x}{\sqrt{x} + \sqrt{x + \delta x}}$$

Divide by δx and take limit $\delta x \rightarrow 0$

$$\lim_{\delta x \rightarrow 0} \frac{\delta y}{\delta x} = \lim_{\delta x \rightarrow 0} \frac{-\delta x}{\delta x(\sqrt{x} + \sqrt{x + \delta x})} = \lim_{\delta x \rightarrow 0} \frac{-1}{(\sqrt{x} + \sqrt{x + \delta x})}$$

$$\frac{dy}{dx} = \frac{-1}{\sqrt{x} + \sqrt{x + 0}} = \frac{-1}{\sqrt{x} + \sqrt{x}} = \frac{-1}{2\sqrt{x}}$$

(iii) $\frac{1}{\sqrt{x}}$

Sol: Let $y = \frac{1}{\sqrt{x}}$

$$\text{Then } y + \delta y = \frac{1}{\sqrt{x + \delta x}}$$

$$\delta y = \frac{1}{\sqrt{x + \delta x}} - y = \frac{1}{\sqrt{x + \delta x}} - \frac{1}{\sqrt{x}} = \frac{\sqrt{x} - \sqrt{x + \delta x}}{\sqrt{(x + \delta x)(x)}}$$

نوٹ:

ان تمام سوالوں میں

1- دی گئی رقم کو y کے برابر لیں۔

2- x کی جگہ x + δ اور y کی جگہ y + δ لگائیں

3- y کو دائیں طرف لے جا کر منفی کریں اور پھر y کی جگہ اس کی

قیمت درج کریں۔

4- تب تک حل کریں جب تک دائیں طرف سے δ x مشترک نہ

آجائے۔

5- دونوں اطراف δ x کے ساتھ تقسیم کریں۔ اور limit لیں۔

$$= \frac{\sqrt{x} - \sqrt{x+\delta x}}{(\sqrt{x+\delta x})\sqrt{x}} \times \frac{\sqrt{x} + \sqrt{x+\delta x}}{\sqrt{x} + \sqrt{x+\delta x}} \quad \left(\begin{array}{l} \text{multiply and divide by} \\ \sqrt{x} + \sqrt{x+\delta x} \end{array} \right)$$

$$= \frac{(\sqrt{x})^2 - (\sqrt{x+\delta x})^2}{\sqrt{x+\delta x}\sqrt{x}(\sqrt{x} + \sqrt{x+\delta x})} = \frac{\cancel{x} - \cancel{x} - \delta x}{\sqrt{x+\delta x}\sqrt{x}(\sqrt{x} + \sqrt{x+\delta x})}$$

Divide by δx and take limit $\delta x \rightarrow 0$

$$\lim_{\delta x \rightarrow 0} \frac{\delta y}{\delta x} = \lim_{\delta x \rightarrow 0} \frac{1}{\cancel{\delta x} \sqrt{x+\delta x}\sqrt{x}(\sqrt{x} + \sqrt{x+\delta x})} \cdot \cancel{-\delta x}$$

$$\frac{dy}{dx} = \lim_{\delta x \rightarrow 0} \frac{-1}{\sqrt{x+\delta x}\sqrt{x}(\sqrt{x} + \sqrt{x+\delta x})} = \frac{-1}{\sqrt{x+0}\sqrt{x}(\sqrt{x} + \sqrt{x+0})}$$

$$\frac{dy}{dx} = \frac{-1}{x(2\sqrt{x})} = -\frac{1}{2x^{3/2}}$$

(vi) $x(x-3)$

Sol: Let $y = x(x-3) = x^2 - 3x$

$$\text{Then } y + \delta y = (x + \delta x)^2 - 3(x + \delta x)$$

$$\delta y = x^2 + 2x\delta x + (\delta x)^2 - 3x - 3\delta x - y$$

$$= \cancel{x^2} + 2x\delta x + (\delta x)^2 - \cancel{3x} - 3\delta x - \cancel{x^2} + \cancel{3x} = \delta x(2x + \delta x - 3)$$

Divide by δx and take limit $\delta x \rightarrow 0$

$$\lim_{\delta x \rightarrow 0} \frac{\delta y}{\delta x} = \lim_{\delta x \rightarrow 0} \frac{\cancel{\delta x}(2x + \delta x - 3)}{\cancel{\delta x}}$$

$$\frac{dy}{dx} = 2x + 0 - 3 = 2x - 3$$

Q.2: Find $\frac{dy}{dx}$ from first principle and find gradient of the curve at the given point.

(i) $\sqrt{x+2}$ at $x=6$

(ii) $\frac{1}{\sqrt{x+a}}$ at $x=a$

(i) $(\sqrt{x+2})$ at $x=6$

Sol: Let $y = (x+2)^{1/2}$

$$\text{Then } y + \delta y = (x + \delta x + 2)^{1/2}$$

$$\delta y = (x+2 + \delta x)^{1/2} - y$$

$$\delta y = ((x+2) + \delta x)^{1/2} - (x+2)^{1/2} = ((x+2) + \delta x)^{1/2} - (x+2)^{1/2}$$

$$= (x+2)^{1/2} \left[\left(1 + \frac{\delta x}{x+2} \right)^{1/2} - 1 \right] = (x+2)^{1/2} \left[1 + \frac{1}{2} \left(\frac{\delta x}{x+2} \right) + \frac{1}{2} \left(\frac{1}{2} - 1 \right) \left(\frac{\delta x}{x+2} \right)^2 + \dots - 1 \right]$$

$$= (x+2)^{1/2} \cdot \left(\frac{\delta x}{x+2} \right) \left[\frac{1}{2} + \frac{\left(\frac{1}{2} - 1 \right)}{2!} \left(\frac{\delta x}{x+2} \right) + \dots \right]$$

$$\lim_{\delta x \rightarrow 0} \frac{\delta y}{\delta x} = \lim_{\delta x \rightarrow 0} \frac{(x+2)^{\frac{1}{2}-1} \cdot \delta x}{\delta x} \left[\frac{1}{2} + \frac{1}{2} \left(\frac{\frac{1}{2}-1}{2} \right) \left(\frac{\delta x}{x+2} \right) + \dots \right]$$

$$\Rightarrow \frac{dy}{dx} = (x+2)^{-\frac{1}{2}} \left(\frac{1}{2} + 0 \right) \Rightarrow \frac{dy}{dx} = \frac{1}{2} (x+2)^{-\frac{1}{2}} = \frac{1}{2\sqrt{x+2}}$$

$$\text{At } x=6 \therefore m = \left. \frac{dy}{dx} \right|_{x=6} = \frac{1}{2} (6+2)^{-\frac{1}{2}} = \frac{1}{2} (8)^{-\frac{1}{2}} = \frac{1}{2\sqrt{8}} = \frac{1}{2\sqrt{4 \times 2}} = \frac{1}{2 \times 2\sqrt{2}} = \frac{1}{4\sqrt{2}}$$

(ii) $y = \frac{1}{\sqrt{x+a}} = (x+a)^{-1/2}$ at $x = a$

Sol:

$$y + \delta y = (x + \delta x + a)^{-1/2}$$

$$\delta y = (x+a+\delta x)^{-1/2} - y = (x+a+\delta x)^{-1/2} - (x+a)^{-1/2}$$

$$\delta y = (x+a)^{-1/2} \left[\left(1 + \frac{\delta x}{x+a} \right)^{-\frac{1}{2}} - 1 \right]$$

$$\frac{\delta y}{\delta x} = \frac{(x+a)^{-\frac{1}{2}} \left[1 + \left(-\frac{1}{2} \right) \left(\frac{\delta x}{x+a} \right) + \frac{\left(-\frac{1}{2} \right) \left(-\frac{1}{2} - 1 \right)}{2!} \left(\frac{\delta x}{x+a} \right)^2 + \dots - 1 \right]}{\delta x}$$

$$\lim_{\delta x \rightarrow 0} \frac{\delta y}{\delta x} = \lim_{\delta x \rightarrow 0} \frac{(x+a)^{-\frac{1}{2}} \cdot \frac{\delta x}{(x+a)} \left[-\frac{1}{2} + \frac{\left(-\frac{1}{2} \right) \left(-\frac{3}{2} \right)}{2!} \left(\frac{\delta x}{x+a} \right) + \dots \right]}{\delta x}$$

$$\Rightarrow \frac{dy}{dx} = (x+a)^{-\frac{3}{2}} \left(-\frac{1}{2} + 0 \right) \Rightarrow \frac{dy}{dx} = -\frac{1}{2} (x+a)^{-\frac{3}{2}}$$

$$m = \left. \frac{dy}{dx} \right|_{x=a} = -\frac{1}{2} (x+a)^{-\frac{3}{2}} = -\frac{1}{2} \frac{1}{(2a)^{\frac{3}{2}}}$$

$$= -\frac{1}{2} \frac{1}{2\sqrt{2} \cdot a^{\frac{3}{2}}} = -\frac{1}{4\sqrt{2} a^{\frac{3}{2}}}$$

$$m = \left. \frac{dy}{dx} \right|_{x=a} = -\frac{1}{2} (a+a)^{-\frac{3}{2}} = -\frac{1}{2} (2a)^{-\frac{3}{2}} = -\frac{1}{2} \frac{1}{2(2a)^{\frac{3}{2}}} = -\frac{1}{2} \cdot \frac{1}{2\sqrt{2} \cdot a^{\frac{3}{2}}} = -\frac{1}{4\sqrt{2} a^{\frac{3}{2}}}$$

Q.3. i. Find the derivative of $x^{\frac{2}{3}}$ at $x = 8$ from the first principle.

Sol: Let $f(x) = x^{\frac{2}{3}}$. Then

$$f(x+\delta x) = (x+\delta x)^{\frac{2}{3}} = x^{\frac{2}{3}} \left(1 + \frac{\delta x}{x} \right)^{\frac{2}{3}}$$

$$f(x+\delta x) - f(x) = x^{\frac{2}{3}} \left[1 + \left(\frac{2}{3} \right) \left(\frac{\delta x}{x} \right) + \frac{2 \left(\frac{2}{3} - 1 \right)}{2!} \left(\frac{\delta x}{x} \right)^2 + \dots \right] - x^{\frac{2}{3}}$$

$$f(x+\delta x) - f(x) = x^{\frac{2}{3}} \left[\cancel{x} + \left(\frac{2}{3}\right)\left(\frac{\delta x}{x}\right) + \frac{2\left(\frac{2}{3}-1\right)}{2!}\left(\frac{\delta x}{x}\right)^2 + \dots - \cancel{x} \right]$$

$$\frac{f(x+\delta x) - f(x)}{\delta x} = \frac{x^{\frac{2}{3}} \left(\frac{\delta x}{x}\right) \left[\left(\frac{2}{3}\right) + \frac{2\left(\frac{2}{3}-1\right)}{2!}\left(\frac{\delta x}{x}\right) + \dots \right]}{\cancel{\delta x}}$$

$$\lim_{\delta x \rightarrow 0} \frac{f(x+\delta x) - f(x)}{\delta x} = \lim_{\delta x \rightarrow 0} x^{\frac{2}{3}-1} \left[\left(\frac{2}{3}\right) + \frac{2\left(\frac{2}{3}-1\right)}{2!}\left(\frac{\delta x}{x}\right) + \dots \right]$$

$$f'(x) = x^{\frac{2}{3}-1} \left[\left(\frac{2}{3}\right) + 0 + 0 + \dots \right] \Rightarrow \boxed{f'(x) = \frac{2}{3}x^{-\frac{1}{3}}}$$

Put $x = 8$

$$f'(8) = \frac{2}{3}8^{-\frac{1}{3}} = \frac{2}{3}(2)^{-\frac{1}{3}} = \frac{2}{3}(2)^{-1} = \frac{2}{3} \times \frac{1}{2} = \frac{1}{3}$$

Q3.ii Find the derivative of $x^2 + 2x + 3$ by definition.

Sol: Let $y = x^2 + 2x + 3$. Then

(i) $y + \delta y = (x + \delta x)^2 + 2(x + \delta x) + 3$ and

(ii) $\delta y = [(x + \delta x)^2 + 2(x + \delta x) + 3] - [x^2 + 2x + 3]$

$$\delta y = x^2 + 2x\delta x + 5x^2 + \cancel{2x} + 2\delta + \cancel{3} - x^2 - \cancel{2x} - \cancel{3}$$

(iii) $\frac{\delta y}{\delta x} = \frac{(\cancel{\delta x})(2x + \delta x + 2)}{(\cancel{\delta x})}$

(iv) $\lim_{\delta x \rightarrow 0} \frac{\delta y}{\delta x} = \lim_{\delta x \rightarrow 0} (2x + \delta x + 2) \Rightarrow \frac{dy}{dx} = 2x + 2$

Q.4. Find from first principle, the derivatives of the following expressions w.r.t their respective independent variables.

(i) $(3x - 2)^{-2}$

Sol. Let $y = (3x - 2)^{-2}$

Then $y + \delta y = [3(x + \delta x) - 2]^{-2} = (3x + 3\delta x - 2)^{-2}$

$$\delta y = (3x - 2 + 3\delta x)^{-2} - (3x - 2)^{-2} = (3x - 2)^{-2} \left[\left(1 + \frac{3\delta x}{3x - 2}\right)^{-2} - 1 \right]$$

$$\delta y = (3x - 2)^{-2} \left[\cancel{1} + \frac{(-2)3\delta x}{(3x - 2)} + \frac{(-2)(-2-1)}{2!} \left(\frac{3\delta x}{3x - 2}\right)^2 + \dots - \cancel{1} \right]$$

$$\delta y = (3x+2)^{-2} \left[\frac{(-2)3\delta x}{(3x+2)} + \frac{(-2)(-3)}{2!} \left(\frac{3\delta x}{3x+2} \right)^2 + \dots \right]$$

$$\delta y = (3x+2)^{-2} \cdot \left(\frac{3\delta x}{3x+2} \right) \left[(-2) + \frac{(-2)(-3)}{2!} \left(\frac{3\delta x}{3x+2} \right) + \dots \right]$$

Divide by δx and take limit $\delta x \rightarrow 0$

$$\lim_{\delta x \rightarrow 0} \frac{\delta y}{\delta x} = \lim_{\delta x \rightarrow 0} \frac{(3x+2)^{-2-1} \cancel{3\delta x} \left[-2 + \frac{(-2)(-3)}{2!} \left(\frac{3\delta x}{3x+2} \right) + \dots \right]}{\cancel{\delta x}}$$

$$\Rightarrow \frac{dy}{dx} = (3x+2)^{-3} \cdot 3(-2+0) = -6(3x+2)^{-3}$$

(ii) $(2t+3)^5$

Sol. Let $y = (2t+3)^5$

Then $y + \delta y = [2(t + \delta t) + 3]^5 = (2t + 2\delta t + 3)^5 = (2t + 3 + 2\delta t)^5$

$\delta y = (2t + 3 + 2\delta t)^5 - y = (2t + 3 + 2\delta t)^5 - (2t + 3)^5$

$$\delta y = (2t+3)^5 \left[\left(1 + \frac{2\delta t}{2t+3} \right)^5 \right] - (2t+3)^5$$

$$\delta y = (2t+3)^5 \left[\left(1 + \frac{2\delta t}{2t+3} \right)^5 - 1 \right]$$

$$\delta y = (2t+3)^5 \left[1 + 5 \left(\frac{2\delta t}{2t+3} \right) + \frac{5(5-1)}{2!} \left(\frac{2\delta t}{2t+3} \right)^2 + \dots - 1 \right]$$

$$\delta y = (2t+3)^5 \left[5 \left(\frac{2\delta t}{2t+3} \right) + \frac{5(4)}{2!} \left(\frac{2\delta t}{2t+3} \right)^2 + \dots \right]$$

$$\delta y = (2t+3)^5 \left(\frac{2\delta t}{2t+3} \right) \left(5 + \frac{5(4)}{2!} \left(\frac{2\delta t}{2t+3} \right) + \dots \right)$$

Divide by δt and take lim $\delta t \rightarrow 0$

$$\lim_{\delta t \rightarrow 0} \frac{\delta y}{\delta t} = \lim_{\delta t \rightarrow 0} \frac{(2t+3)^{5-1} \cdot 2 \cancel{\delta t} \left[5 + \frac{5(4)}{2!} \left(\frac{2\delta t}{2t+3} \right) + \dots \right]}{\cancel{\delta t}}$$

$$\frac{dy}{dt} = (2t+3)^4 \cdot 2(5+0) = 5 \times 2(2t+3)^4 = 10(2t+3)^4$$

(iii) $(aw+b)^7$

Sol. Let $y = (aw+b)^7$

Then $y + \delta y = (a(w + \delta w) + b)^7$

$y + \delta y = (aw + a\delta w + b)^7$

$$y + \delta y = (aw + b + a\delta w)^7$$

$$\delta y = (aw + b + a\delta w)^7 - y$$

$$\delta y = (aw + b + a\delta w)^7 - (aw + b)^7$$

$$\delta y = (aw + b)^7 \left(1 + \frac{a\delta w}{aw + b} \right)^7 - (aw + b)^7$$

$$\delta y = (aw + b)^7 \left[1 + 7 \left(\frac{a\delta w}{aw + b} \right) + \frac{7(7-1)}{2!} \left(\frac{a\delta w}{aw + b} \right)^2 + \dots \right]$$

$$\delta y = (aw + b)^7 \cdot \left(\frac{a\delta w}{aw + b} \right) \left[7 + \frac{7(6)}{2} \left(\frac{a\delta w}{aw + b} \right) + \dots \right]$$

$$\lim_{\delta w \rightarrow 0} \frac{\delta y}{\delta w} = \lim_{\delta w \rightarrow 0} \frac{(aw + b)^6 \cdot a\delta w \left[7 + 7(3) \left(\frac{a\delta w}{aw + b} \right) + \dots \right]}{a\delta w}$$

$$\frac{dy}{dw} = (aw + b)^6 (a)(7 + 0)$$

$$\frac{dy}{dw} = 7a(aw + b)^6$$

Q.5. Find the gradient and equation of the tangent line to $y = 3x^2 - 4x + 1$ at $x = 2$.

Sol: Given $y = 3x^2 - 4x + 1$

$$\frac{dy}{dx} = \frac{d}{dx}(3x^2 - 4x + 1)$$

$$\frac{dy}{dx} = 3(2x) - 4(1) + 0$$

$$\frac{dy}{dx} = 6x - 4(1) + 0 = 6x - 4$$

$$m = \left. \frac{dy}{dx} \right|_{x=2} = 6(2) - 4$$

$$m = 12 - 4$$

$$m = 8$$

$$y = 3(2)^2 - 4(2) + 1$$

$$y = 3(4) - 8 + 1$$

$$y = 12 - 8 + 1 = 5$$

The point of tangency is $(2, 5)$.

Using the point-slope form:

$$y - y_1 = m(x - x_1)$$

Substituting $m = 8$ and $(x_1, y_1) = (2, 5)$:

$$y - 5 = 8(x - 2)$$

$$y - 5 = 8x - 16$$

$$y = 8x - 16 + 5$$

$$y = 8x - 11$$

The gradient of the tangent is 8 and the equation of tangent line is $y = 8x - 11$.

Q.6. For the function $f(x) = 2x^3 + x$, Calculate the equation of the tangent line at $x = -1$.

Sol: Given $f(x) = 2x^3 + x$

$$f'(x) = 2 \cdot 3x^2 + 1$$

$$f'(x) = 6x^2 + 1$$

$$f'(-1) = 6(-1)^2 + 1$$

$$f'(-1) = 6(1) + 1$$

$$f'(-1) = 6 + 1$$

$$m = 7$$

$$f(-1) = 2(-1)^3 + (-1)$$

$$f(-1) = 2(-1) + (-1)$$

$$f(-1) = -2 - 1$$

$$f(-1) = -3$$

The point of tangency is $(-1, -3)$

The point-slope form is $y - y_1 = m(x - x_1)$, where (x_1, y_1) is the point of tangency and m is the slope.

$$y - (-3) = 7(x - (-1))$$

$$y + 3 = 7(x + 1)$$

$$y + 3 = 7x + 7$$

$$y = 7x + 7 - 3$$

$$y = 7x + 4$$

The equation of the tangent line is

$$y = 7x + 4$$

Q.7. Find the coordinates of the point of tangency and the equation of the tangent line for $f(x) = x^3 - 2x + 1$ at $x = 1$

Sol: Given $f(x) = x^3 - 2x + 1$

$$f(1) = (1)^3 - 2(1) + 1$$

$$f(1) = 1 - 2 + 1$$

$$f(1) = 0$$

Take derivative w.r.t. x

$$f'(x) = \frac{d}{dx}(x^3 - 2x + 1)$$

$$f'(x) = 3x^2 - 2$$

Put $x = 1$ to find $f'(1)$

$$f'(1) = 3(1)^2 - 2$$

$$f'(1) = 3 - 2$$

$$f'(1) = 1$$

The slope of the tangent line is 1.

Use the point-slope form with point $(1, 0)$ and slope 1:

$$y - 0 = 1(x - 1)$$

$$y = x - 1$$

The coordinates of the point of tangency are $(1, 0)$ and the equation of the tangent line is $y = x - 1$.

Q.8. Find the gradient of the curve $f(x) = 3x^2 + 2x$ at $x = 1$

Sol: Given $f(x) = 3x^2 + 2x$

Take derivative w.r.t. x

$$f'(x) = 3(2x) + 2(1) = 6x + 2$$

Put $x = 1$

$$f'(1) = 6 + 2 = 8$$

The gradient of the curve $f(x) = 3x^2 + 2x$ at $x = 1$ is 8.

Q.9. Find the gradient and an equation of tangent line to the graph of $f(x) = \sqrt{x}$ at $x = 9$.

Sol: $f(x) = \sqrt{x} = x^{\frac{1}{2}}$ _____ (i)

$$f'(x) = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$$

$$f'(9) = \frac{1}{2\sqrt{9}} = \frac{1}{2 \cdot 3} = \frac{1}{6}$$

$$f(9) = \sqrt{9} = 3$$

The point is $(9, 3)$

$$y - y_1 = m(x - x_1)$$

$$y - 3 = \frac{1}{6}(x - 9)$$

$$y = \frac{1}{6}x - \frac{9}{6} + 3$$

$$y = \frac{1}{6}x - \frac{3}{2} + \frac{6}{2}$$

$$y = \frac{1}{6}x + \frac{3}{2}$$

The gradient of the tangent line is $\frac{1}{6}$ and the

equation of the tangent line is $y = \frac{1}{6}x + \frac{3}{2}$

Q.10. The position of a car after t hours is given by: $s(t) = 2t^3 - 3t^2 + t$ (in kilometers)

i. Find the average velocity over the interval $[1, 4]$

ii. Find the instantaneous velocity at $t = 2$

Sol: Given $s(t) = 2t^3 - 3t^2 + t$

$$s(1) = 2(1)^3 - 3(1)^2 + 1 = 2 - 3 + 1 = 0$$

$$s(4) = 2(4)^3 - 3(4)^2 + 4 = 2(64) - 3(16) + 4 = 1$$

$$v_{\text{avg}} = \frac{s(4) - s(1)}{4 - 1}$$

$$v_{\text{avg}} = \frac{84 - 0}{3}$$

$$v_{\text{avg}} = 28$$

Differentiate w.r.t t

$$v(t) = s'(t) = \frac{d}{dt}(2t^3 - 3t^2 + t)$$

$$v(t) = 6t^2 - 6t + 1$$

$$v(2) = 6(2)^2 - 6(2) + 1 = 6(4) - 12 + 1$$

$$v(2) = 24 - 12 + 1 = 13$$

The average velocity over the interval $[1, 4]$ is 28

km/h and the instantaneous velocity at $t = 2$ is 13 km/h.

Q.11. A stone is thrown upwards and its height after t seconds is given by: $s(t) = -16t^2 + 32t + 10$ (in feet), Find the instantaneous velocity at $t = 1$.

Sol: Given $s(t) = -16t^2 + 32t + 10$

$$v(t) = s'(t) = -16(2t) + 32(1) + 0$$

$$v(t) = -32t + 32$$

Evaluate $v(t)$ at $t = 1$

$$v(1) = -32(1) + 32$$

$$v(1) = -32 + 32 = 0$$

The instantaneous velocity at $t = 1$ is 0.

Q.12. The outdoor temperature (in°C) over time is modeled by: $T(t) = -t^2 + 12t + 10$ where t is the time in hours. Find the instantaneous rate of change at $t = 2$.

Sol: $\frac{d}{dt}T(t) = \frac{d}{dt}(-t^2 + 12t + 10)$

$$\frac{d}{dt}T(t) = -2t + 12 + 0$$

$$T'(t) = -2t + 12$$

Substitute $t = 2$ into the derivative:

$$T'(2) = -2(2) + 12 = -4 + 12$$

$$\Rightarrow T'(2) = 8$$

The instantaneous rate of change of the temperature at $t = 2$ is 8.