Q.1. Differentiate w.r.t 'x'.

i. $x^4+2x^3+x^2$

Sol. Let $y=x^4+2x^3+x^2$

Then $\frac{dy}{dx}=\frac{d}{dx}x^4+2\frac{d}{dx}x^3+\frac{d}{dx}x^2=4x^3+2.3x^2+2x$

$$\frac{dy}{dx}=4x^3+6x^2+2x.$$

ii. $x^{-3}+2x^{-\frac{3}{2}}+3$

Sol. Let $y=x^{-3}+2x^{-3/2}+3$ then

$$\frac{dy}{dx}=\frac{d}{dx}x^{-3}+2\frac{d}{dx}x^{-3/2}+\frac{d}{dx}3=-3x^{-4}+2\left(-\frac{3}{2}\right)x^{-3/2-1}+0$$

$$\frac{dy}{dx}=-3x^{-4}-3x^{-5/2}=\frac{-3}{x^4}-\frac{3}{x^{5/2}}=-3\left(\frac{1}{x^4}+\frac{1}{x^{5/2}}\right)$$

iii. $\frac{2x-3}{2x+1}$

Sol. Let $y=\frac{2x-3}{2x+1}$ *then* $\Rightarrow\frac{dy}{dx}=\frac{d}{dx}\left(\frac{2x-3}{2x+1}\right)$

$$\frac{dy}{dx}=\frac{(2x+1)\frac{d}{dx}(2x-3)-(2x-3)\frac{d}{dx}(2x+1)}{(2x+1)^2}=\frac{(2x+1)(2-0)-(2x-3)(2+0)}{(2x+1)^2}$$

$$\frac{dy}{dx}=\frac{4x+2-4x+6}{(2x+1)^2}=\frac{8}{(2x+1)^2}$$

iv. $\frac{(1+\sqrt{x})\left(x-x^{\frac{3}{2}}\right)}{\sqrt{x}}$

Sol. Let $y=\frac{(1+\sqrt{x})(x-x^{3/2})}{\sqrt{x}}$

$$y=\frac{x(1+\sqrt{x})(1-x^{1/2})}{\sqrt{x}}=\frac{\sqrt{x}\times\sqrt{x}(1+\sqrt{x})(1-\sqrt{x})}{\sqrt{x}}$$

$$y=\sqrt{x}\left((1)^2-(\sqrt{x})^2\right)=\sqrt{x}(1-x)=(x)^{1/2}-x^{1/2+1}$$

$$y=x^{1/2}-x^{3/2}\Rightarrow\frac{dy}{dx}=\frac{d}{dx}x^{1/2}-\frac{d}{dx}x^{3/2}$$

$$\frac{dy}{dx}=\frac{1}{2}x^{1/2-1}-\frac{3}{2}x^{3/2-1}=\frac{1}{2}x^{-1/2}-\frac{3}{2}x^{1/2}=\frac{1}{2\sqrt{x}}-\frac{3\sqrt{x}}{2}$$

$$\frac{dy}{dx}=\frac{1-3x}{2\sqrt{x}}$$

v. $\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2$

Sol. Let $y=\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2=\left(\sqrt{x}\right)^2+\left(\frac{1}{\sqrt{x}}\right)^2-2\sqrt{x}\,\frac{1}{\sqrt{x}}$

$y=x+\frac{1}{x}-2 \Rightarrow \quad y=x+x^{-1}-2 \Rightarrow \quad \frac{dy}{dx}=\frac{d}{dx}(x)+\frac{d}{dx}x^{-1}-\frac{d}{dx}(2)$

$$\frac{dy}{dx}=1+(-1)x^{-2}-0=1-\frac{1}{x^2}=\frac{x^2-1}{x^2}$$

vi. $(x-5)(3-x)$

Sol. Let y = (x−5) (3−x) then $\Rightarrow \frac{dy}{dx}=\frac{d}{dx}(x-5)(3-x)$

$$\frac{dy}{dx}=(x-5)\frac{d}{dx}(3-x)+(3-x)\frac{d}{dx}(x-5)=(x-5)(0-1)+(3-x)(1-0)$$

$$\frac{dy}{dx}=-x+5+3-x=8-2x$$

vii. $\frac{\left(x^2+1\right)^2}{x^2-1}$

Sol. Let $y=\frac{\left(x^2+1\right)^2}{x^2-1}$ *then*

$$\frac{dy}{dx}=\frac{\left(x^2-1\right)\frac{d}{dx}\left(x^2+1\right)^2-\left(x^2+1\right)^2\frac{d}{dx}\left(x^2-1\right)}{\left(x^2-1\right)^2}$$

$$\frac{dy}{dx}=\frac{\left(x^2-1\right)2\left(x^2+1\right)^1(2x)-\left(x^2+1\right)^2 2x}{\left(x^2-1\right)^2}$$

$$\frac{dy}{dx}=\frac{2x\left(x^2+1\right)\left[2\left(x^2-1\right)-\left(x^2+1\right)\right]}{\left(x^2-1\right)^2}$$

$$\frac{dy}{dx}=\frac{2x\left(x^2+1\right)\left(2x^2-2-x^2-1\right)}{\left(x^2-1\right)^2}=\frac{2x(x^2+1)(x^2-3)}{(x^2-1)^2}$$

viii. $\frac{x^2+1}{x^2-3}$

Sol. Let $y=\frac{x^2+1}{x^2-3}$ *then*

$$\frac{dy}{dx}=\frac{\left(x^2-3\right)\frac{d}{dx}\left(x^2+1\right)-\left(x^2+1\right)\frac{d}{dx}\left(x^2-3\right)}{\left(x^2-3\right)^2}=\frac{(x^2-3)(2x)-(x^2+1)(2x)}{\left(x^2-3\right)^2}=\frac{2x\left(x^2-3-x^2-1\right)}{\left(x^2-3\right)^2}$$

$$\frac{dy}{dx}=\frac{2x(-4)}{\left(x^2-3\right)^2}=\frac{-8x}{\left(x^2-3\right)^2}$$

ix. $\dfrac{2x-1}{\sqrt{x^2+1}}$

Sol. Let $y = \dfrac{2x-1}{\sqrt{x^2+1}}$

$$\frac{dy}{dx} = \frac{(x^2+1)^{1/2}\frac{d}{dx}(2x-1) - (2x-1)\frac{d}{dx}(x^2+1)^{1/2}}{\left[(x^2+1)^{1/2}\right]^2}$$

$$\frac{dy}{dx} = \frac{\sqrt{(x^2+1)}(2) - (2x-1)\frac{1}{2}(x^2+1)^{-1/2}(2x)}{(x^2+1)}$$

$$\frac{dy}{dx} = \left[2\sqrt{(x^2+1)} - \frac{x(2x-1)}{\sqrt{x^2+1}}\right]\frac{1}{(x^2+1)}$$

$$\frac{dy}{dx} = \frac{1}{(x^2+1)}\left[\frac{2(x^2+1)-(2x^2-x)}{\sqrt{x^2+1}}\right] = \frac{\cancel{2x^2}+2-\cancel{2x^2}+x}{(x^2+1)(x^2+1)^{1/2}}$$

$$\frac{dy}{dx} = \frac{x+2}{(x^2+1)^{3/2}}$$

x. $\sqrt{\dfrac{a-x}{a+x}}$

Sol. Let $y = \sqrt{\dfrac{a-x}{a+x}} = \dfrac{(a-x)^{1/2}}{(a+x)^{1/2}}$

$$\frac{dy}{dx} = \frac{(a+x)^{1/2}\frac{d}{dx}(a-x)^{1/2} - (a-x)^{1/2}\frac{d}{dx}(a+x)^{1/2}}{\left[(a+x)^{1/2}\right]^2}$$

$$\frac{dy}{dx} = \frac{(a+x)^{1/2}\frac{1}{2}(a-x)^{-1/2}(-1) - (a-x)^{1/2}\frac{1}{2}(a+x)^{-1/2}(1)}{(a+x)}$$

$$\frac{dy}{dx} = \left[\sqrt{a+x}\,\frac{-1}{2\sqrt{a-x}} - \frac{\sqrt{a-x}}{2\sqrt{a+x}}\right]\frac{1}{(a+x)}$$

$$= \frac{1}{(a+x)}\left[\frac{-(a+x)-(a-x)}{2\sqrt{a-x}\sqrt{a+x}}\right] = \frac{-a-\cancel{x}-a+\cancel{x}}{2\sqrt{(a-x)}(a+x)^{3/2}}$$

$$\frac{dy}{dx} = \frac{-\cancel{2}a}{\cancel{2}\sqrt{a-x}(a+x)^{3/2}} = \frac{-a}{\sqrt{(a-x)}(a+x)^{3/2}}$$

xi. $\dfrac{\sqrt{x^2+1}}{\sqrt{x^2-1}}$

Sol. Let $y = \dfrac{\sqrt{x^2+1}}{\sqrt{x^2-1}} = \dfrac{(x^2+1)^{1/2}}{(x^2-1)^{1/2}}$

Then $\dfrac{dy}{dx}=\dfrac{(x^2-1)^{1/2}\dfrac{d}{dx}(x^2+1)^{1/2}-(x^2+1)^{1/2}\dfrac{d}{dx}(x^2-1)^{1/2}}{\left[(x^2-1)^{1/2}\right]^2}$

$$\frac{dy}{dx}=\frac{1}{(x^2-1)}\left[\sqrt{x^2-1}\,\frac{1}{2}(x^2+1)^{-1/2}(2x)-\sqrt{x^2+1}\,\frac{1}{2}(x^2-1)^{-1/2}(2x)\right]$$

$$\frac{dy}{dx}=\frac{1}{(x^2-1)}\left[\frac{x\sqrt{x^2-1}}{\sqrt{x^2+1}}-\frac{x\sqrt{x^2+1}}{\sqrt{x^2-1}}\right]=\frac{1}{(x^2-1)}\left[\frac{x(x^2-1)-x(x^2+1)}{\sqrt{x^2+1}\sqrt{x^2-1}}\right]$$

$$=\frac{x}{(x^2-1)}\left[\frac{x^2-1-x^2-1}{\sqrt{x^2+1}(x^2-1)^{1/2}}\right]=\frac{-2x}{\sqrt{x^2+1}(x^2-1)^{3/2}}$$

Q.2. **Find $\dfrac{dy}{dx}$ if $y=\dfrac{(\sqrt{x}+1)\left(x^{\frac{3}{2}}-1\right)}{x^{\frac{1}{2}}-1}$, $(x\neq 1)$**

Sol: Given $y=\dfrac{(\sqrt{x}+1)(x^{3/2}-1)}{x^{1/2}-1}=\dfrac{(\sqrt{x}+1)[(\sqrt{x})^3-(1)^3]}{\sqrt{x}-1}$

$$=\frac{(\sqrt{x}+1)\,(\sqrt{x}-1)\,(x+1+\sqrt{x})}{(\sqrt{x}-1)}=(\sqrt{x}+1)\,(x+1+\sqrt{x})$$

$$=x\sqrt{x}+\sqrt{x}+(\sqrt{x})^2+x+1+\sqrt{x}$$

$$=x\left(x^{1/2}\right)+2\sqrt{x}+2x+1$$

$$\frac{dy}{dx}=\frac{d}{dx}(x^{3/2}+2x+2x^{1/2}+1)=\frac{d}{dx}\left(x^{3/2}\right)+\frac{d}{dx}(2x)+\frac{d}{dx}\left(2x^{1/2}\right)+\frac{d}{dx}(1)$$

$$=\frac{3}{2}x^{1/2}+2(1)+2\cdot\frac{1}{2\sqrt{x}}+0=\frac{3}{2}\sqrt{x}+2+\frac{1}{\sqrt{x}}$$

Q.3. **Differentiate $\dfrac{(\sqrt{x}+1)\left(x^{\frac{3}{2}}-1\right)}{x^{\frac{3}{2}}-x^{\frac{1}{2}}}$ with respect to x.**

Sol: Let $y=\dfrac{(\sqrt{x}+1)(x^{3/2}-1)}{x^{3/2}-x^{1/2}}$

$$=\frac{(\sqrt{x}+1)\left[(\sqrt{x})^3-(1)^3\right]}{\sqrt{x}(x-1)}=\frac{(\sqrt{x}+1)(\sqrt{x}-1)(x+\sqrt{x}+1)}{\sqrt{x}(x-1)}=\frac{(x-1)(x+\sqrt{x}+1)}{\sqrt{x}(x-1)}$$

$$=\frac{x+\sqrt{x}+1}{\sqrt{x}}=\frac{x}{\sqrt{x}}+\frac{\sqrt{x}}{\sqrt{x}}+\frac{1}{\sqrt{x}}=\frac{\sqrt{x}\times\sqrt{x}}{\sqrt{x}}+1+\frac{1}{\sqrt{x}}=x^{1/2}+1+x^{-1/2}$$

Differentiating with respect to x, we have

$$\frac{dy}{dx}=\frac{1}{2}x^{\frac{1}{2}-1}+0+(-\frac{1}{2})x^{\frac{-1}{2}-1}$$

$$\frac{dy}{dx}=\frac{1}{2}x^{\frac{-1}{2}}-\frac{1}{2}x^{\frac{-3}{2}}=\frac{1}{2x^{1/2}}-\frac{1}{2x^{3/2}}=\frac{x-1}{2x^{3/2}}$$

Q.4. If $y=\sqrt{x}-\frac{1}{\sqrt{x}}$, show that $2x\frac{dy}{dx}+y=2\sqrt{x}$

Sol. $y=\sqrt{x}-\frac{1}{\sqrt{x}}=x^{1/2}-x^{-1/2}$

$$\frac{dy}{dx}=\frac{1}{2}x^{-1/2}-\left(-\frac{1}{2}\right)x^{-3/2}$$

$$\frac{dy}{dx}=\frac{1}{2\sqrt{x}}+\frac{1}{2x^{3/2}}$$

$$\text{L.H.S}=2x\frac{dy}{dx}+y=2x\left(\frac{1}{2\sqrt{x}}+\frac{1}{2x^{3/2}}\right)+\sqrt{x}-\frac{1}{\sqrt{x}}\quad (\textit{Put value of } y \textit{ and } \frac{dy}{dx})$$

$$=\sqrt{x}+\frac{1}{\sqrt{x}}+\sqrt{x}-\frac{1}{\sqrt{x}}=2\sqrt{x}$$ Hence Proved.

Q.5. If $y=x^4+2x^2+2$, prove that $\frac{dy}{dx}=4x\sqrt{y-1}$

Sol. Let $y=x^4+2x^2+2$ ____ I

$$\frac{dy}{dx}=4x^3+2.2x+0=4x^3+4x$$

$$\frac{dy}{dx}=4x(x^2+1)\text{ \_\_\_\_ II}$$

From I $y=x^4+2x^2+2$

or $y-1=x^4+2x^2+2-1$

or $y-1=x^4+2x^2+1$

or $y-1=(x^2+1)^2$

or $\sqrt{y-1}=(x^2+1)$

Put value of x^2+1 in II then

$$\frac{dy}{dx}=4x\sqrt{y-1}$$

hence proved