STUDENT'S LEARNING OUTCOMES (SLO's)

After studying this unit, the students will be able to:

- Make reasonable estimates of value of physical quantities [of those quantities that are discussed in the topics of this grade].
- Use the conventions for indicating units, as set out in the SI units.
- Express derived units as products or quotients of the SI base units
- Analyze the homogeneity of physical equations [Through dimensional analysis]
- Derive formulae in simple cases [Through using dimensional analysis]
- Analyze and critique the accuracy and precision of data collected by measuring instruments
- Justify why all measurements contain some uncertainty.
- Assess the uncertainty in a derived quantity [By simple addition of absolute, fractional or percentage uncertainties]
- Quote answers with correct scientific notations, number of significant figures and units in all experimental and numerical results.

INTRODUCTION

1. What is Physics? Describe its importance and comforts?

Ans. Physics is the branch of science that deals with the study of matter, energy, and the fundamental forces of nature.

It explores how objects move, interact, and behave under various conditions. Physics seeks to understand the laws and principles that govern the universe, from the smallest particles to the largest galaxies.

Importance of Physics

(i) **Understanding the Universe:** Physics helps us comprehend the fundamental workings of the natural world, including the origin and structure of the universe.

(ii) **Technological Advancements:** Many modern technologies—such as electricity, computers, smartphones, and medical imaging devices—are based on principles of physics.

(iii) **Everyday Applications:** Physics explains everyday phenomena like gravity, sound, light, and heat, making it essential for solving practical problems.

(iv) **Foundation for Other Sciences:** Physics forms the basis of many other scientific fields, including chemistry, astronomy, and engineering.

(v) **Innovation and Development:** Knowledge of physics drives innovation in industries like space exploration, renewable energy, transportation, and communication.

Physics is crucial for scientific understanding, technological progress, and improving our quality of life.

Comforts Provided by Physics

(i) **Electricity and Appliances:** Physics principles allow us to generate and use electricity, powering lights, fans, refrigerators, air conditioners, and other household devices.

(ii) **Communication:** Technologies like mobile phones, television, and the internet are made possible through physics-based innovations in waves and electromagnetism.

(iii) **Transportation:** Cars, airplanes, and trains operate on the principles of mechanics and thermodynamics, which are branches of physics.

(iv) Medical Advancements: Tools like X-rays, MRI machines, and radiation therapy in medicine are developed through physics.

Physics is not only essential for understanding the universe but also for providing the comforts and technologies we rely on in daily life.

1.1 PHYSICAL QUANTITIES AND THEIR UNITS

2. What are physical quantities? Explain.

Ans. Physical Quantities

Definition

A physical quantity is a property of a material or system that can be measured and expressed using a number (magnitude) and a unit.

Examples: All base and derived quantities.

Base Quantities

Definition

Base quantities are the fundamental physical quantities that are independent and from which other quantities are derived.

Examples: Length, mass, time, temperature, etc.

Derived Quantities

Definition

Derived quantities are the physical quantities that are derived from the base quantities using mathematical relationships.

Examples: Force, work, power, pressure, etc.

Areas of Physics	Interdisciplinary areas of Physics
Mechanics	Astrophysics
Heat & thermodynamics	Biophysics
Electromagnetism	Chemical physics
Optics	Engineering physics
Sound	Geophysics
Hydrodynamics	Medical physics
Special relativity	Physical oceanography
General relativity	Physics of music
Quantum mechanics	
Atomic physics	
Molecular physics	
Nuclear physics	
Solid state physics	
Particle physics	
Superconductivity	
Superfluidity	
Magnetohydrodynamics	
Space physics	

1.2 INTERNATIONAL SYSTEM OF UNITS

3. What is International System of units? Describe base quantities and derived quantities.

Ans. International System of Units

In 1960, an international committee agreed on a set of definitions and standards to describe the physical quantities. The system that was established is called the System International (SI).

The International system of units (SI) is the globally accepted standard for measurement. It is used in science, industry, and everyday life to ensure consistency and clarity in measurements across the world.

Base Units

Definition

A base unit is a unit of measurement adopted for a base quantity.

There are seven base units for physical quantities namely: length, mass, time, temperature, electric current, light or luminous intensity and amount of substance (with special reference to the number of particles). Prefixes such as milli, micro, kilo, etc. may be used with them to express smaller or larger quantities.

The names of base units for these physical quantities together with symbols are given in table (1.1).

Table 1.1: Base Units		
Physical Quantity	**SI Unit**	**Symbol**
Length	Metre	m
Mass	Kilogram	kg
Time	Second	s
Electric current	Ampere	A
Thermodynamic temperature	Kelvin	K
Intensity of light	Candela	Cd
Amount of substance	Mole	mol

Derived Units

Definition

A *derived unit* is a unit that result from mathematical combination of SI base units.

The units of plane angle and solid angle have also been included in the list of derived units since 1995.

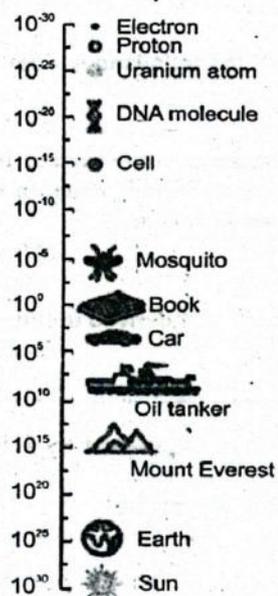| Table 1.2: Derived Units			
Physical Quantity	**Unit**	**Symbol**	**In terms of base units**
Plane angle	radian	rad	Dimensionless
Solid angle	steradian	sr	dimensionless
Force	newton	N	$kg\ m\ s^{-2}$
Work	joule	J	$N\ m = kg\ m^2\ s^{-2}$
Power	watt	W	$J\ s^{-1} = kg\ m^2\ s^{-3}$
Electric charge	coulomb	C	$A\ s$
Pressure	pascal	Pa	$N\ m^{-2} = kg\ m^{-1}\ s^{-2}$

4. In addition to base and derived units, which other units does System International permit?

Ans. The SI unit permits the use of certain additional units in addition to base and derive units. Such as:

- the traditional mathematical units for measuring angles (degree, arc minute, and arc second).
- the traditional units for standard time are (minute, hour, day, and year).
- the logarithmic units bel (and its multiples, such as the decibel).
- two metric units commonly used in ordinary life: the litre for volume and the tonne (metric ton) for large masses.
- two non-metric scientific units are atomic mass unit (m) and the electron volt (eV).
- the nautical mile and knot; units traditionally used at sea and in meteorology.
- the acre and hectare, common metric units of land area.
- the bar is a unit of pressure and it is commonly used as the millibar in meteorology and the kilobar in engineering.
- the angstrom and the barn, units used in physics and astronomy.

Interesting Information

10^{-30}	• Electron / o Proton
10^{-25}	Uranium atom
10^{-20}	DNA molecule
10^{-15}	• Cell
10^{-10}	
10^{-5}	Mosquito
10^{0}	Book
10^{5}	Car
10^{10}	Oil tanker
10^{15}	Mount Everest
10^{20}	
10^{25}	Earth
10^{30}	Sun

Order of magnitude of some masses.

MCQ's

1. Mass is a which quantity:
(A) Derived
(B) Base ✓
(C) Both derived and base
(D) None of these

2. Which one is not a base unit in SI?
(A) kilogram
(B) joule ✓
(C) kelvin
(D) ampere

3. Unit used for the factor $\sqrt{\dfrac{l}{g}}$ may be:
(A) metre
(B) second ✓
(C) kilogram
(D) radian

4. Which is the base quantity?
(A) Charge ✓
(B) Area
(C) Force
(D) Electric Current

5. Which of the *following* is not base quantity in the SI system?
(a) Electric current
(b) Temperature
(c) Luminous intensity
(d) Speed ✓

6. Which of the following is the base unit in the SI system?
(a) joule
(b) second ✓
(c) newton
(d) watt

7. Which of the following pairs both represent base quantities?
(a) Acceleration and time
(b) Mass and speed
(c) Length and temperature ✓
(d) Force and area

8. Physical quantity "pressure" in term of base unit is:
(A) $kg^{-1}ms^{-2}$ ✓
(B) kg^2ms^{-3}
(C) kg^2m^2s
(D) $kgm^{-1}s^{-2}$

9. Solid angle is:
(A) One dimensional
(B) Two dimensional
(C) Three dimensional ✓
(D) Four dimensional

10. **Which of the following is a derived quantity?**
(A) Mass
(B) Velocity ✓
(C) Length
(D) Time

11. **Which is base quantity?**
(A) Time ✓
(B) Force
(C) Power
(D) Velocity

12. **An example of base quantity is:**
(A) Area
(B) Length ✓
(C) Velocity
(D) Volume

13. **The SI unit of intensity of light is:**
(A) ampere
(B) mole
(C) candela ✓
(D) joule

14. **Supplementary units are:**
(A) 2 ✓
(B) 3
(C) 4
(D) 5

15. **The SI unit of power in terms of base unit is:**
(A) $kgm^{-1}s^{-2}$
(B) $kgm^{-1}s^{-3}$
(C) $kgms^{-2}$
(D) kgm^2s^{-3} ✓

16. **Which of the following is a derived quantity?**
(A) Force ✓
(B) Mass
(C) Length
(D) Time

17. **The solid angle subtended at the centre of sphere by an area of its surface equal to the square of radius of the sphere is called:**
(A) degree
(B) radian
(C) minute
(D) steradian ✓

18. **The example of derived unit is:**
(A) candela
(B) ampere
(C) coulomb ✓
(D) none of these

19. **Which pair has same units?**
(A) Work and power
(B) Momentum and impulse ✓
(C) Force and torque
(D) Torque and power

20. **Which is the base quantity?**
(A) Area
(B) Volume
(C) Length ✓
(D) Density

21. **Solid angle subtended at the centre of a sphere of radius "r" in steradian is:**
(A) 2π
(B) 4π ✓
(C) 6π
(D) 8π

22. **Which is not a base unit in SI units?**
(A) kilogram
(B) joule ✓
(C) ampere
(D) kelvin

23. **Which of the following is derived unit?**
(A) newton ✓
(B) metre
(C) candela
(D) mole

24. **The unit of work in base units is:**
(A) $kgms^2$
(B) $kgms^2$
(C) kgm^2s^{-2} ✓
(D) $kgm^{-1}s^2$

25. **The total base units are:**
(A) 7 ✓
(B) 6
(C) 5
(D) 3

26. **The unit of solid angle is:**
(A) Radian
(B) Degree
(C) Steradian ✓
(D) Revolution

27. **π-radian is equal to:**
(A) $0°$
(B) $90°$
(C) $180°$ ✓
(D) 360

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5. What is meant by Scientific notation and prefixes? Explain.

Ans. Scientific Notation

Definition

Scientific Notation is a way of expressing numbers that are too large or too small to be conveniently written in decimal form.

It may be referred to as scientific form or standard form.

Numbers are expressed in standard form called scientific notation, which employs powers of ten. The internationally accepted practice is that there should be only one non-zero digit left of decimal. Thus, the number 134.7 should be written as 1.347×10^2 and 0.0023 be expressed as 2.3×10^{-3}.

Prefixes

Definition

Prefixes are letters or groups of letters added to the beginning of words to change their meaning.

In measurement prefixes are used to indicate multiples or fractions of a unit, making it easier to express quantities across different scales.

For example, "kilo" means a thousand times the base unit (e.g., kilogram) = 1000 grams, while "milli" denotes one – thousandth of the base unit e.g., millimetre = 1/1000th of a metre.

Table 1.3 Some Prefixes for Powers of Ten		
Factor	**Prefix**	**Symbol**
10^{-18}	atto	a
10^{-15}	femto	f
10^{-12}	pico	p
10^{-9}	nano	n
10^{-6}	micro	μ
10^{-3}	milli	m
10^{-2}	centi	c
10^{-1}	deci	d
10^1	deca	da
10^2	hector	h
10^3	kilo	k
10^6	mega	M
10^9	giga	G
10^{12}	tera	T
10^{15}	peta	P
10^{18}	exa	E

6. State the conventions for using SI units in detail.

Ans. Conventions of Using SI Unit

Use of SI units requires special care, more particularly in writing prefixes. Some points to be noted are:

(i) Each SI unit is represented by a symbol not an abbreviation. These symbols are the same in all languages. Hence, correct use of the symbol is very important.

   For example: For ampere, we should use "A" not "amp"; for seconds, "s" not "sec", SI not S.I.

(ii) Full name of a unit does not begin with capital letter.

   For example: newton, metre, etc., except Celsius.

(iii) Symbols appear in lower case.

   For example: "m" for metre, "s" for second, etc., exception "L" for litre.

(iv) Symbols named after scientists have initial letters capital.

   For example: "N" for newton, "Pa" for pascal, "W" for watt.

(v) Symbols and prefixes are printed in upright (roman) style regardless of the type style in surrounding text.

   For example: a distance of 50 m.

(vi) Symbols do not take plural form.

   For example: 1 mm, 100 mm, 1 kg, 60 kg.

(vii) No fullstop or dot is placed after the symbol except at the end of the sentence.

(viii) Prefix is written before and without space to base unit.

   For example: "mL" not m L or "ms" not m s.

(ix) Base units are written one space apart. Leave a space even between the number (value) and the symbol.

   For example: 1 kg, 10 m s^{-1}, etc.

(x) Compound prefixes are not allowed:

   For example: 1$\mu\mu$F should be 1 pF.

(xi) When base unit of multiple is raised to a power, the power applies to whole multiple and not to base unit alone.

   For example: $1 \text{ km}^2 = 1 \text{ (km)}^2 = 1 \times (10^3 \text{ m})^2 = 1 \times 10^6 \text{ m}^2$.

(xii) Use negative index notation (m s^{-1}) instead of solidus (m/s).

(xiii) Use scientific notation, that is, one non-zero digit left of decimal.

   For example: $143.7 = 1.437 \times 10^2$.

(xiv) Do not mix symbols and names in the same expression.

   For example: metre per second or m s^{-1}, not metre/sec or m/second.

(xv) Practical work should be recorded in most convenient units depending upon the instruments being used.

   For Example: Measurements using screw gauge should be recorded in mm but the final results must be recorded to the appropriate base units.

(xvi) System International do not allows the use of former CGS System units such as dyne, erg, gauss, poise, torr , etc.

MCQs

1. The quantity 2.3×10^{-3} can be written as:
   (A) 0.0023 ✓ (B) 0.023
   (C) 0.23 (D) 2.3

2. The term 124.7 can be written in scientific notation as:
   (A) 1.247×10^2 ✓ (B) 1.247×10^3
   (C) 1.247×10^1 (D) 1.247×10^4

3. The prefix femto is equal to:
   (A) 10^{-9} (B) 10^{-12}
   (C) 10^{-14} (D) 10^{-15} ✓

4. The ratio of micro to mega is equal to:
   (A) Femto (B) Pico ✓
   (C) Tera (D) Peta

5. How many nanometres in a metre?
   (A) 10^{-19} (B) 10^{19}
   (C) 10^9 ✓ (D) 10^{-9}

6. One femto is equal to:
   (A) 10^{-12} (B) 10^{-13}
   (C) 10^{-14} (D) 10^{-15} ✓

7. One giga is equal to:
   (A) 10^9 ✓ (B) 10^{-9}
   (C) 10^{12} (D) 10^{-12}

8. The ratio of 1 femtometre to 1 nanometre is:
   (A) 10^{-6} ✓   (B) 10^6
   (C) 10^{-7} (D) 10^8

9. The quantity of 1 $(km)^2$ is equals to:
   (A) $1 \times 10^6 m^2$ ✓   (B) $1 \times 10^5 m^2$
   (C) $1 \times 10^7 m^2$ (D) $1 \times 10^4 m^2$

10. One peta is equal to:
    (A) 10^{18} (B) 10^{15} ✓
    (C) 10^{12} (D) 10^9

11. The ratio of one nanometre to one attometre is:
    (A) 10^{-12} (B) 10^{-15}
    (C) 10^9 ✓   (D) 10^{12}

12. The prefix pico is equal to:
    (A) 10^{-12} ✓   (B) 10^{-15}
    (C) 10^{-9} (D) 10^{12}

13. Which of the following is the least multiple?
    (A) Atto ✓   (B) Femto
    (C) Nano   (D) Pico

14. One tera is equal to:
    (A) 10^{10} (B) 10^9
    (C) 10^{12} ✓   (D) 10^{11}

1.3 UNCERTAINTY IN MEASUREMENT

7. What is meant by uncertainty in a measurement? How the uncertainty in a digital instrument is indicated?

Ans. Uncertainty in Measurement

Definition

The uncertainty of measurement describes the approximate value by which the actual value differs from the true value.

Explanation

Every instrument is calibrated to a certain smallest division mark on it and this fact puts a limit regarding its accuracy. When you take a reading with one instrument, its limit of measurement is the smallest division or graduation on its scale. Hence, every measured quantity has some uncertainty about its value. When a measurement is made, it is taken to the nearest graduation or marking on the scale. You can estimate the maximum uncertainty as being one smallest division of the instrument. This is called **absolute uncertainty**. It is one millimetre on a metre rule that is graduated in millimetres.

For example, if one edge of the book coincides with 10 cm mark and the other with 33.5 cm, then the length with uncertainty is given by

(33.5 ± 0.05) cm $-$ (10.0 ± 0.05) cm $= (23.5 \pm 0.1)$ cm

It means that the true length of the book is in between 23.4 cm and 23.6 cm. Hence, the maximum uncertainty is ± 0.05 cm, which is equivalent to an uncertainty of 0.1 cm. Infact, it is equal to least count of the metre rule. Uncertainty may be recorded as:

$$\text{Fractional uncertainty} = \frac{\text{Absolute uncertainty}}{\text{Measured value}}$$

or $$\text{Percentage uncertainty} = \frac{\text{Absolute uncertainty}}{\text{Measured value}} \times 100\%$$

Uncertainty in Digital Instruments

Some modern measuring instruments have a digital scale. We usually estimate one digit beyond what is certain: with a *digital scale*, this is reflected in some fluctuations of the last digit. If the last digit fluctuates by 1 or 2, write down that last digit. If fluctuation is more than 2 or so in the last digit, it may mean that the reading is being influenced by some factor such as air currents. Regardless of the reason, a large fluctuation may mean that the displayed digit is not really significant.

8. How to find the uncertainty in a timing experiment such as the time period of a simple pendulum?

Ans. To find the uncertainty in a timing experiment, such as measuring the time period of a simple pendulum, we typically consider both random errors (e.g., due to reaction time or stopwatch resolution) and systematic error (e.g., air resistance, amplitude effects).

Let us know how we can calculate the uncertainty in a standard school or laboratory experiment.

1. **Use Multiple Oscillations**

To reduce uncertainty, we usually time multiple oscillations (e.g., 10 or 20 rather than just one).

Example: We measure the time for 10 oscillations:

$t_{10} = 15.8$ s, then the time period T is:

$$T = \frac{t_{10}}{10} = \frac{15.8 \text{ s}}{10} = 1.58 \text{ s}$$

2. Estimate Uncertainty in Time Measurement

Uncertainty in time measurement can come from:

(i) **Stopwatch resolution** (e.g., ±0.01 s or ±0.1 s)

(ii) **Human reaction time** (typically ±0.2 s for starting and stopping combined)

If timing 10 oscillations:

Uncertainty in t_{10} = ±0.2 s (reaction time)

Then uncertainty in the period T is:

$$\Delta T = \frac{\Delta t_{10}}{10} = \frac{0.2 \text{ s}}{10} = \pm 0.02 \text{ s}$$

So, result could be written as:

$$T = 1.58 \pm 0.02 \text{ s}$$

3. Alternate: Standard Deviation (If we have Multiple Trials)

If we repeat the experiment several times, calculate the mean and standard deviation of those values.

Example: Suppose we measure 10 oscillations in 5 times and get periods.

1.57 s, 1.59 s, 1.58 s, 1.60 s , 1.56 s

Then   Mean T = $\dfrac{(1.57 + 1.59 + 1.58 + 1.60 + 1.56) \text{ s}}{5}$ = 1.58 s

Standard deviation ≈ 0.016 s

Uncertainty in mean (standard error) $\Delta T = \dfrac{\text{Standard deviation}}{\sqrt{n}} = \dfrac{0.016 \text{ s}}{\sqrt{5}} \approx 0.007 \text{ s}$

So, the period would be: $T = 1.58 \pm 0.01 \text{ s}$

1.4 USE OF SIGNIFICANT FIGURES

9. What are significant figures? Describe their use.

Ans. Significant Figures

Definition

Significant figures are the digits in a number that carry meaning contribution to its precision.
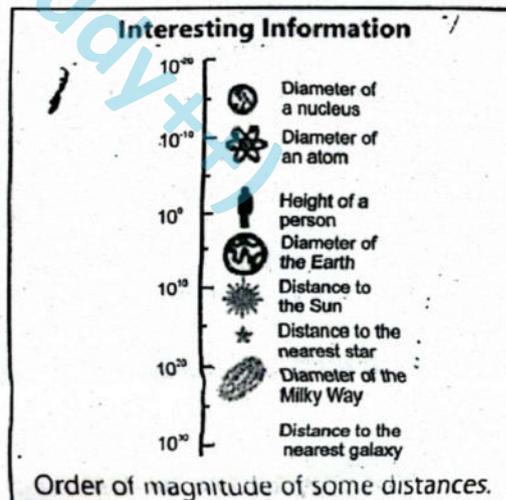
This includes all non-zero digits, any zeros between them, and any trailing zeros in a decimal number.

Examples

(i)   123.45 has 5 significant figures.

(ii)   0.00450 has three significant figures.
(The leading zeros are not significant)

Use of Significant Figures

(i)   **Precision Representation:** They show how precise a measurement is. More significant figures mean higher precision.

(ii)   **Measurement Communication:** They indicate the reliability of the measurement equipment. For instance, a metre rule marked in millimetres allows more significant figures than one marked in centimetres.

(iii)   **Calculation Rules:** When doing calculations (addition, subtraction, multiplication, or division), significant figure rules help to maintain appropriate precision.

- **Multiplication / Division:** The result should have the same *number of significant figures* as the number with the fewest.
- **Addition / Subtraction:** The result should be rounded to the same decimal as the least precise value.

Interesting Information

10^{∞}	
	Diameter of a nucleus
10^{-10}	Diameter of an atom
10^{0}	Height of a person
	Diameter of the Earth
10^{10}	Distance to the Sun
	Distance to the nearest star
10^{20}	Diameter of the Milky Way
10^{30}	Distance to the nearest galaxy

Order of magnitude of some distances.

Importance

Using the correct number of significant figures prevent false precision and ensures consistency in scientific and technical communication.

Reasons for Using Significant Figures in Measurements

1. **To Reflect Measurement Precision**

   Significant figures indicate the precision of a measurement by showing how many digits are known reliably based on the instrument used. This helps to convey the level of uncertainty in the measurement.

2. **To Ensure Consistency in Calculations**

   Using significant figures in calculations ensures that results are not reported with unrealistic precision. It maintains consistency and prevents misleading calculations based on overly precise or inaccurate numbers.

10. What rules should be observed while counting significant figures?

Ans. **Working with Significant Figures**

Counting significant digits

(i) All digits 1,2,3,4,5,6,7,8,9 are significant. However, zeros may or may not be significant. In case of zeros, the following rules may be adopted:

(ii) A zero between two significant figures is itself significant.

(iii) Zeros to the left most significant figure are not significant. For example, none of the zeros in 0.00467 or 02.59 are significant.

(iv) Zeros to the right of a significant figure may or may not be significant. In decimal fraction, zeros to the right of a significant figure are significant. For example, all the zeros in 3.570 or 7.4000 are significant. However, in integers such as 8, 000 kg, the number of significant zeros is determined by the precision of the measuring instrument. If the measuring scale has a least count of 1 kg, then there are four significant figures written in scientific notation as 8.000×10^3 kg. If the least count of the scale is 10 kg, then the number of significant figures will be 3 written in scientific notation as 8.00×10^3 kg and so on.

(v) When a measurement is recorded in scientific notation or standard form, the figures other than the powers of ten are significant figures. For example, a measurement recorded as 8.70×10^4 kg has three significant figures.

11. Describe the rules observed for significant figures while multiplying or dividing numbers.

Ans. **Multiplying or Dividing Numbers**

Keep the number of significant figures in the product or quotient not more than that contained in the least accurate factor i.e., the factor containing the least number of significant figures. For example, the computation of the following using a calculator, gives:

$$\frac{5.348 \times 10^{-2} \times 3.64 \times 10^4}{1.336} = 1.45768982 \times 10^3$$

As the factor 3.64×10^4, the least accurate in the above calculation has three significant figures, the answer should be written to 3 significant figures only. The other figures are insignificant and should be deleted. While deleting the figures, the last significant figure to be retained is rounded off for which the following rules are followed:

(i) If the first digit dropped is less than 5, the last digit retained should remain unchanged.

(ii) If the first digit dropped is more than 5, the digit to be retained is increased by one.

(iii) If the digit to be dropped is 5, the previous digit which is to be retained is increased by one if it is odd and retained as such if it is even. For example, the following numbers are rounded off to three significant figures as follows. The digits are deleted one by one.

43.75	is rounded off as	43.8
56.8546	is rounded off as	56.9
73.650	is rounded off as	73.6
64.350	is rounded off as	64.4

Following this rule, the correct answer of the computation given in section (ii) is 1.46×10^3.

12. What rules should be followed in adding and subtracting numbers?

Ans. **In Adding or Subtracting Numbers**

The number of decimal places retained in the answer should be equal to the smallest number of decimal places in any of the quantities being added or subtracted. In this case, the number of significant figures is not important. For example, we add the following quantities:

(i) 72.1
     3.42
     0.003
     75.523
Correct answer is 75.5 m

(ii) 2.7543
     4.10
     1.273
     8.1273
     8.13 m

In case (i), the number 72.1 has the smallest decimal places, thus the answer is rounded off to the same position which is then 75.5 m. In case (b), the number 4.10 has the smallest number of decimal places and hence, the answer is rounded off to the same decimal position which is then 8.13 m.

Quick Quiz

1. Give the correct number of significant figures for 0.0054 m, 0.03030 m, 40.0 m, 0.5 m, 8.20×10^3 m.
   Ans. 2, 4, 3, 3
2. Give the answer to the appropriate number of significant figures.
   2602 kg + 36.02 kg + 54.1 kg = ?
   Ans. 2692
3. Give the answer to the appropriate number of significant figures.
   3.54 kg − 2.4 kg = ?
   Ans. 1.1
4. Give the answer to the appropriate number of significant figure.
   2.45×10^3 m × 2.46 m / 3.6 m = ?
   Ans. 1.7×10^3

13. What are the limitations of significant figures?

Ans. **Limitations of Significant Figures**

Significant figures are a useful tool in representing the precision of measurements, in science and engineering, but they do have several limitations:

(i) **Lack of Clarity with Uncertainty:** Significant figures give a rough indication of precision, but they do not explicitly show the uncertainty in a measurement. For more precise work, uncertainty should be expressed with error margins (e.g., ± 0.02).

(ii) **Ambiguity with Zeros:** It is sometimes unclear how many significant figures a number has, especially when written without scientific notation. For example; 1500 could have two, three, or four significant figures depending on the context.

(iii) **Not Suitable for all Calculations:** Using significant figures in complex calculations (like trigonometric functions) can be misleading if rounding is done too early or improperly.

(iv) **Over-simplification:** Significant figures provide a simplified rule for rounding but may not reflect the actual measurement process or instrument precision accurately.

(v) **Inconsistent with Statistical Methods:** In data analysis and experimental science, statistical methods (e.g., standard deviation, confidence intervals) and much more accurate and meaningful for expressing precision than significant figures.

(vi) **Depending on Measurement Conventions:** The rule of significant figures are conventions and do not universally apply in every scientific or engineering context, particularly in computer science or where exact values (like counting numbers) are used.

MCQs

1. Significant figures in 0.0010 are:
   (A) 1   (B) 2 ✓
   (C) 3   (D) 4

2. Which one of the following is the correct record for the diameter of a wire when measured with a screw gauge of least count of 0.001 cm?
   (A) 2.3 cm   (B) 2.31 cm
   (C) 2.312 cm ✓ (D) 2.3124 cm

3. Significant figures in 0.00567 are:
   (A) 2   (B) 3 ✓
   (C) 4   (D) 5

4. The significant figures in 0.04060 are:
   (A) 2   (B) 4 ✓
   (C) 5   (D) 6

5. Significant figures in 0.0004813 are:
   (A) 8   (B) 7
   (C) 4 ✓   (D) 3

6. The significant figures in 34.676 are:
   (A) 2   (B) 4
   (C) 3   (D) 5 ✓

7. A student added three figures 72.1, 3.32 and 0.003. The correct answer regarding the rules of the addition of the significant figures will be:
(A) 75.423   (B) 75.42
(C) 75.4 ✓   (D) 75.5

8. The number of significant figures in 0.0173 are:
(A) 2   (B) 3 ✓
(C) 4   (D) 5

9. Number of significant figures in 01.020mm are:
(A) 2   (B) 3
(C) 4 ✓   (D) 5

10. 275.00 has the significant digits:
(A) 2   (B) 3
(C) 4   (D) 5 ✓

11. Significant figures in 0.0010 are:
(A) 1   (B) 2 ✓
(C) 3   (D) 4

12. The number of significant figures in 0.00232 are:
(A) 6   (B) 5
(C) 3 ✓   (D) 4

13. In $5.47 \times 19.89 = 108.7983$, answer should be written as:
(A) 108.8   (B) 108.9
(C) 109 ✓   (D) 108.79

14. Significant figures in 0.0045 are:
(A) 1   (B) 3
(C) 4   (D) 2 ✓

15. Which of the following number has three significant figures?
(a) 0.0450 ✓   (b) 4500
(c) 0.0045   (d) 450.00

16. The result of the calculation 4.56×1.4 should be reported as:
(a) 6.38   (b) 6.4 ✓
(c) 6.384   (d) 6.3

17. The number of significant figures in 0.00407 are:
(A) 2   (B) 3 ✓
(C) 4   (D) 5

18. If we round off 64.34546 up to three significant figures, the best answer is:
(A) 64.3   (B) 64.4 ✓
(C) 64.5   (D) 64.6

19. The significant figures in 0.0004813 are:
(A) 8   (B) 7
(C) 4 ✓   (D) 3

20. The correct answer of $\dfrac{5.348 \times 10^{-2} \times 3.64 \times 10^4}{1.336}$ is:
(A) 1.46×10^3 (B) 1.451×10^3
(C) 1.457×10^3 ✓   (D) 1.5×10^3

21. The quantity 0.00467 has significant figures:
(A) 3 ✓   (B) 4
(C) 5   (D) 6

22. Significant figures in 0.00876 are:
(A) 3 ✓   (B) 4
(C) 5   (D) 6

23. Significant figures in 0.00846 are:
(A) 3 ✓   (B) 4
(C) 6   (D) 7

24. The sum of three numbers 2.7543, 4.10, 1.273 up to correct decimal place is:
(A) 8.12   (B) 8.13 ✓
(C) 8.1273   (D) 8.127

25. In Scientific notation, the number 0.0001 may be written as:
(A) 10^{-2} (B) 10^3
(C) 10^{-4} ✓   (D) 10×10^4

26. Which of the following best expresses a measurement with uncertainty?
(A) 3.6 m   (B) 3.6 ± 0.2m
(C) 3.6 ms^{-2} ✓   (D) 3.6 with no unit

27. If you add two quantities with uncertainties, how do you find the uncertainty in the result?
(a) Add the uncertainties directly. ✓
(b) Multiply the uncertainties
(c) Take the average of the uncertainties
(d) Subtract the uncertainties

1.5 PRECISION AND ACCURACY

14. Differentiate between the terms precision and accuracy with reference to measurement of physical quantities.

Ans. Precision and Accuracy

Definition

Accuracy is how close a given set of measurement are to their true value. Whereas precision is how close the measurement are to each other.

Explanation

The precision of a measurement is determined by the instrument or devicebeing used. The smaller the least count, the more precise is the measurement.

Accuracy is the closeness of a measurement to the exact or accepted value of a physical quantity. It is expressed by the fractional or percentage uncertainty. The smaller the fractional orpercentage uncertainty, the more accurate is the measurement.

For example, the length of an object is recorded as25.5 cm by using a metre rule having smallest division in millimetre. Its precision or absolute uncertainty (least count) = ±0.1 cm.

$$\text{Fractional uncertainty} = \frac{0.1\,cm}{25.5\,cm} = 0.004$$

$$\text{Percentage uncertainty} = \frac{0.1\,cm}{25.5\,cm} \times 100 = 0.4\%$$

Another measurement taken by Vernier Calipers with least count 0.01 cm is recorded as 0.45 cm. It has precision or absolute uncertainty (least count) = ±0.01 cm.

$$\text{Fractional uncertainty} = \frac{0.01\,cm}{0.45\,cm} = 0.02$$

$$\text{Percentage uncertainty} = \frac{0.01\,cm}{0.45\,cm} \times 100 = 2\%$$

Thus, the reading 25.5 cm taken by metre rule is although less precise but is more accurate having less percentage uncertainty or error.

Whereas the reading 0.45 cm taken by Vernier Callipers is more precise but is less accurate. In fact, it is the relative measurement which is important. The smaller a physical quantity, the more precise instrument should be used. Here the measurement 0.45 cm demands that a more precise instrument, such as micrometer screw gauge, with least count 0.001 cm, should have been used. Hence, we can conclude that:

A precise measurement is the one which has less precision or absolute uncertainty and an accurate measurement is the one which has less fractional or percentage uncertainty.

1.6 ASSESSMENT OF TOTAL UNCERTAINTY IN THE FINAL RESULT

15. How is the total uncertainty in the final result assessed? Explain.

Ans. Assessment of Total Uncertainty in the Final Result

By knowing the uncertainties in all the factors involved in a calculation, the maximum possible uncertainty or error in the final result can be found as follows:

(i) For Addition and Subtraction

Absolute uncertainties are added. For example, the distance between two positions $x_1 = 15.4 \pm 0.1$ cm and $x_2 = 25.6$ cm ± 0.1 cm is recorded as:

$$x = x_2 - x_1 = 10.2 \pm 0.2 \text{ cm}$$

and addition of two lengths is:

$$l_1 = 8.5 \pm 0.1 \text{ cm and } l_2 = 12.6 \pm 0.1 \text{ cm recorded as:}$$

$$l = l_1 + l_2 = 21.1 \pm 0.2 \text{ cm}$$

(ii) For Multiplication and Division

Percentage uncertainties are added. For example, the maximum possible uncertainty in the value of resistance R of a conductor determined by the potential difference V applied across the conductor resulting in current flowing through it is estimated as under:

$$\text{Let} \quad V = 3.4 \pm 0.1 \text{ V}$$
$$I = 0.68 \pm 0.05 \text{ A}$$

$$\text{Using} \quad R = \frac{V}{I}$$

$$\text{Percentage uncertainty in} \quad V = \frac{0.1\,V}{3.4\,V} 100 = 3\%$$

$$\text{Percentage uncertainty in} \quad I = \frac{0.05\,A}{0.68\,A} \times 100 = 7\%$$

Hence, total percentage uncertainty in the value of R is 3 + 7 = 10%

The value of R will be written as:

$$R = \frac{3.4\,V}{0.68\,A} = 5.0 \text{ ohm}$$

Hence, $R = 5.0 \pm 0.5$ ohms, uncertainty being an estimate only, is recorded by one significant figure.

For Your Information Travel time of light		
Moon to Earth	1 min 20 s	
Sun to Earth	8 min 20 s	
Pluto to Earth	5 h 20 s	

(iii) For Power Factor:

The percentage uncertainty is multiplied by the power factor in the formula. For example, the calculation of cross-sectional area of a cylinder of radius $r = 1.25$ cm using formula for Area $A = \pi r^2$ is given by the %age uncertainty which is $A = 2 \times$ %age uncertainty in radius r. As uncertainty is multiplied by power factor, it increases the precision demand of measurement. When the radius of a small sphere is measured as 1.25 cm by Vernier Callipers with least count 0.01 cm, then

The radius r is recorded as $\qquad r = 1.25 \pm 0.01$ cm

%age uncertainty in radius r is $\qquad r = \dfrac{0.01}{1.25} \times 100 = 0.8\%$

Total percentage uncertainty in area $\quad A = 2 \times 0.8 = 1.6\%$

Thus $\qquad\qquad\qquad\qquad A = \pi r^2$

$\qquad\qquad\qquad\qquad = 3.14\,(1.25)^2 = 4.906$ cm^2 with 1.6% uncertainty.

Thus, the result should be recorded as $A = 4.91 \pm 0.08$ cm^2.

Example 1.1: The length, breadth and thickness of a metal sheet are 2.03 m, 1.22 m and 0.95 cm respectively. Calculate the volume of the sheet correct up to the appropriate significant digits.

Solution:

Given that;

Length $\quad l = 2.03$ m

Breadth $\quad b = 1.22$ m

Thickness $\quad h = 0.95$ cm $= 0.95 \times 10^{-2}$ m

To Find:

$\qquad V = ?$

Calculations:

As Volume $\quad V = l \times b \times h = 2.03$ m $\times 1.22$ m $\times 0.95 \times 10^{-2}$ m

$\qquad\qquad\qquad = 2.35277 \times 10^{-2}$ m^3

As the factor 0.95 cm has minimum number of significant figures equal to two, therefore, volume is recorded up to 2 significant figures, hence, $V = 2.4 \times 10^2$ m^3.

Example 1.2: The mass of a metal box measured by a lever balance is 3.25 kg. Two silver coins of masses 10.01 g and 10.02 g measured by a beam balance are added to it. What is now the total mass of the box correct up to the appropriate precision?

Solution:

Total mass when silver coins are added to box $= 3.25$ kg $\times 0.01001$ kg $+ 0.01002$ kg

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad = 3.27003$ kg

Since least precise mass is 3.25 kg, having two decimal places, hence, total mass should be reported to 2 decimal places which is the appropriate precision.

Thus $\qquad$ Total mass $= 3.27$ kg

Example 1.3: The diameter and length of a metal cylinder measured with the help of Vernier Callipers of least count 0.01 cm are 1.25 cm and 3.35 cm, respectively. Calculate the volume V of the cylinder and uncertainty in it.

Solution:

Given that;

Diameter $\quad d = 1.25$ cm with least count 0.01 cm

Length $\quad l = 3.35$ cm with least count 0.01 cm

Absolute uncertainty in length $= 0.01$ cm

To Find:

$\qquad V = ?$

Calculations:

%age uncertainty in length $= (0.01$ cm $/ 3.35$ cm$) \times 100\% = 0.3\%$

Absolute uncertainty in diameter $= 0.01$ cm

%age uncertainty in diameter $= (0.01$ cm $/ 1.25$ cm$) \times 100\% = 0.8\%$

$$\text{Volume} \qquad = \pi r^2 l = \pi \frac{d^2}{4} l$$

Total uncertainty in $V = 2$ (%age uncertainty in diameter) + (%age uncertainty in length)

$= 2 \times 0.8\% + 0.3\% = 1.9\%$

Then $\qquad V = 3.14 \times (1.25 \text{ cm})^2 \times 3.35 \text{ cm}^4 = 4.1089842 \text{ cm}^3$ with 1.9% uncertainty

Thus $\qquad V = (4.11 \pm 0.08) \text{ cm}^3$

where 4.11 cm^3 is calculated volume and 0.08 cm^3 is the uncertainty in it.

mcQs ☑

1. The percentage uncertainty in mass and velocity are 2% and 3% the maximum uncertainly in the measurement of kinetic energy is:
   (A) 11%   (B) 8% ✓
   (C) 6%   (D) 1%

2. If $x_1 = 10.5 \pm 0.1$ cm and $x_2 = 26.8 \pm 0.1$ cm $x_2 - x_1$, is given by:
   (A) 16.3 ± 0.1 cm   (B) 16.3 ± 0.2 cm ✓
   (C) 16.1 ± 0 cm   (D) 16.3 ± 0.3 cm

3. The percentage uncertainly in radius of a sphere is 2%. The total percentage uncertainly in the volume of the sphere is:
   (A) 2%   (B) 4%
   (C) 6% ✓   (D) 8%

4. For total assessment of uncertainty in the final result obtained by multiplication we add:
   (A) Absolute uncertainties
   (B) Fractional uncertainties
   (C) Percentage uncertainties ✓
   (D) Error

5. Error in the measurement of radius of sphere is 1%. The error in the calculated value of its volume is:
   (A) 7%   (B) 5%
   (C) 3% ✓   (D) 1%

6. I $V = 5.2 \pm 0.1$ volt. The percentage uncertainly of V^3 will be:
   (A) 2%   (B) 4%
   (C) 6% ✓   (D) 1%

7. A precise measurement is the one which has:
   (A) Greater Precision ✓   (B) Less Precision
   (C) Medium Precision   (D) More % error

8. If error in measurement of radius of circle is 2% then permissible error in its area will be:
   (A) 1%   (B) 2%
   (C) 4% ✓   (D) 8%

9. There are four readings of a micrometer to measure the diameter of a wire in mm are 1.21, 1.23, 1.25, 1.23. The mean of deviations is:
   (A) 0.02mm   (B) 0.01 mm ✓
   (C) 0.10mm   (D) 0.20mm

10. Absolute uncertainty in a measuring instrument is equal to:
    (A) Least count ✓   (B) Accuracy
    (C) Fractional uncertainties (D) Percentage uncertainties

11. A measurement taken by Verniercalliper with least count as 0.01cm is recorded as 0.45cm, it has fractional uncertainty.
    (A) 0.01   (B) 0.02 ✓
    (C) 0.03   (D) 0.45

12. In order to reduce the uncertainty in finding time period of a vibrating body, it is advised to count:
    (A) Small number of swings
    (B) Large number of swings
    (C) Infinite number of swings ✓
    (D) Both A and C

13. The uncertainty in the time period of a vibrating body is:
    (A) Least count × No. of vibrations
    (B) Least count + No. of vibrations
    (C) Least count ÷ No. of vibrations ✓
    (D) Least count − No. of vibrations

14. If $r = 2.25 \pm 0.01$ cm them (%) percentage uncertainty in r is:
    (A) 0.225%   (B) 22.5%
    (C) 0.2%   (D) 0.4% ✓

15. Length of an object is recorded as 25.5 cm by using a meter rod having smallest division in millimeter. The fractional uncertainty is:
    (A) 0.400   (B) 2.550
    (C) 0.004 ✓   (D) 0.100

16. Which of the following measurement is more precise:
    (A) 3127 s   (B) 312.7 s
    (C) 31.27 s   (D) 3.327 s ✓

17. The percentage error in the measurement of voltage, current, resistance and power are 2%, 6%, 8% and 12% respectively. The accurate measurement is of:
    (A) Current   (B) Voltage
    (C) Resistance   (D) Power ✓

18. In a simple pendulum experiment percentage uncertainty in L are 0.5% and 1.5% respectively. The maximum percentage uncertainty in LT^2 is:
    (A) 2.5%   (B) 2%
    (C) 3.5%   (D) 4% ✓

19. The fractional uncertainty in the measurement of radius $r = 2.25 \pm 0.01$ cm is:
    (A) 0.4   (B) 0.04
    (C) 0.004 ✓   (D) 0.0004

20. The percentage uncertainties in length and width of a rectangle are 2% and 3%. Its area has percentage uncertainty.
    (A) 1% ✓   (B) 5%
    (C) 6%   (D) 2%

20. The percentage uncertainties in length and width of a rectangle are 2% and 3%. Its area has percentage uncertainty.
(A) 1% (B) 5% ✓
(C) 6% (D) 2%

21. Velocity of an object is 1% uncertainty and mass has 2% uncertainty. What is the total uncertainty?
(A) 3% (B) 2%
(C) 4% ✓ (D) 1%

22. Which of the following best expresses a measurement with uncertainty?
(A) 3.6 m (B) 3.6 ± 0.2 m ✓
(C) 3.6 ms^{-2} (D) 3.6 with no units

23. A length is measured is; L = 25.4 ± 0.2 cm. What does the value ±0.2 represent?
(A) The resolution (B) The value
(C) The uncertainty ✓ (D) The error

24. If a scale always read 2 kg, heavier that the actual value, the measurements are:
(A) Precise but not accurate ✓
(B) Accurate but not precise
(C) Both precise and accurate
(D) Neither precise nor accurate

25. If a scale gives different weights for the same object every time, it is said to lack:
(A) Accuracy (B) Precision ✓
(C) Uncertainty (D) Resolution

26. Which of the following set, of data best represents high accuracy and high precision if the true value is 50.0?
(A) 49.9, 50.0, 50.1 ✓ (B) 45.0, 55.0, 50.0
(C) 49.0, 49.1, 49.2 (D) 50.5, 50.6, 50.7

27. Which instrument is more precise for measuring small lengths?
(A) Metre rule (B) Measuring tape
(C) Protractor (D) Vernier callipers ✓

1.7 DIMENSIONS OF PHYSICAL QUANTITIES

16. Explain with examples the writing of physical quantities into their dimensions.

Ans. Dimensions of Physical Quantities

Definition

Dimensions of a physical quantity refer to the powers (or exponents) to which the fundamental physical quantities (like mass, length, time, etc.) must be raise to represent that physical quantity.

Explanation

Dimensions deal with the qualitative nature of a physical quantity in terms of fundamental quantities. The quantities such as length, depth, height, diameter, light year are all measured in metre and denoted by the same dimension, basically known as length given by symbol L written within square brackets [L]. Similarly, the other fundamental quantities, mass, time, electric current and temperature are denoted by specific symbols [M], [T], [A] and [θ], respectively. These five dimensions have been chosen as being basic because they are easy to measure in experiments.

The dimensions of other quantities indicate how they are related to the basic quantities and are combination of fundamental dimensions. For example, speed v is measured in metres per second, so it has the dimensions of length [L] divided by time [T].

$$[v] = [L]/[T] = [L][T^{-1}] = [LT^{-1}]$$

As the acceleration $a = \Delta v/\Delta t$

Dimensions of acceleration are

$$[a] = [v]/[T] = [LT^{-1}]/[T] = [LT^{-2}]$$

Also, dimensions of force can be written as

$$[F] = [m][a] = [M].[LT^{-2}] = [MLT^{-2}]$$

By the use of dimensionality, we can check the homogeneity (correctness) of a physical equation, and also, we can derive formula for a physical quantity.

17. Check the homogeneity of the relation; $v = \sqrt{\dfrac{T \times l}{m}}$ where v is the speed of transverse wave on a stretched string of tension T, length l and mass m.

Ans. We have to check the homogeneity of the equation i.e., we have to ensure that both sides of the equation have the same dimensions.

The equation is: $v = \sqrt{\dfrac{T \times l}{m}}$

Step 1: Finding the dimensions of R.H.S

$$R.H.S = \sqrt{\frac{T \times l}{m}}$$

We write the dimensional formula for each quantity:

- $[T]$ = Force = $[MLT^{-2}]$
- $[l]$ = Length = $[L]$
- $[m]$ = Mass = $[M]$

Now

$$[T \times l] = [MLT^{-2}] \times [L] = [ML^2T^{-2}]$$

$$\frac{T \times l}{m} = \frac{[ML^2T^{-2}]}{[M]} = [L^2T^{-2}]$$

Taking the square root

$$\sqrt{[L^2T^{-2}]} = [LT^{-1}]$$

$$R.H.S = [LT^{-1}] \qquad \dots\dots (i)$$
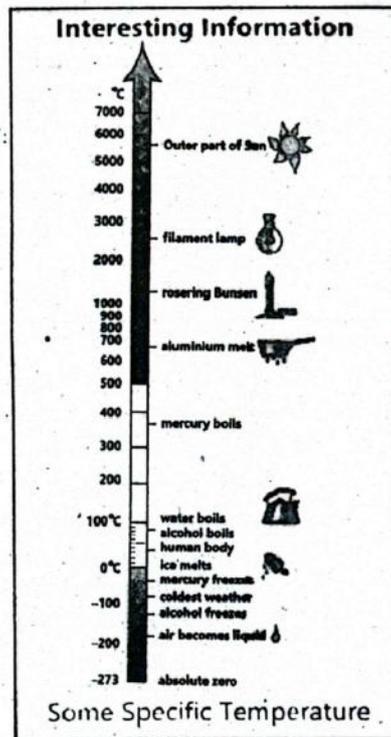
Step 2: Finding the dimensions of L.H.S

- $[v]$ = speed = $[LT^{-1}]$ $\qquad \dots\dots (ii)$

Comparing Eqs. (i) and (ii)

$$L.H.S : [LT^{-1}] = R.H.S : [LT^{-1}]$$

As both sides have the same dimensions, so the relation in homogeneous.

Conclusion: The equation; $v = \sqrt{\frac{T \times l}{m}}$ is dimensionally homogeneous.

Interesting Information

Some Specific Temperature

18. What is meant by homogeneity of physical equations? Explain by giving an example.

Ans. Homogeneity of Physical Equations:

The correctness of an equation can be checked by showing that the dimensions of quantities on both sides of the equation are the same. This is known as principle of homogeneity.

Example: Suppose a car starts from rest ($v_i = 0$) and covers a distance S in time t moving with an acceleration a. The equation of motion is given by,

$$S = v_i t + \frac{1}{2}at^2$$

$$\text{or} \quad S = \frac{1}{2}at^2$$

Numerical factors like 1/2 have no dimensions, so they can be ignored. By putting the dimensions of both sides of the equation:

$$[S] = [a][t^2]$$

Writing the symbols of dimensions

$$[L] = [LT^{-2}][T^2]$$
$$[L] = [LT^{-2}T^2]$$

or

$$[L] = [L]$$

This shows that dimensions on both sides of equation are the same, therefore, the equation is dimensionally correct.

19. What are the benefits of using dimensions of a physical quantity?

Ans. Benefits of Dimensions

The key benefits of using dimensions of a physical quantity are:

(i) Checking the correctness of equations (dimensional analysis).
(ii) Deriving formulae.
(iii) Converting units.
(iv) Simplifying complex problems.
(v) Identifying fundamental and derived quantities.
(vi) Understanding the nature of physical quantities.

20. What are the limitation of dimensional analysis?

Ans. Limitations of Dimensions

The limitations of dimensions of a physical quantity (or dimensional analysis) are:

(i) Cannot determine dimensionless constants.
(ii) Applicable only to dimensionally consistent equations.
(iii) Cannot distinguish between physical quantities with the same dimensions.
(v) Cannot derive equations involving trigonometric exponential, or logarithmic functions.
(vi) Only gives proportionality, not exact equations.
(vii) Limited when empirical data is essential.

Example 1.4: Derive a formula for the centripetal force required to keep an object moving along a circle with uniform speed. Assuming that centripetal force depends on mass of the object, radius of the circle and uniform speed.

Solution: As force depends on mass m of the object, radius r of the circle and uniform speed v, we can write:

$$F \propto m^a\, m^b\, r^c$$
$$F = (\text{constant})\, m^a\, m^b\, r^c$$

where the exponents (powers) a, b and c are to be determined. By the principle of homogeneity, the dimensions on both sides of the equation should be the same. Since, constant has no dimension so by ignoring it, we write the above equation in terms of dimensions as,

$$[F] = [m^a]\,[m^b]\,[r^c]$$
$$[MLT^{-2}] = [M^a]\,[L/T]^b\,[L]^c$$
$$[MLT^{-2}] = [M^a]\,[LT^{-1}]^b\,[L]^c$$
$$[MLT^{-2}] = [M^a\, L^{b+c}\, T^{-b}]$$

Comparing the powers of dimensions on both sides of the above equation, we have

$$a = 1$$
$$b + c = 1$$
$$-b = -2$$

Solving the above equations, we have $a = 1$, $b = 2$, $c = -1$

Putting the values of a, b and c in equation (i), we have

$$F = (\text{constant})\, mv^2\, r^{-1}$$

or

$$F = (\text{constant})\, mv^2\, /\, r$$

The numerical value of the constant cannot be determined by dimensional analysis. However, it can be found by experiments. In the above equation, numerical value of the constant happens to be "1", so the equation reduces to:

$$F = mv^2/r$$

---

mQs

1. Which of the following physical quantities has the same dimensions and impulse?
(a) Force   (b) Work
(c) Momentum ✓   (d) Power

2. Which of the following quantities is dimensionless?
(a) Angular velocity   (b) Strain ✓
(c) Pressure   (d) Temperature

3. The dimensions of power are:
(A) $[ML^{-2}T^{-3}]$ (B) $[MLT^{-2}]$
(C) $[ML^2T^{-3}]$ ✓   (D) $[M^2L^2T^{-1}]$

4. The dimensions of coefficient of viscosity "η" are:
(A) $[ML^{-1}T^{-1}]$ ✓   (B) $[MLT^{-2}]$
(C) $[MLT^{-1}]$ (D) $[ML^{-3}T^{0}]$

5. Which one has the same dimensions?
(A) Work and power   (B) Momentum and energy
(C) Work and torque ✓   (D) Power and pressure

6. Dimensions of the ratio of angular momentum to linear momentum is:
(A) $[M^0LT^0]$ ✓   (B) $[M^1L^1T^1]$
(C) $[M^1L^2T^1]$ (D) $[M^{-1}L^{-1}T^1]$

7. The dimensions of pressure are:
(A) MLT^{-2} (B) ML^2T^{-2}
(C) $ML^{-1}T^{-2}$ ✓   (D) MLT^{-3}

8. The numerical value of constants in any formula cannot be determined by dimensional analysis, however, it can be found by
(A) Addition   (B) Physical Quantities
(C) Experiments ✓   (D) Uncertainly

9. Which pair has the same dimensions?
(A) Work and power   (B) Force and Torque
(C) Torque and power   (D) None of these ✓

10. The dimensions of work are:
    (A) $[MLT^1]$ (B) $[MLT^2]$
    (C) $[ML^2T^{-2}]$ (D) $[MLT]$ ✓

11. The dimension $[M^0LT^0]$ represents the quantity:
    (A) Length ✓ (B) Mass
    (C) Time (D) Velocity

12. The dimensional formula for the quantity of light year is:
    (A) $[LT^{-1}]$ (B) $[T]$
    (C) $[ML^2T^2]$ (D) $[L]$ ✓

13. Which of the following pair has the same dimensions?
    (A) Work and power (B) Momentum and Energy
    (C) Power and Pressure (D) Work and Torque ✓

14. The dimensions $[ML^2T^{-2}]$ belong to:
    (A) Pressure (B) Momentum
    (C) Power (D) Heat energy ✓

15. The dimensions of density are:
    (A) $[ML^{-2}]$ (B) $[M^2L^{-2}]$
    (C) $[ML^{-3}]$ ✓ (D) $[M^{-1}L^{-1}]$

16. $[M^0L^0T^{-1}]$ refers to:
    (A) Velocity (B) Time period
    (C) Frequency ✓ (D) Force

17. The dimensions of torque are:
    (A) $[ML^{-1}T]$ (B) $[ML^2T^1]$
    (C) $[ML^2T^{-2}]$ ✓ (D) $[ML^{-1}T^2]$

18. The dimensions of acceleration due to gravity are:
    (A) $[MLT^2]$ (B) $[MLT]$
    (C) $[LT^{-2}]$ ✓ (D) $[ML^0T^{-2}]$

19. The dimensions of angular velocity are:
    (A) $[LT^{-1}]$ (B) $[LT^{-2}]$
    (C) $[L^{-1}T]$ (D) $[T^{-1}]$ ✓

20. Which pair has same dimensions?
    (A) Work and Power (B) Momentum and impulse ✓
    (C) Force and torque (D) Torque and power

21. The dimensions of $\sqrt{\dfrac{l}{g}}$ are the same as that of:
    (A) Time ✓ (B) Energy
    (C) Velocity (D) Force

22. The dimensions of the relation $\sqrt{\dfrac{F \times l}{m}}$ are equal to the dimensions of:
    (A) Force (B) Momentum
    (C) Acceleration (D) Velocity ✓

23. The dimensions of Einstein equation $E = mc^2$ are:
    (A) $[MLT^2]$ (B) $[ML^{-1}T^2]$
    (C) $[ML^3T^2]$ (D) $[ML^2T^{-2}]$ ✓

24. The dimension of angular acceleration is:
    (A) $[T^{-1}]$ (B) $[LT^{-2}]$
    (C) $[T^{-1}]$ ✓ (D) $[T^{-3}]$

25. The dimensions of kinetic energy are similar to that of:
    (A) Power (B) Torque ✓
    (C) Momentum (D) Pressure

26. $[M^0LT^{-1}]$ are the dimensions of:
    (A) Force (B) Velocity ✓
    (C) Work Done (D) Momentum

27. The dimensions of gravitational constant are:
    (A) $[M^3L^1T^1]$ (B) $[M^{-1}L^3T^{-2}]$ ✓
    (C) $[M^0LT^{-2}]$ (D) $[M^1L^3T^1]$

ADDITIONAL SHORT ANSWER QUESTIONS

SQ 1. Differentiate between base and derived quantities.

Ans.

Base Quantities	Derived Quantities
Definition	
"The minimum number (7) of those physical quantities in terms of which other physical quantities can be defined are called base quantities."	"Physical quantities whose definitions are based on other physical quantities are called derived quantities."
Examples	
Length, mass, time, temperature, electric current, intensity of light and amount of substance.	Speed, velocity, acceleration, force, momentum, Torque etc.

Q.2 How many micro seconds are there in one year?

Ans. One year $= 365 \times 24 \times 60 \times 60$ s
$= 31536000$ s
$= 3.1536 \times 10^7$ s
$= 3.1536 \times 10^7 \times 10^6 \times 10^{-6}$ s
$= 3.1536 \times 10^{13}$ micro s $\because 10^{-6} = \mu$ (micro)

| One year $= 3.1536 \times 10^{13}$ μs |

Q.3 What are derived quantities?

Ans. These are physical quantities that are obtained from combinations of base quantities through mathematical operations.

Q.4   How are derived quantities formed?

Ans. They are formed by multiplying and dividing base units.

Q.5   What is SI system?

Ans. It provides a universal standard for measurements used worldwide in science, industry and everyday life.

Q.6   What are significant figures?

Ans. They are the digits in a number that carry meaningful information about its precision.

Q.7   Why are significant figures important in measurements?

Ans. They show the accuracy and reliability of a measurement.

Q.8   What is the rule for multiplication and division of a measurement?

Ans. The result should have the same number of significant figures as the value with the least number of significant figures.

Q.9   What is the rule for addition and subtraction with significant figures?

Ans. The result should have the same number of decimal places as the value of the least number of decimal places.

Q.10 Using rules of significant figures, compute $\dfrac{5.348 \times 10^{-10} \times 3.64 \times 10^4}{1.336}$ up to appropriate significant figures.

Ans.

$$\frac{5.348 \times 10^{-10} \times 3.64 \times 10^4}{1.336} = \frac{5.348 \times 3.64}{1.336} \times 10^{4-2}$$

$$= 14.5708982 \times 10^2$$

$$= 1.45708982 \times 10^3$$

In multiplication and division, the number of significant figures are not more than that contained in the least accurate factor.

So, according to above rule 1.46×10^3 are appropriate significant figures.

Q.11 What is accuracy and precision?

Ans. Accuracy is how close a measured value is to the true or accepted value. Precision is how close repeated measurements are to each other.

Q.12 Can a measurement be precise but not accurate?

Ans. Yes, if repeated measurements are close to each other but far from the true value.

Q.13 Can a measurement be accurate but not precise?

Ans. Yes, if measurements are close to the true value but vary widely from each other.

Q.14 How can precision be improved?

Ans. By using better instruments and consistent measurement techniques.

Q.15 Isa precise measurement is also an accurate measurement? Explain your answer.

Ans. A precise measurement means closeness of two measured values to each other while an accurate measurement means how close a measured value to the true value.

So, if the measurements are close to each other as well as close to the true value, then such measurements will be precise as well as accurate.

Q.16 What is uncertainty in measurement?

Ans. Uncertainty is the doubt or possible error in the result of any measurement.

Q.17 What causes uncertainty in measurement?

Ans. Instrument limitations, human error and environmental conditions.

Q.18 What is absolute uncertainty?

Ans. The margin of error in the measured value, expressed with the same result.

Example: 20.0 ± 0.5 cm

Q.19 What is relative uncertainty?

Ans. It is the ratio of absolute uncertainty to the measured value.

$$\text{Relative uncertainty} = \frac{\text{Absolute uncertainty}}{\text{Measured value}}$$

Q.20 Why is it important to report uncertainty in measurements?

Ans. It indicates the reliability and precision of the measurement.

Q.21 How uncertainty can be reduced in a timing experiment?

Ans. Timing uncertainty can be reduced by taking large number of readings and find their mean value.

Example:

Let, the time period of 30 vibrations of the simple pendulum be 54.6 s.

Then time for one vibration is $T = \dfrac{54.6}{30} = 1.82\ s$

Q.22 Give that; V = (5.2 ± 0.1) volt. Find its percentage uncertainty.

Ans. V = (5.2 ± 0.1) volt

Absolute uncertainty = 0.1

Measured value = 5.2

$$\%\ \text{age uncertainty in V} = \dfrac{\text{Absolute uncertainty}}{\text{Measured Value}} \times 100$$

$$\%\ \text{age uncertainty in V} = \dfrac{0.1}{5.2} \times 100 = 1.92\% \approx 2\%$$

$\boxed{\%\ \text{age uncertainty in V} \approx 2\%}$

Q.23 Find the percentage uncertainty in the volume of a cylinder, if the percentage uncertainties in length and diameter of the cylinder are 0.3% and 0.6% respectively.

Ans.

$$\text{Volume of a cylinder} = V = \pi r^2 l = \pi \left(\dfrac{d}{2}\right)^2 l$$

$$\text{Volume of a cylinder} = V = \dfrac{\pi d^2 l}{4} \qquad \left(\because r = \dfrac{d}{2}\right)$$

Data:

% age uncertainty in length (l) = 0.3 %

% age uncertainty in diameter (d) = 0.6 %

So % age uncertainty in Volume (V) = (% age uncertainty in l) + 2 (% age uncertainty in d)

= 0.3 % + 2 (0.6 %)

= 0.3 % + 1.2 %

$\boxed{\%\ \text{age uncertainty in volume} \quad = 1.5\ \%}$

Q.24 The volume of a sphere is V = 47.689 cm³ with 1.2% uncertainty. What is the correct range of volume measurement?

Ans.

$$\text{Volume of a sphere} = V = \dfrac{4}{3}\pi r^3$$

Data: Measured value = 47.689 cm³

Percentage uncertainty in V = 1.2

As $\text{Percentage uncertainty in V} = \dfrac{\text{Absolute uncertainty}}{\text{Measured value}} \times 100\%$

$$1.2 = \dfrac{\text{Absolute uncertainty}}{47.689} \times 100\%$$

$$\dfrac{1.2 \times 47.689}{100} = \text{Absolute uncertainty}$$

Absolute uncertainty = 0.6 cm³

So, the correct range of volume measurement is

$\boxed{V = (47.7 \pm 06)\ cm^3}$

Q.25 What are dimensions of a physical quantity?

Ans. Dimensions show how a physical quantity depends on the base quantities like mass, length, time, etc.

Q.26 What are dimensions important in physics?

Ans. They help in checking the correctness of equations and converting units.

Q.27 What is dimensional analysis?

Ans. It is a method of using dimensions to check the correctness of equations to derive relationship.

Q.28 Can dimensions help in deriving formulae?

Ans. Yes, using the principle of homogeneity of dimensions.

Q.29 Calculate the dimension of Physical quantities, if possible, 2π and rupees hundred.

Ans. Both 2π and rupees hundred have no dimensions.

Because 2π has no unit and rupees hundred is not a physical quantity. So both can't be expressed in terms of dimension. Because dimensional analysis is the method to express the physical quantities in terms of most fundamental quantities.

Q.30 Write the dimensions of:

(i) Force	(ii) Energy	(iii) Angular momentum
(iv) angular Velocity	(v) Work	(vi) Torque

Ans.

(i)   Dimensions of Force : $[F] = [ma] = [m]\,[a] = [M]\,[LT^{-2}] = [MLT^{-2}]$

(ii)   Dimensions of Energy : $[W] = [Fd] = [F]\,[d] = [MLT^{-2}]\,[L] = [ML^2T^{-2}]$

(iii)   Dimensions of angular momentum: $[L] = [mvr] = [m][v][r] = [M][LT^{-1}][L] = [ML^2T^{-1}]$

(iv)   Dimensions of angular velocity: $[\omega] = \dfrac{[\theta]}{[t]} = \dfrac{[1]}{[T]} = [T^{-1}]$

(v)   Work: $[W] = [F.d] = [MLT^{-2}]\,[L] = [ML^2T^2]$

(vi)   Torque: $[\tau] = [r]\,[F] = [L]\,[ma] = [L]\,[m]\,[a] = [L]\,[M]\,[LT^2] = [LMLT^{-2}] = [MLT^{-2}]$

Q.31 Differentiate between dimensional and non-dimensional variables.

Ans.

Dimensional Variables	Non-Dimensional Variables
Definition	
The physical quantities having dimensions are called dimensional variables.	The physical quantities having no dimensions are called non-dimensional variables.
Examples	
Length, Force, Velocity, Momentum etc.	Plain angle, Solid angle, Strain etc.

Q.32 Differentiate between dimensional and non-dimensional constants.

Ans.

Dimensional Constants	Non-Dimensional Constants
Definition	
The physical constants having dimensions are called dimensional constants.	The physical constants having no dimensions are called non-dimensional constants.
Examples	
Gravitation constant, g, k, η, h, etc.	Refractive index, ε, π, e, etc.

Q.33 Find percentage uncertainty in the volume of a cylinder if percentage uncertainties in length and diameter of cylinder are 0.2 and 0.8% respectively.

Ans.   Volume of Cylinder $= V = \pi r^2 l = \pi \left[\dfrac{d}{2}\right]^2 l$

$$= V = \dfrac{\pi d^2 l}{4} \qquad\qquad \because r\,\dfrac{d}{2}$$

% age uncertainty in length (l) = 0.2 %

% age uncertainty in diameter (d) = 0.8 %

So,% age uncertainty in volume = (% uncertainty in l) + 2(% uncertainty in$d$)

$$= 0.2\% + 2(0.8\%)$$

$$= 0.3\% + 1.6\%$$

% age uncertainty in V   = 1.9%

Q.34 Check correctness of E = hf dimensionally.

Ans. E = hf

L.H.S:

$E = w = Fd$

$[E] = [MLT^{-1}][L]$

$[E] = [ML^2T^{-2}]$ .... (i)

R.H.S:

hf

$$hf = \frac{[E]}{[v]} = \frac{[ML^2T^{-2}]}{[T^{-1}]}$$

$hf = [ML^2T^{-2}]$ .... (ii)

From eq (i) and (ii) we conclude that

$E = hf$ is dimensionally correct.

Q.35 Why do we find it useful to have two units for the amount of substance, the kilogram and the mole?

Ans. Both kilogram and mole are used as units for the amount of a substance.

Kilogram	Mole
(i) Kilogram is used for ordinary measurement of mass.	(i) Mole is used for scientific work.
(ii) Kilogram is used for large-scale mass of a substance i.e., at macro level.	(ii) Mole is used for small mass of a substance i.e, at micro level.
(iii) One kilogram of different substances contains different number of molecules.	(iii) One mole of all substances contains the same number of atoms/molecules i.e., Avogadro's number. (i.e, $N_A = 6.022 \times 10^{23}$)
(iv) Kilogram is used when number of molecules are not required but weight of the substance is required.	(iv) Mole is used when number of atoms / molecules are required. 1 mole $= 6.02 \times 10^{23}$ atoms / molecules

Q.36 Three students measured the length of a needle with a scale on which minimum division is 1 mm and recorded as (i) 0.2145 m (ii) 0.21 m (iii) 0.214 m. Which record is correct and why?

Ans. 0.214 is correct record

Reason:

Least count of the scale = 1 mm = 0.001 m, which is in three decimal points.

Therefore, the third reading 0.214 m is correct because this reading is in three decimal points.

Q.37 Does a dimensional analysis give any information on constant of proportionality that may appear in an algebraic expression? Explain.

Ans. A dimensional analysis does not give any information about the value of constant of proportionality present in the algebraic expression. This constant can be determined theoretically or practically. The formula for time period of simple pendulum is:

$$T = 2\pi \sqrt{\frac{l}{g}}$$

where dimension analysis does not give any information about 2π

However, *dimensions* of some constants of proportionality can be determined by the dimensional analysis.

Example: Dimensions of G, the gravitational constant are given by,

$[G] = [M^{-1}L^3T^{-2}]$

Q.38 Write the dimensions of:

(i) Pressure (ii) Density.

Ans. (i)   Dimensions of Pressure

Pressure is defined by relation,

$$P = \frac{Force}{Area} = \frac{F}{A}$$

Dimensions of pressure $= [P] = \dfrac{[F]}{[A]} = \dfrac{[MLT^{-2}]}{[l^2]} = [ML^{-1}T^{-2}]$

$$\boxed{\text{Dimension of pressure} = ML^{-1}T^{-2}}$$

(ii) Dimension of Density:

Density is defined by relation,

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}} = \frac{M}{V}$$

$$\text{Dimensions of Density} = \frac{[M]}{[V]} = \frac{[M]}{[L^3]} = [ML^{-3}]$$

$$\boxed{\text{Dimensions of Density} = [ML^{-3}]}$$

Q.39 The wavelength λ of a wave depends on the speed v of the wave and its frequency f. knowing that

$$[\lambda] = [L] \quad , \quad [v] = [LT^{-1}] \quad \text{and} \quad [f] = [T^{-1}]$$

Decide which of the following is correct, $f = v\lambda$ or $f = \frac{v}{\lambda}$.

Ans. Dimension of wavelength $= [\lambda] = [L]$

Dimension of speed $= [v] = [LT^{-1}]$

Dimension of frequency $[f] = [T^{-1}]$

(i) For $f = v\lambda$, Dimensions of L.H.S $= [f] = [T^{-1}]$

Dimensions of R.H.S $= [v][\lambda] = [LT^{-1}][L] = [L^2T^{-1}]$

Since, dimensions of L.H.S $\neq$ dimensions of R.H.S, therefore, the equation $f = v\lambda$ is not correct.

(ii) For $f = v/\lambda$, Dimensions of L.H.S $= [f] = [T^{-1}]$

Dimensions of R.H.S $= \frac{[v]}{[\lambda]} = \frac{[LT^{-1}]}{[L]} = \frac{[L][T^{-1}]}{[L]} = [T^{-1}]$

Since, dimensions of L.H.S = dimensions of R.H.S, therefore, the equation $f = v/\lambda$ is correct.

Q.40 How many nano seconds in 1 year?

Ans. Given: Time = 1 year

To Find:

No. of nanoseconds in 1 years $= ?$

Calculations:

Seconds in one year $= (365 \times 24 \times 60 \times 60)$ s

One year $= 3.1536 \times 10^7$ s

One second $= 10^9$ nanoseconds $= 10^9$ ns

$= 3.1536 \times 10^7$ s

$= 3.1536 \times 10^7 \times 10^9$ ns

$= 3.1536 \times 10^{16}$ ns

Q.41 Dimension of coefficient of viscosity.

Ans. As; $F\Sigma = 6\pi\eta rv$

or $\eta = \frac{F}{6\pi rv}$

6π is dimensionless, because it has numeric value so,

$$[\eta] = \frac{[F]}{[r][v]} = \frac{[MLT^{-2}]}{[L][LT^{-1}]}$$

$$= [ML^{-1}T^{-1}]$$

$$\boxed{\textbf{SOLVED EXERCISE}}$$

$$\boxed{\textbf{MULTIPLE CHOICE QUESTIONS}}$$

Tick ($\checkmark$) the correct answer.

1.1 The purpose of study and discoveries in Physics is:
   (a) the *probing of interstellar spaces*
   (b) *the betterment of mankind* $\checkmark$
   (c) the development of destructive technology in warfare
   (d) development in aesthetics for the world

1.2 The length of a steel pipe is in between 0.7 m to 0.8 m. Identify from the following, the appropriate instrument to be used for an accuracy of 0.001 m.
(a) A micrometer screw gauge
(b) A metre rule
(c) A ten metres measuring tape
(d) A Vernier Callipers ✓

1.3 The diameter of a steel ball is measured using a Vernier Callipers and its reading is shown in the figure. What is the diameter of the steel ball?
(a) 1.30 cm
(b) 1.39 cm ✓
(c) 1.40 cm
(d) 1.31 cm
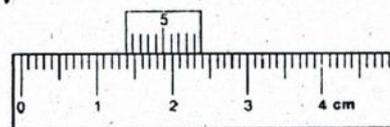
1.4 The figure shows the reading on a micrometer screw gauge used to measure diameter of a thin rod. One complete turn of the thimble is 0.50 mm and there are 50 lines on the circular scale. The diameter of the rod is:
(a) 3.67 mm ✓
(b) 3.17 mm
(c) 4.17 mm
(d) 4.20 mm
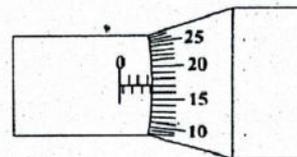
1.5 The number of significant figures of a measurement are defined as:
(a) they reflect the accuracy of the observation in a measurement
(b) they are the figures which are reasonably reliable
(c) they are the accurately known digits and the first doubtful digit of a measurement
(d) all of the above ✓

1.6 The number of significant figures in the measured mass 2500.0 kg is:
(a) two
(b) three
(c) four
(d) five ✓

1.7 The sum 12 kg + 2.02 kg + 5.1 kg according to appropriate precision is:
(a) 19 kg
(b) 19.0 kg
(c) 19.1 kg ✓
(d) 19.12 kg

1.8 The answer to appropriate precision for the subtraction (1.126-0.97268) is:
(a) 0.15
(b) 0.153 ✓
(c) 0.1533
(d) 0.15332

1.9 The answer of the product (2.8723 × 1.6) to the appropriate number of significant figures is:
(a) 4.59568
(b) 4.595
(c) 4.59
(d) 4.6 ✓

1.10 The answer to the mathematical division (45.2 ÷ 6.0) in appropriate number of significant figures is:
(a) 7.5 ✓
(b) 7.53
(c) 7.533
(d) 7.5333

1.11 The answer to the following mathematical operation 24.4 m × 100 m / 5.0 m to the appropriate number of significant figures is:
(a) 4880 m
(b) 4900 m
(c) 4.88×10^3 m
(d) 4.9×10^2 m ✓

1.12 The ratio of the dimensions of force and energy is:
(a) T
(b) T^{-1}
(c) L
(d) L^{-1} ✓

1.13 Identify which pair from the following does not have identical dimensions.
(a) Work and torque
(b) Angular momentum and Planck's constant
(c) Moment of inertia and moment of force ✓
(d) Impulse and momentum

1.14 The following figures are of the same Vernier Callipers. Figure (1) shows the reading when the jaws are closed while Fig. (2) shows the reading when a solid cylinder is placed between the jaws. The length of the cylinder is:
(a) 3.26 cm ✓
(b) 3.30 cm
(c) 3.34 cm
(d) 4.20 cm

1.15 The least count of an instrument determines:
(a) precision of a measurement ✓
(b) accuracy of a measurement
(c) fractional uncertainty of a measurement
(d) percentage uncertainty of a measurement

1.16 A measuring tape has been graduated with a minimum scale division of 0.2 cm. The allowed reading using this tape may be:
(a) 80.5 cm
(b) 80.6 cm ✓
(c) 80.65 cm
(d) 80.7 cm

1.1   What are base units and derived units? Give some examples of both these units.

Ans.   Base Units: Base units are the fundamental units of measurement in the International System of Units (SI) that are defined independently. These units are used to measure base quantities like length, mass, time, electric current, temperature, amount of substance, and luminous intensity.

Examples of base units

- Meter (m) – for length
- Second (s) – for time
- Kelvin (K) – for temperature
- Candela (Cd) – for luminous intensity
- Kilogram (kg) – for mass
- Ampere (A) – for electric current
- Mole (mol) – for the amount of substance

Derived Units: Derived units are units that are formed from the combination of base units according to physical laws and relationships. These units express quantities that involve more than one base unit.

Examples of derived units

- Metre per second (ms^{-1}) – for velocity
- Kilogram per cubic metre (kgm^{-3}) – for density
- newton (N) – for force (1 N = 1 $kgms^{-2}$)
- Joule (J) – for energy (1 J = 1 Nm)
- pascal (Pa) – for pressure (1 Pa = 1 Nm^{-2})

1.2   How many significant figures should be retained in the following?
(i) Multiplying or dividing several numbers   (ii) Adding or subtracting numbers

Ans. The rules for significant figures depend on whether you are multiplying/dividing or adding-subtracting numbers.

(i)   Multiplying or Dividing Several Numbers
When multiplying or dividing several numbers, the result should be rounded to the same number of significant figures as the number with the least significant figures in the calculation.
Example: 4.56 (3 significant figures) × 1.4 (2 significant figures) = 6.384. The result should be rounded to 2 significant figures (the least among 3 and 2), giving 6.4.

(ii)   Adding or Subtracting Numbers
When adding or subtracting numbers, the result should be rounded to the same decimal place as the number with the least number of decimal places.
Example: 12.11 (2 decimal places) + 18.0 (1 decimal place) = 30.11. The result should be rounded to 1 decimal place (since 18.0 has 1 decimal place), giving 30.1.

1.3   How is the Vernier scale related to the main scale of a Vernier Callipers? What is meant by L.C. of the Vernier Callipers?

Ans. The Vernier scale is a secondary scale on a Vernier Callipers that slides along the main scale to provide more precise measurements. It is designed to measure fractional parts of the smallest unit on the main scale, thus improving the precision of measurements.

How they are related?

Main Scale: The main scale provides the primary measurement, typically in millimetres or centimetres. The smallest division on the main scale is usually 1 mm or 0.1 cm.

Vernier Scale: The Vernier scale is graduated in such a way that it allows measurement of fractions of the smallest division on the main scale. The number of divisions on the Vernier scale is usually chosen so that each division on the Vernier scale is slightly shorter or longer than one division on the main scale. This difference allows the user to read more accurately by comparing the position of the zero point of the Vernier scale to the divisions on the main scale.

For example, if the main scale measures up to 1 mm, the Vernier scale might be divided into 10 parts, each representing 0.1 mm. When the zero point on the Vernier scale aligns with a point on the main scale, it allows for measurements that are more precise than the main scale's smallest division. By reading where the divisions on the Vernier scale match up with divisions on the main scale, the user can measure to a fraction of the smallest division on the main scale such as 0.1 mm.

In short, the Vernier scale enhances precision of the main scale by enabling readings that are more precise than what the main scale alone would allow.

Least Count

The least count of a Vernier Callipers is the smallest length that can be measured with the instrument. It is determined by the difference between the value of one main scale reading and one Vernier scale reading. Mathematically, it is calculated as:

Least count = Value of one main scale division – Value of on Vernier scale division

For example, if the main scale has divisions of 1 mm and the Vernier scale has 10 divisions over 9 mm, the least count would be:

$$\text{Least count} = 1\text{ mm} - \frac{9\text{ mm}}{10} = 0.1\text{ mm}$$

Thus, the least count of the Vernier Callipers would be 0.1 mm.

1.4 Write the following numbers in scientific notation:

(a) 143.7   (b) 206.4×10^2

Ans. (a) 1.437×10^2 (b) 2.064×10^4

1.5 Write the following numbers using correct prefixes:

(a) 580×10^2 g   (b) 0.45×10^{-5} s

Ans. (a) 58,000 g = 58 kg   (b) 4.5×10^{-6} s = 4.5 μs

1.6 **Kinetic energy of a body of mass m moving with speed v is given by $1/2\ mv^2$. What are the dimensions of kinetic energy?**

Ans. The formula for kinetic energy is given by

$$\text{K.E.} = \frac{1}{2}mv^2$$

To determine the dimensions of kinetic energy, we need to find the dimensions of each quantity involved.

- Mass (m) has the dimension of mass, denoted as [M].
- Velocity (v) has the dimension of length divided by time, denoted as $[LT^{-1}]$.

Now, the dimensions of: $\text{K.E} = \frac{1}{2}mv^2$ are:

$$[\text{K.E.}] = [M]\,[LT^{-1}]^2 = [M]\,[L^2T^{-2}]$$

So, the dimensions of kinetic energy are: $[ML^2T^{-2}]$.

This corresponds to the dimensions of energy or work.

1.7 **How many significant figures are there in the following measurements?**

(i) 37 km   (ii) 0.002953 m   (iii) 7.50034 cm   (iv) 200.0 m

Ans. (a) 2 significant figures

(37 is whole number with no decimal point, and both digits are non-zero)

(b) 4 significant figures

(Leading zeroes are not significant, only 2, 9, 5 and 3 counts)

(c) 6 significant figures

(All digits are significant, including zeros between and after non-zero digits)

(d) 4 significant figures

(The trailing zero after the decimal point is significant, including precision)

1.8 **Write the dimensions of: (i) Planck's constant (ii) angular velocity.**

Ans. (i) Planck's constant has the dimensions of action, which is energy multiplied by time.

$$\text{Energy} = \frac{[\text{Mass}] \times [\text{Length}]^2}{[\text{Time}]^2} = \frac{[M] \times [L^2]}{[T^2]} = [ML^2T^{-2}]$$

$$\text{Time} = T$$

So, dimensions of Planck's constant $= \dfrac{[ML^2T^{-2}]}{[T]} \times [T] = [ML^2T^{-1}]$

(ii) Angular velocity is the rate of change of angular displacement with respect to time.

Angular displacement is dimensionless quantity, so angular velocity has dimensions of $\dfrac{1}{\text{Time}}$.

CONSTRUCTED RESPONSE QUESTIONS

1.1 Why do we find it useful to have two units for the amount of a substance, the kilogram and the mole?

Ans. Having both the kilogram and the mole as units for the amount of substance is useful because they serve different purposes in different contexts.

(i) Kilogram (kg) is a unit of mass, which is used to quantify the amount of material in terms of its physical weight. It is a practical unit when dealing with larger quantities of matter or when mass is directly measured in experiments or real-world scenarios.

(ii) Mole (mol) is a unit of the amount of substance, used to count entities like atoms, molecules, or ions. The mole allows scientists to relate the macroscopic (bulk) amount of material to the microscopic (atomic or molecular) scale. One mole corresponds to Avogadro's number (approximately (6.022×10^{23})) of entities.

The two units are connected, as the mass of one mole of a substance is equal to its molar mass (in grams). This relationship allows chemists to move seamlessly between macroscopic quantities (kg) and microscopic quantities (moles) when conducting experiments, performing calculations, or designing reactions.

Thus, the kilogram is practical for measuring the total mass of a sample, while the mole is fundamental for understanding chemical reactions and molecular behaviour.

1.2 Three students measured the length of a rod with a scale on which minimum division is 1 mm and recorded as: (i) 0.4235 m (ii) 0.42 m (iii) 0.424 m. Which record is correct and why?

Ans. To determine which student's measurement is correct, we must consider the least count of the measuring scale and the concept of significant figures.

Given: The scale has divisions of 1 mm = 0.001 m. So, the least count (smallest measurable unit) is 0.001 m. Let us analyze each measurement.

(i) 0.4235 m

This has 4 decimal places, i.e., precision up to 0.0001 m. But the scale can only measure up to 0.001 m, so this is more precise than the scale allows.

Incorrect due to over-precision.

(ii) 0.42 m

This has two decimal places, i.e., precision up to 0.01 m. This is less precise than the scale's least count.

Acceptable, though not using the full precision of the instrument.

(iii) It has 3 decimal places (precision up to 0.001 m). This matches the least count of the instrument. **This is the correct measurement.**

Conclusion

(iii) 0.424 m is the correct record because it reflects the appropriate number of significant figures based on the least count (1 mm or 0.001 m) of the scale.

1.3 Why is the kilogram (not the gram), the base unit of mass.

Ans. The kilogram is the base unit of mass in the International System of Units (SI) because of its historical role in standardizing mass measurement. Initially, in the late 18th century, the kilogram was defined as the mass of a liter of water, which was a practical and reproducible reference.

However, over time, the kilogram was later defined by a physical object as a platinum-iridium cylinder kept at the International Bureau of Weights and Measures (BIPM) in France. This was known as the "International Prototype of the Kilogram."

The decision to use the kilogram, rather than the gram, as the base unit arose from the need for a convenient unit that could be easily scaled up or down for practical use. Since mass measurements often involve objects of various sizes, starting with the kilogram as a standard made more sense than using the gram, which would have *required frequent conversions for everyday applications.*

In 2019, the kilogram was redefined based on fundamental constants, specifically Planck's constant, rather than a physical object, making it more accurate and stable in terms of scientific measurements. But the kilogram remains the base unit for mass because of its historical and practical significance.

1.4 **Consider the equation:** $P = Q + R$

If Q and R both have the dimensions of [MLT], what are the dimensions of P? What are the units of P in SI? If the dimensions of Q were different from those of R, could we determine dimensions of P?

Ans. Given that the dimensions of both P and Q are R (Mass × Length × Time), we can infer the following:

(i) **Dimensions of P**

Since P = Q + R and both Q and R have the same dimensions of [MLT], the dimensions of P will be the same as the dimensions of Q and R, i.e., [MLT].

Dimensions of P = [MLT]

(ii) **Units of P in SI**

The units of P, like Q and R, will depend on the dimensions [MLT]. In the SI system:
- M (Mass) has units of kilograms (kg),
- L (Length) has units of meters (m),
- T (Time) has units of seconds (s).

So, the units of P in SI will be:

Unit of P = kg m s

(iii) **If the dimensions of Q were different from those of R:**

If Q and R have different dimensions, it would be impossible to directly add them because the physical quantities would not be of the same kind. In that case, we cannot determine the dimensions of Q + R in a straightforward manner, since adding quantities with different dimensions does not have a clear physical meaning.

1.5 **What is the least count of a clock if it has:**
   (a) Hour's hand, minute's hand and second's hand
   (b) Hour's hand and minute's hand

Ans. The least count of a clock refers to the smallest time interval that can be measured by the clock's hands. Let's look at both scenarios:

(a) Clock with hour's hand, minute's hand, and second's hand:
The second's hand moves one second per tick, so the least count is 1 second.

(b) Clock with hour's hand and minute's hand:
The minute's hand moves one minute per tick, but the hour's hand moves slower. The smallest interval that can be measured is the movement of the minute's hand, which corresponds to 1 minute. So, the least counts are:

(a) 1 second   (b) 1 minute

1.6 **How can the diameter of a round pencil be measured using metre rule with the same accuracy as that of Vernier Callipers? Describe.**

Ans. To measure the diameter of a round pencil using a metre rule with the same accuracy as a Vernier Callipers, follow these steps:

(i) Place several identical pencils side by side on a flat surface so that they are tightly packed in a straight line. For example, align 10 pencils to create a larger width that can be measured more accurately.

(ii) Use the metre rule to measure the total width of the packed pencils. Ensure the metre rule is placed perpendicular to the pencils for an accurate reading. Note the total length, ensuring you measure to the smallest possible division on the metre rule.

(iii) Divide the total width by the number of pencils to obtain the average diameter of one pencil.

(iv) By measuring the combined width of multiple pencils, any error in reading is averaged out across the total number of pencils. This effectively increases the accuracy of the measurement, approaching the precision of a Vernier Callipers.

Example: If 10 pencils measure a total width of 7.5 cm, the diameter of one pencil is:

$$\text{Diameter} = \frac{7.5 \text{ cm}}{10} = 0.75 \text{ cm}$$

This technique reduces human error and the limitations of the meter rule's precision by leveraging the larger measurement range.

1.7 How would be the readings differ if the screw gauge is used instead of a Vernier Callipers to measure the thickness of a glass plate?

Ans. When measuring the thickness of a glass plate with a screw gauge instead of a Vernier Callipers, the key difference lies in the precision and the method of measurement:

(i) Precision

A screw gauge typically provides higher precision than a Vernier Callipers. While a Vernier Callipers can measure up to 0.01 cm or 0.1 mm, a screw gauge can measure to 0.001 cm or 0.01 mm, offering more accurate readings.

(ii) Measurement Method

Vernier Callipers: The measurement is taken by reading the scale directly on the jaws that clamp around the object (in this case, the glass plate).

Screw Gauge: The measurement is taken by rotating the screw mechanism until the spindle touches the object, and the reading is taken from the thimble scale and the main scale.

(iii) Reading Differences

When using a screw gauge, the reading will typically involve interpreting both the main scale (usually a linear scale) and the thimble scale (which gives the finer measurement), whereas the vernier calliper has a main scale and a sliding vernier scale to read the measurement.

(iv) Glass Plate Measurement

Vernier Callipers: You would place the glass plate between the jaws and clamp it, and then read the scale directly. However, the glass plate may not fit easily into the jaws if it's too thick.

Screw Gauge: The plate would be placed between the spindle and anvil, and the micrometer screw would measure the thickness more precisely. However, if the glass plate is thick, it may require several measurements to ensure accuracy, as screw gauges are typically better for measuring smaller objects like wires or thin plates.

1.8 Write the correct reading of the length of a solid cylinder as shown in the figure if there is an error of +0.02 cm in the Vernier Callipers.
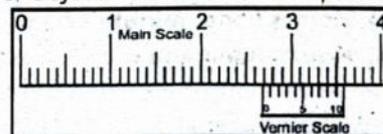
Ans.

Main scale reading	= 2.6 cm
Vernier scale line coinciding with the M.S reading	= 9
Vernier constant	= 0.01
Observed length of solid cylinder	= M.S reading + Vernier scale reading
	= 2.6 cm + 0.01 cm × 9
Observed length	= 2.69 cm
Zero error	= +0.02 cm
Zero correction	= −0.02 cm
Hence Actual length	= 2.69 cm − 0.02 cm
	= 2.67 cm

1.9 There are 50 divisions on the circular scale of a screw gauge. If the head (thimble) of the screw is given 10 revolutions, then the spindle advances by 5 mm. There is also zero error as the 2nd division of the circular scale coincides with the datum line and zero of circular scale is below the datum line. What is the thickness of a glass slab as measured by the described screw gauge shown in the figure?
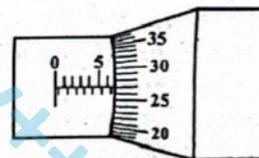
Ans.

Reading completed on the main scale = 6.5 mm

$$\text{Least count} = \frac{0.5 \text{ mm}}{50} = 0.01 \text{ mm}$$

Zero error = +0.01 mm

Zero correction = −0.01 mm

It is because on closing, the screw gauge still shows 0.1 mm reading, so correction should be −0.01 mm. Therefore, total thickness of glass slab is:

$$d = 6.5 \text{ mm} + 0.27 \text{ m} = 6.77 \text{ mm}$$

As 27th line coincides with datum line, thus, corrected thickness will be:

$$6.77 \text{ mm} - 0.01 \text{ cm} = 6.76 \text{ mm}$$

**1.10 What is meant by a dimensionless quantity? Give one example.**

Ans. A dimensionless quantity is a physical quantity that has no units and no physical dimensions. It is a pure number resulting from the ratio or combination of quantities where the units cancel out.

Example: The refractive index is a dimensionless quantity. It is the ratio of the speed of light in vacuum to the speed of light in a medium, $n = \dfrac{c}{v}$

Since both c and v have the same units (ms⁻¹), the units cancel out making dimensionless.

Other common example of a dimensionless quantity is angle in radians, since it is the ratio of arc length to radius.

1.11 A student uses a screw gauge to determine the thickness of a sheet of paper. The student folds the paper three times and measures the total thickness of the folded sheet. Assume that there is no zero error in the screw gauge. The reading of screw gauge is shown in the figure. Find the thickness of the sheet.
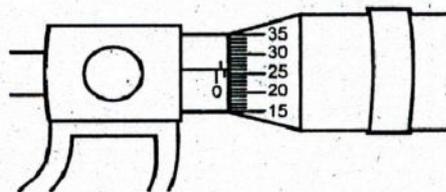
Ans. Least count = $\dfrac{0.5\ mm}{50}$ = 0.01 mm

Reading completed on the main scale is:

1 mm + 26 × 0.01 mm   = 1.26 mm

As 26ᵗʰ line coincides with datum line.

If zero correction is zero, then corrected thickness of 3 pages is 1.26 mm.

Thickness of a single page will be:

$\dfrac{1.26\ mm}{8}$ = 0.158 mm

1.12 Round off each of the following numbers to 3 significant figures and write your answer in scientific notation.

    (a) 0.02055   **(b) 4656.5**

Ans. (a) 0.02055

Rounding to 3 significant figures:

    0.0206

In scientific notation, it can be written as:

    2.06×10^2

(b) 4656.5

Rounding to 3 significant figures:

    4660

In scientific notation, it can be written as:

    4.66×10^3

COMPREHENSIVE QUESTIONS

1.1 What is meant by uncertainty in a measurement? How the uncertainty in a digital instrument is indicated?

Ans. See Q. 7

1.2 Differentiate between the terms precision and accuracy with reference to measurement of physical quantities.

Ans. See Q. 14

1.3 (a) What is meant by significant figures? Write two reasons for using them in measurements. How to find the uncertainty in a timing experiment such as the time period of a simple pendulum?

Ans. See Q. 8 and Q. 9

(b) The mass of a solid cylinder is 12.85 g. Its length is 3.35 cm and diameter is 1.25 cm. Find the density of its material expressing the uncertainty in the density.

Ans. The density ρ is given by

For a solid cylinder, the volume is:

$$V = \pi r^2 h = \pi \left(\frac{d}{2}\right)^2 h$$

Given values:

Mass $m = 12.85$ g

Length $h = 3.35$ cm

Diameter $= 1.25$ cm

We assume standard instrument uncertainties.

- Mass $= 0.01$ g
- Length and diameter $= \pm 0.01$ cm

As $r = \dfrac{d}{2} = \dfrac{1.25\ \text{cm}}{2} = 0.625$ cm

Volume $V = \pi (0.625\ \text{cm})^2 (3.35\ \text{cm}) \approx 3.14 \times 0.3906 \times 3.35 \approx 4.111\ \text{cm}^3$

Density $\rho = \dfrac{12.85\ \text{g}}{4.111\ \text{cm}^3} \approx 3.126\ \text{g cm}^{-3}$

Now

Uncertainty in Volume:

$$\frac{\Delta V}{V} = \frac{2\Delta d}{d} + \frac{\Delta h}{h}$$

$$\frac{\Delta V}{V} = 2 \times \frac{0.01}{1.25} + \frac{0.01}{3.35} \approx 0.016 + 0.003 \approx 0.019$$

Uncertainty in Density:

$$\frac{\Delta \rho}{\rho} = \frac{\Delta m}{m} + \frac{\Delta V}{V}$$

$$\frac{\Delta \rho}{\rho} = \frac{0.01}{12.85} + \frac{0.078}{4.111} \approx 0.0008 + 0.019 \approx 0.0198$$

$$\Delta \rho = 0.0198 \times 3.126 \approx 0.062\ \text{g cm}^{-3}$$

Final answer is:

$$\rho = 3.13 \pm 0.06\ \text{g cm}^{-3}$$

1.4 Explain with examples the writing of physical quantities into their dimensions. Write its two benefits.

Ans. See Q. 16 and Q. 19

1.5 Check the homogeneity of the relation:

COMPREHENSIVE QUESTIONS

where v is the speed of transverse wave on a stretched string of tension T, length l and mass m.

Ans. See Q. 17

NUMERICAL PROBLEMS

1.1 Astronomers usually measure astronomical distances in light years. One light year is the distance that light travels in one year. If the speed of light is 3 ×10⁸ ms⁻¹, what is one light year in metres?

Solution:

Given Data:

Time $= t = 1$ year

1.11 A student uses a screw gauge to determine the thickness of a sheet of paper. The student folds the paper three times and measures the total thickness of the folded sheet. Assume that there is no zero error in the screw gauge. The reading of screw gauge is as shown in the figure. Find the thickness of the sheet.

Ans. Least count $= \dfrac{0.5\ \text{mm}}{50} = 0.01$ mm

Reading completed on the main scale is:

$1\ \text{mm} + 26 \times 0.01\ \text{mm} = 1.26$ mm

As 26th line coincides with datum line.

If zero correction is zero, then zero corrected thickness of 3 pages is 1.26 mm.

Thickness of a single page will be:

$\dfrac{1.26\ \text{mm}}{8} = 0.158$ mm

1.12 Round off each of the following numbers to 3 significant, answer in scientific notation.

(a) 0.02055   (b) 4656.5

Ans. (a) 0.02055

Rounding to 3 significant figures:

0.0206

In scientific notation, it can be written as:

2.06×10^{-2}

(b) 4656.5

Rounding to 3 significant figures:

4660

In scientific notation, it can be written as:

4.66×10^{3}

1.1 What is meant by sensitivity? How is sensitivity of a digital instrument is indicated?

Ans. See Q. 7

1.2 Differentiate between the terms... with reference to measurement of physical quantities.

Ans. See Q. 14

1.3 (a) What is meant by significant figures? Write two examples such as the uncertainty in measuring the...

Ans. See Q. 8 and Q. 3

(d) The mass of a solid cylinder is 12.85 g. Its length is 3.35 cm and diameter is 1.25 cm. Find the density of its material expressing the uncertainty in the density.

Ans. The density ρ is given by:

$= 365 \times 24 \times 60 \times 60$ s

$= 31536000$ s

$= 3.1536 \times 10^{7}$ s

To Find: Distance $= S = ?$

Formula: $S = vt = ct$

Calculations:

$S = 3 \times 10^{8}\ \text{ms}^{-1} \times 3.1536 \times 10^{7}$ s

One light year = 9.15×10^{15} m **Ans.**

1.2 Write the estimated answer of the following in standard form.
(a) How many seconds are there in 1 year?
(b) How many years are in 1 second?

Solution:

(a) Given Data:

Time = 1 year

To Find:
Number of seconds in 1 year = ?

Calculations:

One year = 365 × 24 × 60 × 60 s

= 31536000 s

= 3.1536×10^7 s

$\boxed{\text{one year} = 3.2 \times 10^7 \text{ s}}$

(b) Given Data:

Time = 1 second

To Find:
Number of years in 1 second = ?

Calculations:

1 year = 3.1536×10^7 s

or 3.1536×10^7 s = 1 year

or $1 s = \dfrac{1}{3.1536 \times 10^7}$ years

$\boxed{\text{one second} = 3.1 \times 10^{-8} \text{ years}}$

1.3 The length and width of a rectangular plate are measured to be 18.3 cm and 14.60 cm, respectively. Find the area of the plate and state the answer to correct number of significant figures.

Solution:

Given data:

Length = L = 18.3 cm (3 significant figures)
Width = W = 14.60 cm (4 significant figures)

To find:

Area = A = ?

Calculations:

Area = Length × Width

or A = L × W

or A = 18.3 cm × 14.60 cm = 267.18 cm²

When multiplying or dividing, the result should be reported with the same number of significant figures as the value with the fewest significant figures. So, the result should be given to 3 significant figures.

Thus A = 267 cm²

1.4 Find the sum of the masses given in kg upto appropriate precision:

(i) 3.197   **(ii) 0.068**

(iii) 13.9   **(iv) 3.28**

Sol. The given masses are:

3.197 (3 decimal places)
0.068 (3 decimal places)
13.9 (1 decimal place)
3.28 (2 decimal places)

Adding the numbers:

3.197 + 0.068 + 3.26 + 3.28 = 20.445

Rounding off the result:

The result must be rounded off to 1 decimal place, as the least precise measurement is 13.9 kg. Therefore

$\boxed{20.4 \text{ kg}}$ **Ans.**

1.5 The diameter and length of a metal cylinder measured with the help of a Vernier Callipers of least count 0.01 cm are 1.22 cm and 5.35 cm respectively. Calculate its volume and uncertainty in it.

Solution:

Given data:

Diameter = d = 1.22 cm
Length = h = 5.35 cm
Least count of Vernier Callipers = 0.01 cm
So, uncertainty in d and h; Δd = Δh = 0.01 cm

Calculations:

To calculate volume and its uncertainty for a cylinder, we will use the formula:

$$V = \pi \left(\dfrac{d}{2}\right)^2 h$$

Putting the values

$$V = \dfrac{3.14 \times (1.22 \text{ cm})^2 \times 5.35 \text{ cm}}{4}$$

$$V = \dfrac{3.14 \times 1.488 \text{ cm}^2 \times 5.35 \text{ cm}}{4}$$

$$V \approx \dfrac{25.0015 \text{ cm}^3}{4} \approx 6.25 \text{ cm}^3$$

For uncertainty in volume:

$$\dfrac{\Delta V}{V} = 2 \times \dfrac{\Delta d}{d} + \dfrac{\Delta h}{h}$$

$$\dfrac{\Delta V}{V} = 2 \times \dfrac{0.01}{1.22} + \dfrac{0.01}{5.35}$$

$$\approx 2 \times 0.0082 + 0.00187$$

$$\approx 0.0164 + 0.00187$$

$$\approx 0.01827$$

Now, calculating the absolute uncertainty:

$$\Delta V = 0.01827 \times 6.25 \text{ cm}^3$$

$$\approx 0.1142 \text{ cm}^3$$

Thus V = (6.25 ± 0.11) cm³

1.6 Show that the expression; $v_f^2 - v_i^2 = 2aS$ is dimensionally correct, where v is the initial velocity, a is the acceleration and v_f is the velocity after covering a distance S. Show that the famous "Einstein equation" $E = mc^2$ is dimensionally consistent.

Solution:

Given that:

$$v_f^2 - v_i^2 = 2aS \qquad \text{...... (i)}$$

Velocity 'v' has the dimensions: $[v] = [LT^{-1}]$

Acceleration 'a' has the dimensions: $[a] = [LT^{-2}]$

Δ Displacement or distance 'S' has the dimensions: $[S] = [L]$

Dimensions of left hand side $(v_f^2 - v_i^2)$ of Eq. (i) are:

Since both v_f and v_i have the same dimensions, so

$$[v_f^2] = [v_i^2] = [LT^{-1}]^2 = [L^2T^{-2}]$$

Therefore, the L.H.S has dimensions: $[L^2T^{-2}]$

Dimensions of R.H.S of Eq. (1) are:

In 2aS, the constant '2' is dimensionless, so we only consider 'a' and 'S'.

$$[a][S] = [LT^{-2}][L] = [L^2T^{-2}]$$

Since both sides have same dimensions, therefore, the equation $v_f^2 - v_i^2 = 2aS$ is dimensionally correct.

1.7 Show that the famous "Einstein equation" $E = mc^2$ is dimensionally consistent.

Solution:

Given Data:

$$E = mc^2$$

Calculations:

L.H.S

As

$$E = W = Fd$$

So

$$[E] = [MLT^{-1}][L]$$

or

$$[E] = [ML^2T^{-2}] \qquad ...\text{(i)}$$

R.H.S

$$[mc^2] = [M][LT^{-1}]^2$$

or

$$[mc^2] = [ML^2T^{-2}] \qquad ...\text{(ii)}$$

From equation (i) and (ii), we have

$$E = mc^2$$

As the dimensions of both sides of the equation are the same, therefore, the above equation is dimensionally consistent.

1.8 Derive a formula for the time period of a simple pendulum using dimensional analysis. The various possible factors on which the time period T may depend are:

(i) length of the pendulum

(ii) mass of the bob m

(iii) angle θ which the thread makes with the vertical

(iv) acceleration due to gravity g.

Solution:

Assume $T \propto l^a m^b g^c \theta^d \qquad \text{...... (i)}$

Angle θ is dimensionless (it has no physical units), so it cannot be included in dimensional analysis. Equation (i) becomes as:

$$T = k\, l^a m^b g^c \qquad \text{...... (ii)}$$

where k is dimensionless constant.

Writing the dimensions of all the quantities:

$$T_{(time\ period)} = [T]$$
$$l_{(length)} = [L]$$
$$m_{(mass)} = [M]$$
$$g_{(gravitational\ acceleration)} = [T]$$

Substituting dimensions in Eq. (ii)

$$[T] = [L]^a [M]^b [LT^{-2}]^c = L^a M^b L^c T^{-2c}$$
$$[T] = [L^{a+c} M^b T^{-2c}]$$

Now, equating powers of fundamental dimensions on both sides;

Power of L: $a + c = 0$

Power of M: $b = 0$

Power of T: $-2c = 0$

Now solving the equations

From $b = 0$

It means that mass has no effect on time period.

From $-2c = 1$

or $c = \dfrac{-1}{2}$

From $a + c = 0$

or $a = -c = \dfrac{1}{2}$

For finding the final expression:

$$T = k \times l^{1/2} g^{-1/2} = k \times \sqrt{\dfrac{l}{g}}$$

This gives:

$$T = k\sqrt{\dfrac{l}{g}}$$

or

$$\boxed{T = \text{constant} \sqrt{\dfrac{l}{g}}}$$