Exercise 14.4

Q.1. Find the volume of the parallelepiped for which the given vectors are three edges.

Sol: (i) $\underline{u} = 3\underline{i} + 2\underline{k},\ \underline{v} = \underline{i} + 2\underline{j} + \underline{k},\ \underline{w} = -\underline{j} + 4\underline{k}$

Volume of parallelepiped $= \underline{u}.\underline{v}\times\underline{w}$

$$= \begin{vmatrix} 3 & 0 & 2 \\ 1 & 2 & 1 \\ 0 & -1 & 4 \end{vmatrix} = 3(8+1) - 0 + 2(-1-0) = 27 - 2 = 25$$

(ii) $\underline{u} = \underline{i} - 4\underline{j} - \underline{k},\ \underline{v} = \underline{i} - \underline{j} - 2\underline{k},\ \underline{w} = 2\underline{i} - 3\underline{j} + \underline{k}$

Volume of parallelepiped $= \underline{u}.\underline{v}\times\underline{w}$

$$= \begin{vmatrix} 1 & -4 & -1 \\ 1 & -1 & -2 \\ 2 & -3 & 1 \end{vmatrix} = 1(-1-6) - (-4)(1+4) + (-1)(-3+2) = -7 + 20 + 1 = 14$$

(iii) $\underline{u} = \underline{i} - 2\underline{j} + 3\underline{k},\ \underline{v} = 2\underline{i} - \underline{j} - \underline{k},\ \underline{w} = \underline{j} + \underline{k}$

Volume of parallelepiped $= \underline{u}.\underline{v}\times\underline{w}$

$$= \begin{vmatrix} 1 & -2 & 3 \\ 2 & -1 & -1 \\ 0 & 1 & 1 \end{vmatrix} = 1(-1+1) - (-2)(2+0) + 3(2-0) = 0 + 4 + 6 = 10$$

Q.2. Verify that $\underline{a}.\underline{b}\times\underline{c} = \underline{b}.\underline{c}\times\underline{a} = \underline{c}.\underline{a}\times\underline{b}$

if $\underline{a} = 3\underline{i} - \underline{j} + 5k,\ \underline{b} = 4\underline{i} + 3\underline{j} - 2\underline{k}$ and $\underline{c} = 2\underline{i} + 5\underline{j} + \underline{k}$

Sol: $\underline{a} = 3\underline{i} - \underline{j} + 5\underline{k},\ \underline{b} = 4\underline{i} + 3\underline{j} - 2\underline{k},\ \underline{c} = 2\underline{i} + 5\underline{j} + \underline{k}$

$$\underline{a}.\underline{b}\times\underline{c} = \begin{vmatrix} 3 & -1 & 5 \\ 4 & 3 & -2 \\ 2 & 5 & 1 \end{vmatrix} = 3(3+10) + 1(4+4) + 5(20-6) = 39 + 8 + 70 = 117$$

$$\underline{b}.\underline{c}\times\underline{a} = \begin{vmatrix} 4 & 3 & -2 \\ 2 & 5 & 1 \\ 3 & -1 & 5 \end{vmatrix} = 4(25+1) - 3(10-3) - 2(-2-15) = 104 - 21 + 34 = 117$$

$$\underline{c}.\underline{a}\times\underline{b}=\begin{vmatrix}2 & 5 & 1\\ 3 & -1 & 5\\ 4 & 3 & -2\end{vmatrix}=2(2-15)-5(-6-20)+1(9+4)=-26+130+13=117$$

Hence $\underline{a}.\ \underline{b}\times\underline{c}=\underline{b}.\ \underline{c}\times\underline{a}=\underline{a}\times\underline{b}$

Q.3. Prove that the vectors $\underline{i}-2\underline{j}+3\underline{k},-2\underline{i}+3\underline{j}-4\underline{k}$ and $\underline{i}-3\underline{j}+5\underline{k}$ are coplanar.

Sol: $\underline{u}=\underline{i}-2\underline{j}+3\underline{k},\ \underline{v}=-2\underline{i}+3\underline{j}-4\underline{k},\ \underline{w}=\underline{i}-3\underline{j}+5\underline{k}$

$$\underline{u}.\underline{v}\times\underline{w}=\begin{vmatrix}1 & -2 & 3\\ -2 & 3 & -4\\ 1 & -3 & 5\end{vmatrix}=1(15-12)-(-2)(-10+4)+3(6-3)=3-12+9=12-12=0$$ So given vectors are coplanar.

Q.4. Find the constant α such that the vectors are coplanar.

Sol: (i) $\underline{i}-\underline{j}+\underline{k}\ ,\ \underline{i}-2\underline{j}-3\underline{k}$ and $3\underline{i}-\alpha\underline{j}+5\underline{k}$

let $\underline{u}=\underline{i}-\underline{j}+\underline{k},\underline{v}=\underline{i}-2\underline{j}-3\underline{k}$ *and* $\underline{w}=3\underline{i}-\alpha\underline{j}+5\underline{k}$ are coplanar if

$\underline{u}.\underline{v}\times\underline{w}=0$

$$\underline{u}.\underline{v}\times\underline{w}=\begin{vmatrix}1 & -1 & 1\\ 1 & -2 & -3\\ 3 & -\alpha & 5\end{vmatrix}=0\Rightarrow 1(-10-3\alpha)-(-1)(5+9)+1(-\alpha+6)=0\ \Rightarrow\ -10-3\alpha+14-\alpha+6=0$$

or $-4\alpha+10=0\Rightarrow 4\alpha=10\Rightarrow\alpha=\frac{10}{4}\Rightarrow\alpha=\frac{5}{2}$

(ii) $\underline{i}-2\alpha\underline{j}-\underline{k}\ ,\ \underline{i}-2\underline{j}+2\underline{k}\ ,\ \alpha\underline{i}-2\underline{j}+\underline{k}$

Let $\underline{u}=\underline{i}-2\alpha\underline{j}-\underline{k},\underline{v}=\underline{i}-2\underline{j}+2\underline{k},\underline{w}=\alpha\underline{i}-2\underline{j}+\underline{k}$

$\underline{u},\underline{v},\underline{w}$ are coplanar if $\underline{u}.\underline{v}\times\underline{w}=0$

$$So\ \begin{vmatrix}1 & -2\alpha & -1\\ 1 & -2 & 2\\ \alpha & -2 & 1\end{vmatrix}=1(-2+4)-(-2\alpha)(1-2\alpha)+(-1)(-2+2\alpha)=0$$

$\Rightarrow 2+2\alpha-4\alpha^2+2-2\alpha=0\Rightarrow -4\alpha^2+4=0$

$\alpha^2=\frac{4}{4}=1\Rightarrow\alpha=\pm1$

Q.5. Prove that the points whose position vectors are $A\left(-6\underline{i}+3\underline{j}+2\underline{k}\right),B\left(3\underline{i}-2\underline{j}+4\underline{k}\right),C\left(5\underline{i}+7\underline{j}+3\underline{k}\right),$ $D\left(-13\underline{i}+17\underline{j}-\underline{k}\right)$ are coplanar.

Sol: $\overrightarrow{OA}=-6\underline{i}+3\underline{j}+2\underline{k}\ ,\ \overrightarrow{OB}=3\underline{i}-2\underline{j}+4\underline{k}$

$\overrightarrow{OC}=5\underline{i}+7\underline{j}+3\underline{k}\ ,\ \overrightarrow{OD}=-13\underline{i}+17\underline{j}-\underline{k}$

$\overrightarrow{AB}=\overrightarrow{OB}-\overrightarrow{OA}=3\underline{i}-2\underline{j}+4\underline{k}+6\underline{i}-3\underline{j}-2\underline{k}=9\underline{i}-5\underline{j}+2\underline{k}$

$\overrightarrow{AC}=\overrightarrow{OC}-\overrightarrow{OA}=5\underline{i}+7\underline{j}+3\underline{k}+6\underline{i}-3\underline{j}-2\underline{k}=11\underline{i}+4\underline{j}+\underline{k}$

$\overrightarrow{AD}=\overrightarrow{OD}-\overrightarrow{AD}=-13\underline{i}+17\underline{j}-\underline{k}+6\underline{i}-3\underline{j}-2\underline{k}=-7\underline{i}+14\underline{j}-3\underline{k}$

$$\overrightarrow{AB}\,.\,\overrightarrow{AC}\times\overrightarrow{AD}=\begin{vmatrix}9 & -5 & 2\\ 11 & 4 & 1\\ -7 & 14 & -3\end{vmatrix}=9(-12-14)-(-5)(-33+7)+2(154+28)$$

$$=-234-130+364=0$$

Q.6. a) Find the value of i. $2\underline{i}\times 2\underline{j}.\underline{k}$ ii. $3\underline{j}.\underline{k}\times\underline{i}$ iii. $[\underline{k}\,\underline{i}\,\underline{j}]$ iv. $[\underline{i}\,\underline{i}\,\underline{k}]$

b) Prove that $\underline{u}.(\underline{v}\times\underline{w})+\underline{v}.(\underline{w}\times\underline{u})+\underline{w}.(\underline{u}\times\underline{v})=3\underline{u}.(\underline{v}\times\underline{w})$

Sol: (i) $2\underline{i}\times 2\underline{j}.\underline{k}=4(\underline{i}\times\underline{j}).\underline{k}=4\underline{k}\,.\underline{k}=4(1)=4$

(ii) $3\underline{j}.\underline{k}\times\underline{i}=3\underline{j}.\underline{j}=3(1)=3$

(iii) $[\underline{k}\,\underline{i}\,\underline{j}]=\underline{k}.\underline{i}\times\underline{j}=\underline{k}.\underline{k}=1$

(iv) $[\underline{i}\,\underline{i}\,\underline{k}]=\underline{i}.\underline{i}\times\underline{k}=\underline{i}.(-\underline{j})=-\underline{i}.\underline{j}=0$

b) $\underline{u}.(\underline{v}\times\underline{w})+\underline{v}.(\underline{w}\times\underline{u})+\underline{w}.(\underline{u}\times\underline{v})=3\underline{u}.(\underline{v}\times\underline{w})$

let $\underline{u}=a_1\underline{i}+b_1\underline{j}+c_1\underline{k}$, $\underline{v}=a_2\underline{i}+b_2\underline{j}+c_2\underline{k}$, $\underline{w}=a_3\underline{i}+b_3\underline{j}+c_3\underline{k}$ then $\underline{u}\,.(\underline{v}\times\underline{w})=\begin{vmatrix}a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\\ a_3 & b_3 & c_3\end{vmatrix}$

$$=-\begin{vmatrix}a_2 & b_2 & c_2\\ a_1 & b_1 & c_1\\ a_3 & b_3 & c_3\end{vmatrix}\text{ Interchanging } R_1 \text{ and } R_2$$

$$=(-)(-)\begin{vmatrix}a_2 & b_2 & c_2\\ a_3 & b_3 & c_3\\ a_1 & b_1 & c_1\end{vmatrix}\text{ Interchanging } R_2 \text{ and } R_3$$

$=\underline{v}.(\underline{w}\times\underline{u}) \Rightarrow \underline{u}.(\underline{v}\times\underline{w})=\underline{v}.(\underline{w}\times\underline{u})$ (I)

Similarly we can prove $=\underline{w}.(\underline{u}\times\underline{v})=\underline{u}.(\underline{v}\times\underline{w})$ (II)

Thus $\underline{u}.(\underline{v}\times\underline{w})=\underline{v}.(\underline{w}\times\underline{u})=\underline{w}.(\underline{u}\times\underline{v})$

Now L.H.S $=\underline{u}.(\underline{v}\times\underline{w})+\underline{v}.(\underline{w}\times\underline{u})+\underline{w}.(\underline{u}\times\underline{v})$

$=\underline{u}.(\underline{v}\times\underline{w})+\underline{u}.(\underline{v}\times\underline{w})+\underline{u}.(\underline{v}\times\underline{w})=3\underline{u}.(\underline{v}\times\underline{w})=$ R.H.S

Hence proved.

Q.7. **Find volume of the Tetrahedron with the Vertices**

Sol: (i) $A(0,1,2)$ $B(3,2,1)$ $C(1,2,1)$, $D(5,5,6)$

$\overrightarrow{AB}=(3-0)\underline{i}+(2-1)\underline{j}+(1-2)\underline{k}=3\underline{i}+\underline{j}-\underline{k}$

$\overrightarrow{AC}=(1-0)\underline{i}+(2-1)\underline{j}+(1-2)\underline{k}=\underline{i}+\underline{j}-\underline{k}$

$\overrightarrow{AD}=(5-0)\underline{i}+(5-1)\underline{j}+(6-2)\underline{k}=5\underline{i}+4\underline{j}+4\underline{k}$

$$\overrightarrow{AB}.\overrightarrow{AC}\times\overrightarrow{AD}=\begin{vmatrix}3 & 1 & -1\\ 1 & 1 & -1\\ 5 & 4 & 4\end{vmatrix}=3(4+4)-1(4+5)+(-1)(4-5)=24-9+1=16$$

Volume of tetrahedron $=\frac{1}{6}\left[\overrightarrow{AB}.\overrightarrow{AC}\times\overrightarrow{AD}\right]=\frac{16}{6}=\frac{8}{3}$ cubic Units.

(ii) $A(2,1,8)$ $B(3,2,9)$, $C(2,1,4)$, $D(3,3,10)$

$$\vec{AB}=(3-2)\underline{i}+(2-1)\underline{j}+(9-8)\underline{k}=\underline{i}+\underline{j}+\underline{k}$$

$$\vec{AC}=(2-2)\underline{i}+(1-1)\underline{j}+(4-8)\underline{k}=0\underline{i}+0\underline{j}-4\underline{k}$$

$$\vec{AD}=(3-2)\underline{i}+(3-1)\underline{j}+(10-8)\underline{k}=\underline{i}+2\underline{j}+2\underline{k}$$

$$\vec{AB}.\vec{AC}\times\vec{AD}=\begin{vmatrix}1 & 1 & 1\\ 0 & 0 & -4\\ 1 & 2 & 2\end{vmatrix}=1(0+8)-1(0+4)+1(0-0)=8-4+0=4$$

Volume of tetrahedron $=\frac{1}{6}\left[\vec{AB}.\vec{AC}\times\vec{AD}\right]=\frac{1}{6}(4)=\frac{2}{3}$ cubic Units

Q.8. **Prove that the points whose position vectors are $A(3\underline{i}+2\underline{j}-\underline{k}), B(\underline{i}-2\underline{j}+\underline{k}), C(6\underline{i}+4\underline{j}-2\underline{k}), D(9\underline{i}+6\underline{j}-3\underline{k})$ are coplanar.**

Sol: $\vec{OA}=3\underline{i}+2\underline{j}-\underline{k}$, $\vec{OB}=\underline{i}-2j+k$

$\vec{OC}=6\underline{i}+4\underline{j}-2\underline{k}$, $\vec{OD}=9\underline{i}+6j-3k$

$$\vec{AB}=\vec{OB}-\vec{OA}$$

$$\vec{AB}=\underline{i}-2\underline{j}+\underline{k}-3\underline{i}-2j+k$$

$$\vec{AB}=-2\underline{i}-4\underline{j}+2\underline{k}$$

$$\vec{AC}=\vec{OC}-\vec{OA}=6\underline{i}+4\underline{j}-2\underline{k}-3\underline{i}-2\underline{j}+\underline{k}$$

$$\vec{AC}=3\underline{i}+2\underline{j}-\underline{k}$$

$$\vec{AD}=\vec{OD}-\vec{OA}=9\underline{i}+6\underline{j}-3\underline{k}-3\underline{i}-2\underline{j}+\underline{k}=6\underline{i}+4\underline{j}-2\underline{k}$$

$$\vec{AB}.\vec{AC}\times\vec{AD}=\begin{vmatrix}-2 & -4 & 2\\ 3 & 2 & -1\\ 6 & 4 & -2\end{vmatrix}=-2(-4+4)-(-4)(-6+6)+2(12-12)=0$$ Hence Coplanar

Q.9. **Prove that for any three non-zero $\underline{u},\underline{v}$ and $\underline{w}$ $(\underline{u}+\underline{v}).\left[(\underline{v}+\underline{w})\times(\underline{w}+\underline{u})\right]=2\left[\underline{u}\,\underline{v}\,\underline{w}\right]$**

Sol: L.H.S $(\underline{u}+\underline{v}).\left[(\underline{v}+\underline{w})\times(\underline{w}+\underline{u})\right]$

$=(\underline{u}+\underline{v}).\left[\underline{v}\times\underline{w}+\underline{v}\times\underline{u}+\underline{w}\times\underline{w}+\underline{w}\times\underline{u}\right]=(\underline{u}+\underline{v}).\left[\underline{v}\times\underline{w}+\underline{v}\times\underline{u}+0+\underline{w}\times\underline{u}\right]$

$=\underline{u}.(\underline{v}\times\underline{w})+\underline{u}.(\underline{v}\times\underline{u})+\underline{u}.(\underline{w}\times\underline{u})+\underline{v}.(\underline{v}\times\underline{w})+\underline{v}.(\underline{v}\times\underline{u})+\underline{v}.(\underline{w}\times\underline{u})$

$=\left[\underline{u}\,\underline{v}\,\underline{w}\right]+0+0+0+0+\left[\underline{u}\,\underline{v}\,\underline{w}\right]=2\left[\underline{u}\,\underline{v}\,\underline{w}\right]$

Q.10. **Consider a parallelepiped determined by the vector $\underline{u}=2\underline{i}+4\underline{j}-3\underline{k}$, $\underline{v}=5\underline{i}-3\underline{j}+6\underline{k}$ and $\underline{w}=4\underline{i}-7\underline{j}-2\underline{k}$. If the base of the parallelepiped is defined by the vectors $\underline{u}$ and $\underline{v}$ then find the height of the parallelepiped**

Sol: $\underline{u}=2\underline{i}+4\underline{j}-3\underline{k}$

$\underline{v}=5\underline{i}-3\underline{j}+6\underline{k}$

$\underline{w} = 4\underline{i} - 7\underline{j} - 2\underline{k}$

$$\underline{u} \times \underline{v} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 2 & 4 & -3 \\ 5 & -3 & 6 \end{vmatrix} = 15\underline{i} - 27\underline{j} - 26\underline{k}$$

$$\underline{u} \times \underline{w} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 5 & -3 & 6 \\ 4 & -7 & -2 \end{vmatrix} = 48\underline{i} + 34\underline{j} - 23\underline{k}$$

$$|\underline{u} \times \underline{v}| = \sqrt{(15)^2 + (-27)^2 + (-26)^2} = \sqrt{1630}$$

$$\underline{u}.(\underline{v} \times \underline{w}) = (2\underline{i} + 4\underline{j} - 3\underline{k}).(48\underline{i} + 34\underline{j} - 23\underline{k}) = 301$$

$$\text{Height} = \frac{|\underline{u}.(\underline{v} \times \underline{w})|}{|\underline{u} \times \underline{v}|} = \frac{|301|}{\sqrt{1630}} = \frac{301}{\sqrt{1630}} = \frac{301}{\sqrt{1630}} \times \frac{\sqrt{1630}}{\sqrt{1630}} = \frac{301\sqrt{1630}}{1630}$$

Q.11. A mechanic applies a force of 50 pounds along the positive x-axis on a wrench connected to a bolt. The pivot point of the wrench is at the origin $(0,0,0)$, and the force is applied at the point $(0\ ft, 2\ ft, 3\ ft)$. Determine the torque produced by this force about the pivot point.

Sol: The position vector r from the pivot point to the point where the force is applied is:

$\underline{r} = (0,2,3)\ ft$

The force vector $\underline{F}$ applied along the positive x-axis is:

$\underline{F} = (50,0,0)\ pounds$

The torque $\tau = \underline{r} \times \underline{F} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & 2 & 3 \\ 50 & 0 & 0 \end{vmatrix}$

Expanding the determinant:

$$\tau = \underline{i}\begin{vmatrix} 2 & 3 \\ 0 & 0 \end{vmatrix} - \underline{j}\begin{vmatrix} 0 & 3 \\ 50 & 0 \end{vmatrix} + \underline{k}\begin{vmatrix} 0 & 2 \\ 50 & 0 \end{vmatrix} = 0\underline{i} - (-150)\underline{j} + (-100)\underline{k}$$

$\tau = 0\underline{i} + 150\underline{j} - 100\underline{k}$

$\tau = (0,150,-100)\ ft.pounds$

The torque produced by the force about the pivot point is $(0,150,-100)$ foot-pounds.

$|\tau| = \sqrt{\tau_x^2 + \tau_y^2 + \tau_z^2}$

Substituting the components of our torque vector:

$$|\tau| = \sqrt{(0)^2 + (150)^2 + (-100)^2} = \sqrt{0 + 22500 + 10000} = \sqrt{32500}$$

Now, let's simplify the square root:

$$|\tau| = \sqrt{2500.13} = \sqrt{2500}\ .\ \sqrt{13} = 50\sqrt{13}$$

So, the magnitude of the torque is $50\sqrt{13}$ foot pounds. If you need a decimal approximation:

$|\tau| \approx 50 \times 3.6056$

$|\tau| \approx 180.28\ ft$. Pounds

The magnitude of the torque produced is approximately 180.28 foot-pounds.

Q.12. A drone flies from point $(1,2,5)$ to point $(4,6,9)$, with each unit representing a meter. What is the magnitude of the displacement the drone experienced during this flight?

Sol: Identify the initial and final positions:

Initial position:

$$(x_1, y_1, z_1) = (1,2,5)$$

Final position:

$$(x_2, y_2, z_2) = (4,6,9)$$

Calculate the displacement vector: The displacement vector d is found by subtracting the initial position vector from the final position vector:

$$\underline{d} = (x_2 - x_1, y_2 - y_1, z_2 - z_1)$$

$$\underline{d} = (4-1, 6-2, 9-5)$$

$$\underline{d} = (3,4,4)$$

Determine the magnitude of the displacement vector: the magnitude of a vector $d = (d_x, d_y, d_z)$ is given by formula:

$$|d| = \sqrt{d_x^2 + d_y^2 + d_z^2}$$

Plugging the components of our displacement vector:

$$|d| = \sqrt{(3)^2 + (4)^2 + (4)^2} = \sqrt{9+16+16} = \sqrt{41}$$

So, the magnitude of the displacement the drone experienced is $\sqrt{41}$

Q.13. The vector $\underline{u} = 50\underline{i} + 75\underline{j} + 65\underline{k}$ shows how many belts, pants, and shirts were sold at a store. The vector $\underline{w} = 1500\underline{i} + 3500\underline{j} + 3000\underline{k}$ show the price (in rupees) of each item. Find $\underline{u}.\underline{w}$ and explain what the result tells us in real life.

Sol: Given the vectors $\underline{u} = 50\underline{i} + 75\underline{j} + 65\underline{k}$ and $\underline{w} = 1500\underline{i} + 3500\underline{j} + 3000\underline{k}$, the dot product $\underline{u}.\underline{w}$ is calculated as follows: $\underline{u}.\underline{w} = (50\times1500) + (75\times3500) + (65\times3000)$

$$\underline{u}.\underline{w} = 75000 + 262500 + 195000$$

$$\underline{u}.\underline{w} = 532500$$

The result, 532500, represents the total revenue generated from selling belts, pants, and shirts.
In real life, this calculation tells the store owner the total sales amount from these items.
For example:
Belts: 50 units sold $\times 1500$ rupees / unit $= 75000$ rupees.
Pants: 75 units sold $\times 3500$ rupees / unit $= 262500$ rupees.
Shirts: 65 units sold $\times 3000$ rupees / unit $= 195000$ rupees
Total revenue: 532500 rupees. This information can help the store owner analyze sales performance and make informed decisions.

Q.14. A Force $\underline{F} = (20,-10,30)N$ is applied at a point $P(2,-1,4)$ in $3D$ space. The pivot point is at $M(1,2,-3)$. Calculate the torque produced by this force about the pivot point M

Sol: *To calculate the torque* $(\underline{\tau})$ produced by the force $\underline{F}$ about the pivot point M, we first need to find the vector r from the pivot point M to the point of application of the force P.

The position vector of point P is $\overrightarrow{OP} = 2\underline{i} - \underline{j} + 4\underline{k}$

The position vector of point M is $\overrightarrow{OM} = \underline{i} + 2\underline{j} - 3\underline{k}$

The vector r is given by:

$$\underline{r} = \overrightarrow{OP} - \overrightarrow{OM} = 2\underline{i} - \underline{j} + 4\underline{k} - \underline{i} - 2\underline{j} + 3\underline{k} = \underline{i} - 3\underline{j} + 7\underline{k}$$

The force is given as $\underline{F} = (20, -10, 30)\,N$.

The toque τ is the cross product of the position vector r and the force vector F:

$$\underline{\tau} = r \times F$$

We can calculate the cross product using the determine of a matrix:

$$\underline{\tau} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 1 & -3 & 7 \\ 20 & -10 & 30 \end{vmatrix} = -20\underline{i} + 110\underline{j} + 50\underline{k}$$

So, the torque produced by the force about the pivot point M is $\tau = (-20, 110, 50)\,Nm$.

To find the magnitude of the torque vector $\tau = (-20, 110, 50)\,Nm$, we use the formula for the magnitude of a $3D$ vector:

$$|\underline{\tau}| = \sqrt{(\tau_x)^2 + (\tau_y)^2 + (\tau_z)^2}$$

Substituting the components of the toque vector

$$|\underline{\tau}| = \sqrt{(-20)^2 + (110)^2 + (50)^2} = \sqrt{400 + 12100 + 2500} = \sqrt{15000}$$

To simplify the square root:

$$|\underline{\tau}| = \sqrt{100 \times 150} = 10\sqrt{150} = 10\sqrt{25 \times 6} = 10 \times 5\sqrt{6} = 50\sqrt{6}$$

The magnitude of the toque produced by the force about the pivot point M is $50\sqrt{6}\ Nm$.

We can also approximate the numerical value: $\sqrt{6} \approx 2.449$[1]

$$|\underline{\tau}| \approx 50 \times 2.449 \approx 122.45\,Nm$$

Q.15. **An electric shop sells three types of appliances: Fans, Heaters, and Ovens. The monthly sales quantities are 500 units 300 units of Heaters and 200 units of Ovens. The profit per unit for each appliance is Rs. 500 for Fans, Rs 400 for Heaters, and Rs 2,000 for Ovens.**

a) Represent the monthly sales quantities and the profit per unit as vectors.

b) Calculate the total monthly profit using vector operations.

Sol: **a)** Vector Representation:

Monthly sales quantity vector: $\underline{s} = \begin{pmatrix} 500 \\ 300 \\ 200 \end{pmatrix}$

Profit per unit vector: $\underline{p} = \begin{pmatrix} 500 \\ 400 \\ 2000 \end{pmatrix}$

b) Total Monthly profit calculation (using dot product):

$$Total\ profit = \underline{s}.\underline{p}$$

$$Total\ profit = (500 \times 500) + (300 \times 400) + (200 \times 2000)$$

$$Total\ profit = 250000 + 120000 + 400000$$

$$Total\ profit = 770000$$