



# Exercise 14.2



**Q.1.** Find the cosines of the angle  $\theta$  between  $\underline{u}$  and  $\underline{v}$

(i)  $\underline{u} = 2\underline{i} + 3\underline{j} + \underline{k}$ ,  $\underline{v} = -\underline{i} + 2\underline{j} + 2\underline{k}$

**Sol:**  $\underline{u} \cdot \underline{v} = (2\underline{i} + 3\underline{j} + \underline{k}) \cdot (-\underline{i} + 2\underline{j} + 2\underline{k}) = (2)(-1) + (3)(2) + (1)(2) = -2 + 6 + 2 = 6$

$$|\underline{u}| = \sqrt{(2)^2 + (3)^2 + (1)^2} = \sqrt{4 + 9 + 1} = \sqrt{14}$$

$$|\underline{v}| = \sqrt{(-1)^2 + (2)^2 + (2)^2} = \sqrt{1 + 4 + 4} = \sqrt{9} = 3$$

$$\cos \theta = \frac{\underline{u} \cdot \underline{v}}{|\underline{u}| |\underline{v}|} = \frac{6}{3\sqrt{14}} = \frac{2}{\sqrt{14}}$$

(ii)  $\underline{u} = [-3, 2, 5]$ ,  $\underline{v} = [1, 6, -2]$

**Sol:**  $\underline{u} \cdot \underline{v} = [-3, 2, 5] \cdot [1, 6, -2] = (-3)(1) + (2)(6) + (5)(-2) = -3 + 12 - 10 = -1$

$$|\underline{u}| = \sqrt{(-3)^2 + (2)^2 + (5)^2} = \sqrt{9 + 4 + 25} = \sqrt{38}$$

$$|\underline{v}| = \sqrt{(1)^2 + (6)^2 + (-2)^2} = \sqrt{1 + 36 + 4} = \sqrt{41}$$

$$\cos \theta = \frac{\underline{u} \cdot \underline{v}}{|\underline{u}| |\underline{v}|} = \frac{-1}{\sqrt{38}\sqrt{41}}$$

**Q.2.** If  $\underline{a} + \underline{b} + \underline{c} = \underline{0}$  and  $|\underline{a}| = 3$ ,  $|\underline{b}| = 5$  and  $|\underline{c}| = 7$ . Find the angle between  $\underline{a}$  and  $\underline{b}$ .

**Sol:** Given  $\underline{a} + \underline{b} + \underline{c} = \underline{0}$

$$\Rightarrow \underline{c} = -(\underline{a} + \underline{b})$$

$$|\underline{c}|^2 = (\underline{a} + \underline{b})^2$$

$$|\underline{c}|^2 = (\underline{a} + \underline{b})^2$$

$$|\underline{c}|^2 = |\underline{a}|^2 + |\underline{b}|^2 + 2(\underline{a} \cdot \underline{b})$$

$$(7)^2 = (3)^2 + (5)^2 + 2(\underline{a} \cdot \underline{b})$$

$$49 = 9 + 25 + 2(\underline{a} \cdot \underline{b})$$

$$49 - 34 = 2(\underline{a} \cdot \underline{b})$$

$$\underline{a} \cdot \underline{b} = \frac{15}{2}$$

Using formula

$$\underline{a} \cdot \underline{b} = |\underline{a}| |\underline{b}| \cos \theta$$

$$\frac{15}{2} = (3)(5) \cos \theta$$

$$\frac{15}{2} = 15 \cos \theta \Rightarrow \cos \theta = \frac{1}{2} \Rightarrow \theta = \cos^{-1}\left(\frac{1}{2}\right)$$

$$\theta = 60^\circ = \frac{\pi}{3} \text{ radian}$$



Q.3. If  $|\underline{a}| = 3, |\underline{b}| = 4$  and  $|\underline{a} + \underline{b}| = 5$ . Find the angle between  $\underline{a}$  and  $\underline{b}$

Sol:  $|\underline{a}| = 3, |\underline{b}| = 4, |\underline{a} + \underline{b}| = 5$

$$|\underline{a} + \underline{b}|^2 = (\underline{a} + \underline{b})(\underline{a} + \underline{b}) = |\underline{a}|^2 + |\underline{b}|^2 + 2(\underline{a} \cdot \underline{b})$$

$$(5)^2 = (3)^2 + (4)^2 + 2(\underline{a} \cdot \underline{b})$$

$$25 = 9 + 16 + 2(\underline{a} \cdot \underline{b})$$

$$25 - 25 = 2(\underline{a} \cdot \underline{b}) \Rightarrow \underline{a} \cdot \underline{b} = 0$$

Using formula  $\underline{a} \cdot \underline{b} = |\underline{a}||\underline{b}|\cos\theta$

$$0 = (3)(4)\cos\theta$$

$$0 = 12\cos\theta$$

$$\Rightarrow \cos\theta = 0 \Rightarrow \theta = \cos^{-1}(0) \Rightarrow \theta = 90^\circ = \frac{\pi}{2} \text{ Radian}$$

Q.4. Calculate the projection of  $\underline{a}$  along  $\underline{b}$  and projection of  $\underline{b}$  along  $\underline{a}$ . when

i.  $\underline{a} = 2\underline{i} + 3\underline{j} - \underline{k}, \underline{b} = \underline{i} - 2\underline{j} + 4\underline{k}$

Sol:  $\underline{a} \cdot \underline{b} = (2)(1) + 3(-2) + (-1)(4) = 2 - 6 - 4 = -8$

$$|\underline{a}| = \sqrt{(2)^2 + (3)^2 + (-1)^2} = \sqrt{4 + 9 + 1} = \sqrt{14}$$

$$|\underline{b}| = \sqrt{(1)^2 + (-2)^2 + (4)^2} = \sqrt{1 + 4 + 16} = \sqrt{21}$$

$$\text{Projection of } \underline{a} \text{ along } \underline{b} = \frac{\underline{a} \cdot \underline{b}}{|\underline{b}|} = \frac{-8}{\sqrt{21}}$$

$$\text{Projection of } \underline{b} \text{ along } \underline{a} = \frac{\underline{a} \cdot \underline{b}}{|\underline{a}|} = \frac{-8}{\sqrt{14}}$$

ii.  $\underline{a} = 4\underline{i} - 2\underline{j} + 3\underline{k}, \underline{b} = \underline{i} + \underline{j} + \underline{k}$

Sol:  $\underline{a} = 4\underline{i} - 2\underline{j} + 3\underline{k}, \underline{b} = \underline{i} + \underline{j} + \underline{k}$

$$\underline{a} \cdot \underline{b} = (4)(1) + (-2)(1) + (3)(1) = 4 - 2 + 3 = 5$$

$$|\underline{a}| = \sqrt{(4)^2 + (-2)^2 + (3)^2} = \sqrt{16 + 4 + 9} = \sqrt{29}$$

$$|\underline{b}| = \sqrt{(1)^2 + (1)^2 + (1)^2} = \sqrt{1 + 1 + 1} = \sqrt{3}$$

$$\text{Projection of } \underline{a} \text{ along } \underline{b} = \frac{\underline{a} \cdot \underline{b}}{|\underline{b}|} = \frac{5}{\sqrt{3}}$$

$$\text{Projection of } \underline{b} \text{ along } \underline{a} = \frac{\underline{a} \cdot \underline{b}}{|\underline{a}|} = \frac{5}{\sqrt{29}}$$

Q.5. Find a real number  $\alpha$  so that the vectors  $\underline{u}$  and  $\underline{v}$  are perpendicular:

i.  $\underline{u} = \alpha\underline{i} + 3\underline{j} + \underline{k}, \underline{v} = \underline{i} - 2\underline{j} + \alpha\underline{k}$

Sol:  $\underline{u} \cdot \underline{v} = (\alpha)(1) + 3(-2) + 1(\alpha) = \alpha - 6 + \alpha$

Given  $\underline{u}$  and  $\underline{v}$  are perpendicular i.e.  $\underline{u} \cdot \underline{v} = 0$

$$2\alpha - 6 = 0 \Rightarrow 2\alpha = 6 \Rightarrow \alpha = 3$$

نوٹ:

Projection جس کے along لینی ہے اسی کی magnitude سے تقسیم کرتا ہے۔

$$\text{Projection along } \underline{a} = \frac{\underline{a} \cdot \underline{b}}{|\underline{a}|}$$

ii.  $\underline{u} = \alpha \underline{i} + 2\alpha \underline{j} - \underline{k}$  ,  $\underline{v} = \underline{i} + \alpha \underline{j} + 3\underline{k}$

Sol:  $\underline{u} \cdot \underline{v} = (\alpha)(1) + (2\alpha)(\alpha) + (-1)(3) = \alpha + 2\alpha^2 - 3$   
 $= 2\alpha^2 + \alpha - 3$

Since  $\underline{u}$  and  $\underline{v}$  are perpendicular i.e.  $\underline{u} \cdot \underline{v} = 0$

$$2\alpha^2 + \alpha - 3 = 0$$

$$2\alpha^2 + 3\alpha - 2\alpha - 3 = 0$$

$$\alpha(2\alpha + 3) - 1(2\alpha + 3) = 0$$

$$(2\alpha + 3)(\alpha - 1) = 0$$

$$2\alpha + 3 = 0 \Rightarrow \alpha = -\frac{3}{2} \text{ or } \alpha - 1 = 0 \Rightarrow \alpha = 1$$

Q.6. Find the number  $z$  so that the triangle with vertices  $A(3, 0, -2)$ ,  $B(0, 3, 1)$  and  $C(1, 1, z)$  is a right triangle with right at  $C$ .

Sol: Vertices:  $A(3, 0, -2)$ ,  $B(0, 3, 1)$ ,  $C(1, 1, z)$  right angle at  $C$  implies  $\vec{AC} \perp \vec{CB}$ . Therefore,  $\vec{AC} \cdot \vec{CB} = 0$

$$\vec{AC} = (1-3)\underline{i} + (1-0)\underline{j} + (z+2)\underline{k} = -2\underline{i} + \underline{j} + (z+2)\underline{k}$$

$$\vec{CB} = (0-1)\underline{i} + (3-1)\underline{j} + (1-z)\underline{k} = -\underline{i} + 2\underline{j} + (1-z)\underline{k}$$

$$\vec{AC} \cdot \vec{CB} = (-2\underline{i} + \underline{j} + (z+2)\underline{k}) \cdot (-\underline{i} + 2\underline{j} + (1-z)\underline{k}) = 0$$

$$2 + 2 + (z+2)(1-z) = 0$$

$$z - z^2 + 2 - 2z + 4 = 0$$

$$-z^2 - z + 6 = 0$$

$$z^2 + z - 6 = 0$$

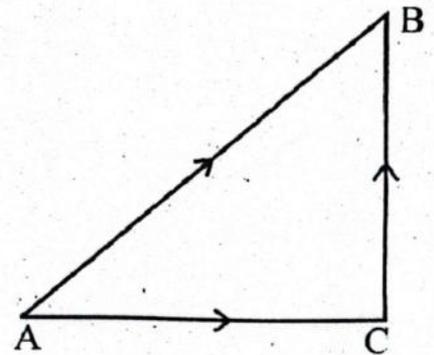
$$z^2 + 3z - 2z - 6 = 0$$

$$z(z+3) - 2(z+3) = 0$$

$$(z-2)(z+3) = 0$$

$$z+3 = 0 \quad \text{or} \quad z-2 = 0$$

$$z = -3 \quad \text{or} \quad z = 2$$



Q.7. if  $\hat{a}$  and  $\hat{b}$  are unit vectors and  $2\theta$  is the angle between them, show that  $\sin \theta = \frac{1}{2} |\hat{a} - \hat{b}|$ .

Sol:  $|\hat{a}| = 1$  ,  $|\hat{b}| = 1$  ,  $\theta' = 2\theta$

$$|\hat{a} - \hat{b}|^2 = (\hat{a} - \hat{b}) \cdot (\hat{a} - \hat{b}) = \hat{a} \cdot \hat{a} + \hat{b} \cdot \hat{b} - 2(\hat{a} \cdot \hat{b}) = |\hat{a}|^2 + |\hat{b}|^2 - 2(\hat{a} \cdot \hat{b}) = 1 + 1 - 2|\hat{a}||\hat{b}|\cos \theta'$$

$$= 2 - 2\cos(2\theta) = 2(1 - \cos 2\theta) = 2(2\sin^2 \theta) = 4\sin^2 \theta \Rightarrow |\hat{a} - \hat{b}| = 2\sin \theta \Rightarrow \sin \theta = \frac{1}{2} |\hat{a} - \hat{b}|$$

Q.8. if  $|\underline{a} + \underline{b}| = |\underline{a} - \underline{b}|$ , then show that  $\underline{a}$  and  $\underline{b}$  are perpendicular.

Sol: Given  $|\underline{a} + \underline{b}| = |\underline{a} - \underline{b}|$

$$|\underline{a} + \underline{b}|^2 = |\underline{a} - \underline{b}|^2$$

$$|a|^2 + |b|^2 + 2(a \cdot b) = |a|^2 + |b|^2 - 2(a \cdot b)$$

$$4(a \cdot b) = 0$$

$a \cdot b = 0 \Rightarrow a$  and  $b$  are perpendicular.

Q.9. i. Show that the vectors  $3\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ ,  $\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$  and  $2\mathbf{i} + \mathbf{j} - 4\mathbf{k}$  form a right triangle.

ii. Show that the set of points  $P(4, -1, 2)$ ,  $Q(1, 3, -1)$  and  $R(-2, 4, 6)$  form a right triangle.

i.  $3\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ ,  $\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$  and  $2\mathbf{i} + \mathbf{j} - 4\mathbf{k}$

Sol:  $\underline{a} = 3\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ ,  $\underline{b} = \mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$ ,  $\underline{c} = 2\mathbf{i} + \mathbf{j} - 4\mathbf{k}$

$$\text{Now, } \underline{b} + \underline{c} = \mathbf{i} - 3\mathbf{j} + 5\mathbf{k} + 2\mathbf{i} + \mathbf{j} - 4\mathbf{k} = 3\mathbf{i} - 2\mathbf{j} + \mathbf{k},$$

$$\underline{b} + \underline{c} = \underline{a}$$

So  $\underline{a}, \underline{b}, \underline{c}$  are sides of triangle

$$\text{Now, } \underline{a} \cdot \underline{c} = (3\mathbf{i} - 2\mathbf{j} + \mathbf{k}) \cdot (2\mathbf{i} + \mathbf{j} - 4\mathbf{k}) = 3(2) + (-2)(1) + (1)(-4) = 6 - 2 - 4 = 6 - 6 = 0$$

Since  $\underline{a}$  and  $\underline{c}$  are perpendicular and  $\underline{a}, \underline{b}, \underline{c}$  are sides of triangle so given sides are of right triangle.

ii.  $P(4, -1, 2)$ ,  $Q(1, 3, -1)$  and  $R(-2, 4, 6)$

Sol:  $P(4, -1, 2)$ ,  $Q(1, 3, -1)$ ,  $R(-2, 4, 6)$

$$\vec{PQ} = (1 - 4, 3 - (-1), -1 - 2) = (-3, 4, -3)$$

$$\vec{QR} = (-2 - 1, 4 - 3, 6 - (-1)) = (-3, 1, 7)$$

$$\vec{PR} = (-2 - 4, 4 - (-1), 6 - 2) = (-6, 5, 4)$$

$$\vec{PQ} + \vec{QR} = (-3, 4, -3) + (-3, 1, 7) = (-6, 5, 4) \Rightarrow \vec{PQ} + \vec{QR} = \vec{PR}$$

Hence  $P, Q, R$  are points of triangle

$$\text{Now } \vec{PQ} \cdot \vec{QR} = (-3, 4, -3) \cdot (-3, 1, 7)$$

$$\vec{PQ} \cdot \vec{QR} = (-3)(-3) + (4)(1) + (-3)(7) = 9 + 4 - 21 = -8$$

Also

$$\vec{PQ} \cdot \vec{PR} = (-3, 4, -3) \cdot (-6, 5, 4) = 18 + 20 - 12 = 26$$

$$\vec{QR} \cdot \vec{PR} = (-3, 1, 7) \cdot (-6, 5, 4) = 18 + 5 + 28 = 51$$

None of product is zero.

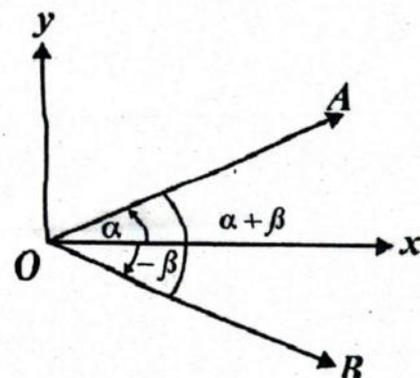
Hence  $P, Q, R$  is not right triangle.

Q.10. Prove that the  $\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$

Sol:  $\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$

Suppose  $\vec{OA}$  and  $\vec{OB}$  are unit vectors.

Then  $\vec{OA} = \cos \alpha \mathbf{i} + \sin \alpha \mathbf{j}$



$$\text{and } \vec{OB} = \cos(-\beta)\underline{i} + \sin(-\beta)\underline{j} = \cos\beta\underline{i} - \sin\beta\underline{j}$$

$$\vec{OA} \cdot \vec{OB} = (\cos\alpha\underline{i} + \sin\alpha\underline{j}) \cdot (\cos\beta\underline{i} - \sin\beta\underline{j})$$

$$\left| \vec{OA} \right| \left| \vec{OB} \right| \cos(\alpha + \beta) = \cos\alpha \cos\beta - \sin\alpha \sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha \cos\beta - \sin\alpha \sin\beta \quad \left( \left| \vec{OA} \right| = \left| \vec{OB} \right| = 1 \right)$$

**Q.11. Prove that in any triangle ABC**

i.  $b = c \cos A + a \cos C$

**Sol:** in any triangle  $\vec{AB} + \vec{BA} + \vec{CA} = 0$  or  $\underline{a} + \underline{b} + \underline{c} = 0$  or  $\underline{b} = \underline{a} + \underline{c}$

Take Dot product by  $\underline{b}$

$$-\underline{b} \cdot \underline{b} = \underline{b} \cdot \underline{a} + \underline{b} \cdot \underline{c}$$

$$-b^2 = |\underline{b}||\underline{a}|\cos(\pi - C) + |\underline{b}||\underline{c}|\cos(\pi - A)$$

**Note:**  $\cos(\pi - A) = -\cos A$  and  $\cos(\pi - C) = -\cos C$

$$-b^2 = ab(-\cos C) + bc(-\cos A) = -ab \cos C - bc \cos A$$

Divide both sides by  $-b$

$$b = c \cos A + a \cos C \quad \text{Hence Proved.}$$

ii.  $c = a \cos B + b \cos A$

**Sol:** in any triangle  $\vec{AB} + \vec{BA} + \vec{CA} = 0$  or  $\underline{a} + \underline{b} + \underline{c} = 0$  or  $-\underline{c} = \underline{a} + \underline{b}$

Taking Dot product by  $\underline{c}$

$$-\underline{c} \cdot \underline{c} = \underline{c} \cdot \underline{a} + \underline{c} \cdot \underline{b}$$

$$-c^2 = |\underline{c}||\underline{a}|\cos(\pi - B) + |\underline{c}||\underline{b}|\cos(\pi - A)$$

$$-c^2 = ca(-\cos B) + cb(-\cos A)$$

$$-c^2 = -ac \cos B + bc \cos A$$

Divide by  $-c$

$$c = a \cos B + b \cos A$$

iii.  $b^2 = c^2 + a^2 - 2ca \cos B$

**Sol:** we know that in any triangle  $\vec{AB} + \vec{BA} + \vec{CA} = 0$  or  $\underline{a} + \underline{b} + \underline{c} = 0$

$$\text{or } -\underline{b} = (\underline{a} + \underline{c}) \quad \text{--- (i)}$$

Taking dot product by  $-\underline{b}$

$$-\underline{b} \cdot (-\underline{b}) = (\underline{a} + \underline{c}) \cdot (-\underline{b})$$

$$\underline{b} \cdot \underline{b} = (\underline{a} + \underline{c}) \cdot (\underline{a} + \underline{c}) \quad \text{use - (i)}$$

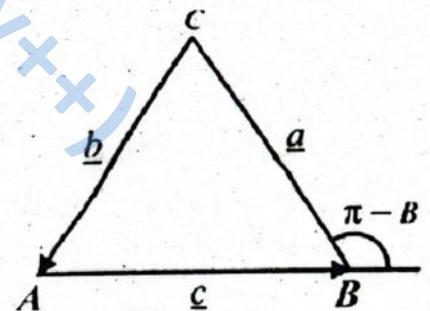
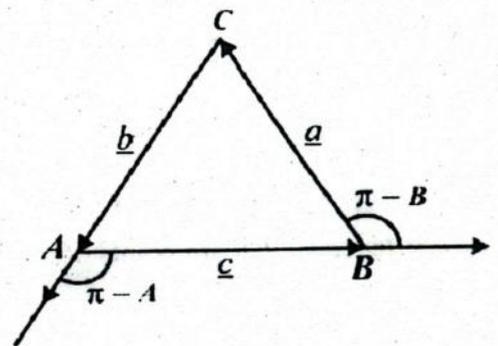
$$b^2 = \underline{a} \cdot \underline{a} + \underline{a} \cdot \underline{c} + \underline{c} \cdot \underline{a} + \underline{c} \cdot \underline{c}$$

$$b^2 = a^2 + 2\underline{a} \cdot \underline{c} + c^2$$

$$b^2 = a^2 + c^2 + 2|\underline{a}||\underline{c}|\cos(\pi - B)$$

$$b^2 = a^2 + c^2 + 2ac(-\cos B)$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$



iv.  $c^2 = a^2 + b^2 - 2ab \cos C$

Sol: we know that in any triangle  $\overline{AB} + \overline{BC} + \overline{CA} = 0$  or  $\underline{a} + \underline{b} + \underline{c} = 0$

or  $-\underline{c} = \underline{a} + \underline{b}$  \_\_\_\_\_ (i)

Taking dot product by  $-\underline{c}$

$-\underline{c} \cdot (-\underline{c}) = (\underline{a} + \underline{b}) \cdot (-\underline{c})$

$c^2 = (\underline{a} + \underline{b}) \cdot (\underline{a} + \underline{b})$  use - (i)

$c^2 = \underline{a} \cdot \underline{a} + \underline{a} \cdot \underline{b} + \underline{b} \cdot \underline{a} + \underline{b} \cdot \underline{b}$

$c^2 = \underline{a}^2 + 2\underline{a} \cdot \underline{b} + \underline{b}^2$  ( $\underline{a} \cdot \underline{b} = \underline{b} \cdot \underline{a}$ )

$c^2 = \underline{a}^2 + \underline{b}^2 + 2|\underline{a}||\underline{b}|\cos(\pi - C)$

$c^2 = \underline{a}^2 + \underline{b}^2 + 2ab(-\cos C)$

$c^2 = \underline{a}^2 + \underline{b}^2 - 2ab \cos C$

Hence Proved

Q.12. Show that for any vectors  $\underline{a}$  and  $\underline{b}$ ,  $\| |\underline{a}| - |\underline{b}| \| \leq |\underline{a} + \underline{b}| \leq |\underline{a}| + |\underline{b}|$

Sol:  $|\underline{a}| = |\underline{a} + \underline{b} - \underline{b}| \leq |\underline{a} + \underline{b}| + |-\underline{b}|$

$|\underline{a}| \leq |\underline{a} + \underline{b}| + |\underline{b}|$

$|\underline{a}| - |\underline{b}| \leq |\underline{a} + \underline{b}|$  \_\_\_\_\_ (I)

Similarly:

$|\underline{b}| = |\underline{b} + \underline{a} - \underline{a}| \leq |\underline{a} + \underline{b}| + |-\underline{a}|$

$|\underline{b}| \leq |\underline{a} + \underline{b}| + |\underline{a}|$

$|\underline{b}| - |\underline{a}| \leq |\underline{a} + \underline{b}|$  \_\_\_\_\_ (II)

Combining both parts: I & II

$\| |\underline{a}| - |\underline{b}| \| \leq |\underline{a} + \underline{b}| \leq |\underline{a}| + |\underline{b}|$

Q.13. Find the work done, if the point at which the constant force  $\underline{F} = 2\underline{i} + 5\underline{j} + 3\underline{k}$  is applied to an object, moves it from  $P_1(2, -3, 1)$  to  $P_2(7, 5, 3)$ .

Sol: Force  $\underline{F} = 2\underline{i} + 5\underline{j} + 3\underline{k}$

Initial point  $P_1(2, -3, 1)$

Final point  $P_2(7, 5, 3)$

Displacement vector  $\underline{d} = \overrightarrow{P_1P_2} = (7-2)\underline{i} + (5-(-3))\underline{j} + (3-1)\underline{k}$

$\underline{d} = 5\underline{i} + 8\underline{j} + 2\underline{k}$

Work done =  $\underline{F} \cdot \underline{d} = (2)(5) + (5)(8) + (3)(2) = 10 + 40 + 6 = 56$

Q.14. A particle, acted by constant forces  $\underline{F}_1 = 3\underline{i} + 4\underline{j} - 3\underline{k}$  and  $\underline{F}_2 = \underline{i} + 4\underline{j} - \underline{k}$ , is displaced from  $A(2, 1, 3)$  to  $B(5, 4, 4)$ . Find the work done.

Sol: Constant forces:

$\underline{F}_1 = 3\underline{i} + 4\underline{j} - 3\underline{k}$

$\underline{F}_2 = \underline{i} + 4\underline{j} - \underline{k}$

Displacement from  $A(2,1,3)$  to  $B(5,4,4)$

$$\text{Resultant force } \underline{F} = \underline{F}_1 + \underline{F}_2$$

$$\underline{F} = (3+1)\underline{i} + (4+4)\underline{j} + (-3-1)\underline{k}$$

$$\underline{F} = 4\underline{i} + 8\underline{j} - 4\underline{k}$$

$$\text{Displacement vector } \underline{d} = \overrightarrow{AB}$$

$$\underline{d} = (5-2)\underline{i} + (4-1)\underline{j} + (4-3)\underline{k}$$

$$\underline{d} = 3\underline{i} + 3\underline{j} + 1\underline{k}$$

$$\text{Work done} = \underline{F} \cdot \underline{d}$$

$$= (4)(3) + (8)(3) + (-4)(1) = 12 + 24 - 4 = 32$$

**Q.15.** A particle is displaced from the point  $A(5, -5, -7)$  to the point  $B(6, 2, -2)$  under the action of constant forces defined by  $10\underline{i} - \underline{j} + 11\underline{k}$ ,  $4\underline{i} + 5\underline{j} + 9\underline{k}$  and  $-2\underline{i} + \underline{j} - 9\underline{k}$ . Show that the total work done by the force is 102 units.

**Sol:** Forces

$$\underline{F}_1 = 10\underline{i} - \underline{j} + 11\underline{k}, \quad \underline{F}_2 = 4\underline{i} + 5\underline{j} + 9\underline{k}, \quad \underline{F}_3 = -2\underline{i} + \underline{j} - 9\underline{k}$$

Points:

$$A(5, -5, -7), \quad B(6, 2, -2)$$

Resultant Force  $\underline{F}$

$$\underline{F} = (10+4-2)\underline{i} + (-1+5+1)\underline{j} + (11+9-9)\underline{k}$$

$$\underline{F} = 12\underline{i} + 5\underline{j} + 11\underline{k}$$

Displacement vector

$$\underline{d} = (6-5)\underline{i} + (2-(-5))\underline{j} + (-2-(-7))\underline{k}$$

$$\underline{d} = 1\underline{i} + 7\underline{j} + 5\underline{k}$$

$$\text{Work Done} = \underline{F} \cdot \underline{d}$$

$$\text{Work done} = (12)(1) + (5)(7) + (11)(5) = 12 + 35 + 55 = 102 \text{ units.}$$

**Q.16.** A force of magnitude 6 units acting parallel to  $4\underline{i} + 3\underline{j} - \underline{k}$  displace the point of application from  $A(2, -1, 3)$  to  $B(7, 3, 2)$  Find the work done.

**Sol:**  $\underline{v} = 4\underline{i} + 3\underline{j} - \underline{k}$

$$|\underline{v}| = \sqrt{(4)^2 + (3)^2 + (-1)^2} = \sqrt{16+9+1} = \sqrt{26}$$

$$\underline{F} = 6 \left( \frac{4\underline{i} + 3\underline{j} - \underline{k}}{\sqrt{26}} \right) = \frac{24}{\sqrt{26}}\underline{i} + \frac{18}{\sqrt{26}}\underline{j} - \frac{6}{\sqrt{26}}\underline{k}$$

$$A(2, -1, 3) \text{ to } B(7, 3, 2)$$

$$\text{Displacement } \underline{d} = (7-2)\underline{i} + (3-(-1))\underline{j} + (2-3)\underline{k} \Rightarrow \underline{d} = 5\underline{i} + 4\underline{j} - 1\underline{k}$$

$$\text{Work Done} = \underline{F} \cdot \underline{d}$$

$$= \left( \frac{24}{\sqrt{26}} \right)(5) + \left( \frac{18}{\sqrt{26}} \right)(4) + \left( \frac{-6}{\sqrt{26}} \right)(-1) = \frac{120}{\sqrt{26}} + \frac{72}{\sqrt{26}} + \frac{6}{\sqrt{26}} = \frac{120+72+6}{\sqrt{26}} = \frac{198}{\sqrt{26}}$$