2024

Wednesday, November 05, 2025 11:56 AM

Fuzzy logic and contr...

1a. Describe how the concept of a fuzzy number fits in with the overall notion of computing with words as pioneered by Lutfi Zadeh.

- In classical set theory, an element belongs to a set or it does not.
- However, many real world concepts involving sets of things are gradual or vague e.g. *tallness* of a person. Some may argue that 5'10" should be considered tall, but others would argue that 6' should be considered tall. The rules of classical set theory are not able to capture this vagueness.
- A fuzzy number is a fuzzy set on the real numbers. It allows the classification of fuzzy sets on numbers e.g. to represent that someone is "*around 30 years old*", fuzzy numbers present a membership function which may look something like this μ_{AGE} = {25:0.3, 29:0.9, 30:1, 32:0.7, 35:0,3} where 30 itself has a full membership, and the membership of the other ages decreases as the numerical value becomes further away from 30. This fuzzy number is also an example of a triangular membership function, which has a peak at the value 30.
- Fuzzy numbers are useful as they allow mathematical operations to be conducted with vague or imprecise values.
- The notion of "Computing With Words" was introduced as, according to Zadeh "computing involves manipulation of numbers and symbols". While humans "arrive at conclusions expressed as words from premises expressed in a natural language".
- Computing with words was introduced as a way of making computations with vague words/ideas.
- Fuzzy sets and fuzzy numbers are used to map these vague concepts to numerical values which can be computed by machines. E.g. *"temperature is medium"* = {20, 25, 30}, *"temperature is hot"* = {28, 30, 34}. We can now conduct arithmetic with these words e.g. *"If the temperature is medium and the humidity is high, set the fan to fast"*
- Fuzzy numbers are implemented in Computing With Words by the fuzzification of the vague notions as above into fuzzy numbers, the deriving of a computational result as per "fan" example, and the defuzzification of the fuzzy numbers back into natural language understanding

How will you describe a positive fuzzy number and distinguish it from a negative fuzzy number? (A paper on fuzzy numbers by Ali et al 2016 is on Blackboard as is Zadeh 1996 on fuzzy logic).

- The idea of positive and negative fuzzy numbers is explored by Ali et.al.
- In order for a fuzzy number to be valid, there must be at least one element of the fuzzy set which has unity membership (1). There is a constraint on the definition of a fuzzy number that fuzzy numbers must be represented by a Normal fuzzy set. A Normal fuzzy set is one such that at least one element is unity.
- The membership values must also decrease away from the peak (unity) e.g. to represent that someone is "*around 30 years old*", fuzzy numbers present a membership function which may look something like this μ_{AGE} = {25:0.3, 29:0.9, 30:1, 32:0.7, 35:0,3} where 30 itself has a full membership, and the membership of the other ages decreases as the numerical value becomes further away from 30.
- Ali also explores the α-cut operation on fuzzy numbers. This is a method of closing the membership function under some bound. This turns the fuzzy set into a crisp set. All elements of the fuzzy set which have a membership greater than (or less than) some specific threshold are included in the crisp set. E.g. if I perform an α-cut on the above AGE fuzzy set with the α-level set to 0.5, the resulting crisp set would be {29, 30, 32} as all these elements have a

membership in the fuzzy set greater than 0.5

- Positive triangular fuzzy numbers as described in Ali's paper are triangular fuzzy numbers such that for all A=a_i in the set, all a_i's are >0
- Negative triangular fuzzy numbers as described in Ali's paper are triangular fuzzy numbers such that for all A=a_i in the set, all a_i's are <0
- Ali also describes the notion of partial negative triangular fuzzy numbers, which is described as triangular fuzzy numbers such that there exists some (but not all) A=a_i in the set A such that a_i <0.
- Trivially, the original example of the fuzzy set AGE could be described as a negative triangular fuzzy set. This is because numbers are infinite and hypothetically the fuzzy set could be extended as follows:
  μ_{AGE} = {-19:0.001 ,17:0.05, 20:0.2, 25:0.3, 30:1, 35:0.3, 45:0.1, 80:0.001}
  This is clearly an unreasonable example with a nonsensical universe of discourse for a fuzzy number describing age, as age cannot be negative.
- Therefore, based on the natural language understanding of the notion of age, it is possible to define the fuzzy number "around 30 years old" as a positive fuzzy number, based of natural language understanding.
- It is clear that fuzzy numbers can be (sometimes trivially) expanded to become partially negative.
- The α-cut operation is an advantageous way to distinguish positive and negative fuzzy numbers from each other. This allows us to reliably check the entire support of the fuzzy set (support = mebership>0 to evaluate if it is a positive or negative fuzzy number)
- This also expands the idea of positive and negative fuzzy numbers past Ali's description of triangular fuzzy numbers. Other types of membership functions can reliably distinguish between positive and negative numbers by validating the support of the fuzzy number after some α-cut operation is applied e.g. sigmoidal or trapezoidal membership functions.

1b. Arithmetic operations involving two fuzzy numbers, P and Q, are defined in terms of the parameters of the membership function of the fuzzy number:
P ≡ P(p1, p2, p3) and Q ≡ Q(q1, q2, q3).
The sum and difference of P and Q, denoted as fuzzy numbers
Sum (p1+q1, p2+q2, p3+q3) = P(p1, p2, p3) + Q(q1, q2, q3);
Diff (p1-q3, p2-q2, p3-q1) = P(p1, p2, p3) - Q(q1, q2, q3).
Consider a pipe that has length P represented by a positive fuzzy number. This number is represented with a parabolic membership function with parameters: $p1 = 14\ cm$, $p2 = 15.5$ cm, $p3 = 17\ cm$
or P (14, 15.5, 17). We cut a fuzzy number Q (6,7.5,9) from the pipe.

NOTE: A parabolic fuzzy number $\boldsymbol{A}$ is defined by three parameters: p_1, p_2, p_3 that describe its two extremities (p_1, p_3) and its height , p_2:

The membership function is defined as

$$\mu_{\tilde{A}}(x) = \begin{cases} \left(\frac{x-p_1}{p_2-p_1}\right)^2, & if\ p_1 \le x \le p_2 \\ 1, & if\ x = p_2 \\ \left(\frac{p_3-x}{p_3-p_2}\right)^2, & if\ p_2 \le x \le p_3 \\ 0, & if\ otherwise \end{cases}$$

(i) Calculate the length of the residual pipe R=P-Q
R = Diff = (14-9, 15.5-7.5, 17-6) = (5,8,11)
(ii) What are crisp number equivalents of P, Q, and R;
Deriving the crisp number equivalents of the fuzzy numbers P,Q,R involves the process of defuzzification.
A common method of defuzzification is finding the centroid of the fuzzy number.
Using the centroid as the method of defuzzification:
P_{crisp} = (14 + 15.5 + 17)/3 = 15.5
Q_{crisp} = (6 + 7.5 + 9)/3 = 7.5
R_{crisp} = (5 + 8 + 11)/3 = 8
All of the centroids of these fuzzy numbers evaluate to the median value, this appears to be a very

evenly distributed fuzzy number

(iii) Draw a graph showing the fuzzy numbers P, Q, and R.

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2016_Comparison ...

Zadeh_Fuzzy_logic_...

2011_Biswas and Gh...

1985 Takagi Sugeno P...

2003 Kwok et al Rule ...

2003Kwok_
RuleBase...

Rule-base derivation for intensive care ventilator control using ANFIS

Abstract

In recent years, much research has been done on the use of fuzzy systems in medicine. The fuzzy rule-bases have usually been derived after extensive discussion with the clinical experts. This takes a lot of time from the clinical experts and the knowledge engineers. This paper presents the use of the adaptive neuro-fuzzy inference system (ANFIS) in rule-base derivation for ventilator control. The change of the inspired fraction of oxygen (FiO_2) advised by eight clinical experts responding to 71 clinical scenarios was recorded. ANFIS and a multilayer perceptron (MLP) were then used to model the relationship between the inputs (the arterial oxygen tension (PaO_2), FiO_2 and the positive end-expiratory pressure (PEEP) level) and the change in FiO_2 suggested. Compared to a previous fuzzy advisor (FAVeM), both the ANFIS and the MLP were found to correlate with the clinicians' decision better (correlation coefficient of 0.694 and 0.701, respectively compared to 0.630). A formerly developed model-based radial basis network advisor (RBN-MB) was used for comparison. Closed-loop simulations showed that the ANFIS, MLP and the RBN-MB's performance were comparable to the clinicians' performance (correlation coefficients of 0.852, 0.962 and 0.787, respectively). The FAVeM's performance differed from the clinicians' performance (correlation coefficient of 0.332) but the resulting PaO_2 was still within safety limits.

1. Introduction

The use of fuzzy systems has been widely investigated in biomedical applications. They are appealing because the system can often be interpreted in linguistic terms which are understandable to clinicians. In recent years, they have been applied to the control of ventilators in intensive care units.

Artificial ventilation of the lungs is one of the major components of intensive care therapy. The aim of the artificial ventilation is to deliver oxygen to the tissues and to remove carbon dioxide when the patient's lungs are not able to function adequately. The clinicians in the ICU adjust the various ventilator settings in order to achieve a reasonable level of oxygenation in the blood. Clinicians make these decisions based upon knowledge of the pathophysiology of the lungs and the patient's condition and past medical history.

Providing adequate oxygenation of the arterial blood is crucial to the maintenance of life. The normal PaO_2 when breathing atmospheric air for individuals aged 20–29 years is 11.2–13.9 kPa [14]. In a ventilated patient, PaO_2 can be controlled by adjusting the FiO_2 and the positive end-expiratory pressure (PEEP) level. The latter increases the surface area of the lung available for gas exchange. Increasing the FiO_2 will increase the PaO_2. However, prolonged therapy with high FiO_2 may cause oxygen toxicity. Therefore, there is sometimes a trade-off between maintaining a normal PaO_2 and avoidance of using a high FiO_2. The target PaO_2 for a patient often depends on his/her age, previous health and disease status.

2. Development of expert advisory systems for intensive care ventilators

Although most of the decision-making is done by the medical team in the intensive care units, there has always been an interest in the development of an expert advisory system for intensive care ventilators. Early developments included systems based on clinical protocols [10] and systems based on artificial intelligent using classical logic [11]. In the recent years, fuzzy logic advisors have been reported [2,12,16]. The input–output relationship of a fuzzy control system is defined by the partitions of the input and output space, the rule-base, the inference methods and the defuzzification methods. Rule-base derivation is therefore an important aspect of fuzzy controller design. There is no single approach to rule-base derivation. It can be developed heuristically, using existing knowledge, via a model-based approach or through self-learning.

There have been three fuzzy logic controllers for intensive care ventilators reported recently. In the Fuzzy Advisor for Ventilator Management (FAVeM) [2], the rule-base was developed after extensive literature survey and consultations with a clinical expert. It was also tested using a model of ventilation, Simulation of Patients under Artificial Ventilation (SOPAVent) [1]. The rule-base was adjusted heuristically based on the simulation result of the model. As there was only one clinical expert involved in the fuzzy membership function derivation and the rule-base derivation, the system may be biased. As shown in [8], the fuzzy membership functions and partitions can vary widely among the clinical experts.

Nemoto et al. [12] developed a fuzzy controller to control the pressure support level. The membership functions were derived heuristically. The controller then classified the patient's condition and its trend based on the membership levels of the inputs. The outputs are be based on the fuzzy membership levels of the patient's condition and its trend. The controller was evaluated by comparing its output with the changes actually made by the clinicians. The correlation coefficient and the regression coefficients were not presented. However, the graphs showed substantial deviation of the controller's output from the actual changes made by the clinicians.

Schuh et al. developed the Fuzzy KBWean system [16]. It is a knowledge-based fuzzy rule-base system. The rule-base was derived by the clinical experts using Fuzzy KBEdit, which is a fuzzy rule-base editor. The system was designed to advise change in the positive inspiratory pressure (PIP) level, the PEEP level, the inspiration time, the expiration time and the FiO_2 level based on the pulse oximeter reading (which gives a continuous reading of arterial oxygen saturation), the blood-gases and the end-tidal carbon dioxide level. The algorithm was tested using retrospective patient data. It was found that in 16 episodes of hyper- or hypo-ventilation, Fuzzy KBWean reacted more promptly than the clinical staff. The system is still being developed and validated.

The three fuzzy control systems for intensive care ventilators used either a pure expert knowledge-based approach or a knowledge-based approach combined with heuristic tuning of the system with an existing model. The knowledge-based approach is commonly used in biomedical fuzzy systems. It is a means to encapsulate the experts' knowledge. However, it is a time-consuming procedure. The size of the rule-base will also increase exponentially with the number of fuzzy partitions. Alternatively, empirical knowledge can be extracted from the domain experts. The tuning of the rule-base is then done heuristically or manually. It may reduce the time required to interview the experts but the tuning may be time-consuming. It is beneficial to develop a rule-base derivation process which is less time-consuming and matches the experts' advice more accurately. This study aims to explore the use of an adaptive network fuzzy inference system (ANFIS) in rule-base derivation in order to reduce the time required for the rule-base derivation. It is part of a project to develop an expert advisory system for intensive care ventilator.

The decision-making process of the domain experts can be viewed as a mapping. Considering the control of PaO_2, clinicians usually try to maintain an acceptable level of PaO_2 by adjusting the FiO_2 or PEEP. The adjustment of FiO_2 depends on the PaO_2 level, the FiO_2 and the PEEP level. Therefore, the adjustment of FiO_2 can be viewed as a mapping of the inputs (PaO_2, FiO_2, PEEP) to the output space which is the change in FiO_2. Fuzzy systems can often be represented using neural networks [3]. Therefore, neural networks can be used to 'discover' this mapping provided sufficient input–output data are available. The adaptive neuro-fuzzy inference system (ANFIS) [5,6] is one such system. It encodes a Sugeno-type fuzzy inference system into a feed-forward neural network. The neural network can then be trained using known input–output data.

Multilayer perceptrons have been shown to be universal approximators of non-linear functions [4]. Hence, they can also be used to model this mapping. Based on the clinicians' proposed change in the FiO_2 during a simulator study, a rule-base was developed using ANFIS. For comparison, a multilayer perceptron was also developed to model the relationship between the PaO_2, FiO_2 and the PEEP level, and the change in FiO_2 proposed. The performance of these two models was compared to that of FAVeM.

Most of the fuzzy systems in biomedicine are knowledge-based systems. A model-based radial basis neural network (RBN-MB) was developed in an earlier study [7] to adjust the FiO_2. The ANFIS and MLP developed in this study are compared to the RBN-MB.

3. Adaptive network-based fuzzy inference system—ANFIS

An ANFIS model [5] is an adaptive neural network which represents a particular type of fuzzy inference system. Three types of fuzzy inference systems can be represented by an ANFIS model.

1. Type 1: A fuzzy inference system whose overall output is the weighted average of each rule's crisp output. The output membership functions are monotonic functions.
2. Type 2: A Mamdani fuzzy inference system where the centroid defuzzification operator is replaced by a discrete version which calculates the approximate centroid of area.
3. Type 3: A Sugeno-type fuzzy inference system whose output is a linear combination of the input variables plus a constant term.

In this study, a special case of Type 3 system was used. It is shown in Fig. 1(a). The consequent output member of each rule is a constant. The output of the system is a weighted average of these constants. It can be represented by the network shown in Fig. 1(b). The diagram shows a two inputs/one output system. The fuzzy inference system used in this study was a 3-inputs/1-output system and there were 3–4 fuzzy partitions for each input variable.

4. Methods

The controllers were developed to adjust the FiO_2 based on the current PaO_2 level, the current FiO_2 and PEEP level of the ventilator. The target PaO_2 was not set in the development of the ANFIS, the MLP and the FAVeM. In the development of the model-based RBN, the target PaO_2 was assumed to be 15 kPa. The PaO_2 from the arterial blood-gas measurement is not taken regularly, therefore the system developed was an event-driven control system. The controllers were also designed to emulate the control delivered by the consultant anaesthetists who worked in the intensive care units regularly.

4.1. Collection of training data

The data of the change in FiO_2 initiated by the clinicians under different clinical scenarios were collected using a patient simulator based on the SOPAVent model. Seventy-one blood-gas measurements from three patients in the intensive care unit of the Royal Hallamshire Hospital, Sheffield were retrieved. The shunt and deadspace at each blood-gas sampling point were calculated based on the blood-gases, the ventilator settings and the haemodynamic measurements. Seventy-one patient scenarios were built using the respiratory measurements, the haemodynamic data, the ventilator settings, the body temperature

(b)

Fig. 1. The Sugeno-type fuzzy inference system which is similar to the one that was used in this study. This diagram shows a two inputs/one output system with two fuzzy partitions for each input variable. (a) Shows the relationship of the output to the input membership functions. (b) Shows the neural network which is equivalent to the fuzzy inference system. Here, x and y are the inputs to the system. A_1, A_2 are the two fuzzy partitions for input x; B_1, B_2 are the two fuzzy partitions for input y; w_1 and w_2 are the membership values of the inputs for the two antecedents; and $\bar{w}_1$ and $\bar{w}_2$ are the normalized membership values. The normalized membership values in the two antecedents become the input from the third layer of the network to the output layer of the network. The layer weights of the output layer, which correspond to the output members k_1 and k_2 of the fuzzy inference system, can be adjusted during training.

and haemoglobin level. Nine consultant anaesthetists who regularly work in the intensive care unit from three hospitals were invited to undertake the simulation. The procedure is shown in Fig. 2. The patient's demographic data and the diagnosis were given in the simulations. To simulate the real clinical situation, the clinicians were not informed of the estimated shunt values. Each clinician was asked to advise on the amount to be changed in the FiO_2 at each blood-gas sampling point. The blood-gas at the next sampling point was calculated based on the new FiO_2 advised by the clinician and the clinical scenario at the next sampling point. The clinician was then asked to adjust the FiO_2 according to the new blood-gas result. The PaO_2, FiO_2 and the PEEP level at each sampling point were used as one input training data point and the amount to be changed in the FiO_2 was used as the corresponding output target level for the neural network or the neuro-fuzzy system.

4.2. Development of the ANFIS for FiO_2 control

The ANFIS model developed was a Sugeno-type fuzzy inference system which had three inputs and one output. The consequent parts were constants. The rules were in the form of:

if (PaO_2 is A_i) and (FiO_2 is A_j) and (PEEP is A_k), then change in FiO_2 is c_n,

where A_i, A_j, A_k are fuzzy sets and c_n is a constant.

The 'product' function was used for logical 'and' and the output derived was the weighted average of the output from all the rules. It was developed using the fuzzy logic toolbox of MATLAB version 5.3. The three inputs were the current PaO_2 level, the current FiO_2 level and the current PEEP level. The output was the amount of FiO_2 to be changed. Firstly, clustering was applied to the training data using MATLAB toolbox. Gaussian membership functions were used during the clustering process. A hybrid training algorithm, which is a combination of the gradient descent algorithm and the least-square estimation algorithm, was used for training. The cost function to be minimized was the mean least-squares error (MSE) (the difference between the outputs and the targets). Data from one of the clinicians were found to be unsuitable because the FiO_2 advised by him deviated from other clinicians substantially. Five hundred and sixty-eight data points were available for training. Two hundred and eighty-four were used as the training data, 142 as the cross-validation data and 142 as the testing data.

After training, 11 rules were derived. As there were overlapping membership functions, pruning was performed manually by merging two membership functions, which had a cross-over point of more than 0.9 and a more than 90% overlap in their supports, into a bell-shaped membership function and the ANFIS was retrained. The system was pruned and retrained until four partitions remained in the PaO_2 domain, three partitions in the FiO_2 domain and three partitions in the PEEP domain.

4.3. Development of the feed-forward multilayer perceptron

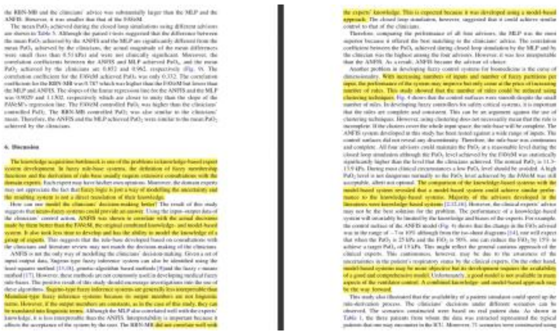
A feed-forward multilayer perceptron (MLP) was developed using the same training dataset. The inputs and output were the same as for the ANFIS model. Using the neural network toolbox of MATLAB 5.3, multilayer perceptrons of two layers with 6, 7, 8 and 9 hidden nodes were constructed. They were trained using the Levenberg–Marquardt algorithm. The MLP with 7 hidden nodes was found to have the smallest MSE for the whole training dataset and was selected.

4.4. Validation and comparison

The ANFIS, MLP, FAVeM and the RBN-MB were first validated using the simulation results by the clinicians. The output from each system for each data point was compared with the clinician's advice using linear regression. Then simulation was performed assuming they were controlling the FiO_2 in a closed-loop manner. The clinical scenarios used were the same as the simulations responded by the clinicians. At each sampling point, the advisors gave a recommended change in FiO_2. A new FiO_2 was calculated. The FiO_2 was limited to the range of 30–100% because in the intensive care scenario, a FiO_2 out of this range is rarely used in mechanical ventilation. The PaO_2 immediately after the change of FiO_2 was calculated based on the new FiO_2 and the patient scenario. This was compared with the PaO_2 achieved by the clinicians. The blood-gas at the next sampling point was calculated based on the new FiO_2 and the new patient scenario.

5. Results

The demographic data of the three patients whose data were used in the simulations are shown in Table 1. The mean FiO_2 and PaO_2 during the simulations by the nine clinicians are shown in Table 2. Clinician two achieved a significantly higher PaO_2 level during the simulation than the rest of his colleagues. The FiO_2 advised by him were also significantly higher than the other clinicians. Therefore, his data were not included in the training set.



Neuro Fuzzy Systems (using a Takagi-Sugeno-Kang type control system)
Clinicians adjust patient's ventilator settings based onf FiO_2 and PEEP parameters with the goal of keeping PaO_2 at a healthy level.
Too low indicates lack of oxygen, too high is toxic.
Clinicians use their <u>expert</u> experience to choose to increase or decrease the input variables.

Fuzzy expert systems have been proposed to help in this task as it follows the IF THEN structure used in fuzzy logic and fuzzy expert systems.
Creating rule bases by interviewing clinicians is <u>not effective</u> as it is for some other fuzzy expert systems e.g. Biswas & Ghose's paper on cocoon grading.
ICU patients ventilation is too high-priority and taking clinicians away to interview them and gain expertise is slow, subjective and inconsistent.

This paper suggests ANFIS (a neuro-fuzzy system) to deal with this challenge of the pitfalls of using a traditional fuzzy expert system.
Instead of the experts deriving the rules manually, ANFIS learns by watching the clinicians decisions.

ANFIS is a Takagi-Sugeno-Kang fuzzy inference system converted to a neural network.
- <u>Fuzzy sets/inputs</u>: IF PaO_2 is low then make FiO_2 high, etc.
- Turns the fuzzy rule base into a neural network layer
- Adjusts membership functions/outputs using gradient descent/least squares
- ANFIS learns fuzzy rules automatically

TSK Control System
For each rule, the output is a constant
The final output is a weighted average of the constants, where the weights come from the fuzzy firing strengths.

Linguistic Variables/Fuzziness
Each rule outputs an adjustment to the FiO_2 level which can be represneted linguistically as {decrease, slight decrease, negative zero, positive zero, slight increase, increase}

Comparison between Mamdani Fuzzy Inference Systems & TSK Fuzzy Inference Systems
<u>Mamdani</u>:
1. Structure: if PaO_2 is low and FiO2 is high then change in FiO_2 is slight increase
2. Both the antecedents and conclusion are using **linguistic fuzzy sets**
3. The output of the rule is also a **fuzzy set**. e.g. slight increase could be a triangular fuzzy set
4. After evaluating many rules, you combine all the fuzzy sets into one large fuzzy output shape
5. Finally, you convert that finalising fuzzy set into a crisp value based on a method such as the centroid

Pros	**Cons**
Very human readable (if-then)	Hard to train automatically (needs expert)

Same shape of reasoning as the cocoon example from the Biswas & Ghose paper	Rule base must be built-up manually
	Defuzzification is computationally expensive
	Not ideal for very large sets of inputs

TSK:
1. Structure: if PaO2 is low and FiO_2 is high then change in FiO_2 is c (where c is some crisp constant, not a fuzzy set)
2. All rules give crisp constants so the final output is a weighted average where the numerical constant output is simply weighted by its firing strength in the fuzzy rule base

Pros	**Cons**
Easy to train	Less linguistic/readable than Mamdani
Fast and numerically stable	Readability has to be mapped back to words like "slight increase"
Membership functions can be tuned automatically	

Why use Takagi-Sugeno-Kang for a neuro fuzzy system?
The authors needed a system that:
- Could learn from real clinical decisions
- Avoids manually interviewing experts
- Can be trained on lots of patient scenarios
- Can be mathematically optimised

Mamdani is not suitable in this scenario because:
- Mamdani output is a fuzzy set which must be interpreted and can't easily be passed into a neural network (no constants)
- Mamdani consequents are linguistic and can't be numerically optimised
- Mamdani rule bases must be manually authored

Kwot used a **first order Sugeno system with constant rule consequents**
This means that the IF part is fuzzy and the THEN part is a constant
It is the simplest TSK form and fits ANFIS suitably
The output being a weight average fits the control system perfectly and makes it easy to map the constant values back to linguistic understanding

Feedback Control system with fuzzy specification
- It takes the current patient measurements (FiO_2, PaO_2)
- Decides how to adjust FiO_2 given PaO_2
- Waits for the patient's lungs to respond
- Takes new measurements
- Adjusts FiO_2 again

This is the definition of a feedback control system: output --> changes --> system response --> new changes --> new input --> new output
This allows the ANFIS system to chase the optimal PaO_2 level

Fuzzy specification is needed
- Clinical decisions about oxygen are: subjective, nonlinear, based on vague linguistics, dependent on combinations of variables, difficult to use crisp input values on
- Fuzzy specification allows: fuzzy inputs, fuzzy rule antecedents, partial truth values

Comparison of Kwok's ANFIs against a Multi-Layer perceptron (MLP)

ANFIS	**MLP**
Explicit fuzzy rules	Purely numerical
Membership functions are interpretable	No linguistic representation
Rule base is human-readable	No interpretable rules
Structure matches clinical reasoning (IF-THEN)	Clinicians cannot see why it made a decision
	Harder to validate clinically

- The ANFIS system was chosen as clinicians cannot trust a black-box neural network in which it cannot see its reasoning.
- Oxygen levels in ICU patients is too high-stakes to use a system which is harder to validate
- Even though MLP may produce more numerically accurate outputs, ANFIS is preferred as it was only very slightly outperformed by MLP, to the point where the readability of ANFIS is the defining trait in making the decision for use.

Zadeh - Computing With Words

Friday 14 November 2025 16:25

- **Fuzzy Logic** is a way to compute using **words** instead of **numbers**
- Computers work on exact numbers, fuzzy logic allows computers to handle vague information --> Zadeh calls this Computing With Words
- Natural language sentences must be transformed into **canonical forms** of the structure *X is R* where
    - X = the variable being constrained
    - R = the fuzzy set / fuzzy meaning of the word
      e.g.
    - X = Mary is young, Carol lives near Mary
    - R = Age(Mary) is young, Distance(Carol, Mary) is near

- **Fuzzy reasoning** moves the **fuzzy constraints** through **fuzzy rule bases**
- If X is A and Y is B, you can combine them
- If X affects Y, you can propagate the fuzziness from X to Y

NB: WHY VAGUENESS/FUZZINESS IS NOT RANDOMNESS. WHY STATISTICAL PROBABILITY IS NOT FUZZY LOGIC

Probability handles randomness.
Fuzzy logic handles vagueness
Computing with words and fuzzy logic allows us to deal with the meanings of words and notions which probability is not capable of representing

Randomness is about not knowing what outcome will occur, despite the fact that the outcomes themselves are clear and well-defined. e.g. the probability of a coin toss. Heads and tails of a coin are crisp notions. Before the coin toss, we know that it will be one of those two outcomes. This is **randomness**, NOT **uncertainty**. There is no **partially heads** or **sort of tails**.

Vagueness deals with concepts that do not have sharp defined boundaries e.g. tallness. Different people may have different interpretations of what it means to be tall. And someone can be "kind of tall" or "a little bit tall". There is no precision to tallness, it is a **vague notion**. This means that there is **degrees** to which the truth can be established by using a membership function.

	Randomness	**Vagueness**
Branch	Probability/Statistics	Fuzzy Logic
Uncertainty	Stochastic uncertainty	Semantic uncertainty
Statement	"Tom is 183cm tall"	"Tom is tall"
Outcome	Could be a true or false value e.g. a 0.7/1 chance of the statement being **completely** true	Degrees of truth to the statement e.g. if a 190cm person is considered "definitely tall" e.g. membership=1, then at 183cm could be **0.8 true**
Function	P(height(tom) is183cm) = 0.7	µ_tall(height(tom)) = 0.7
Meaning	We are uncertain about Tom's actual height	We might know Tom is exactly 183cm but the 0.7 comes from the vague boundary of being tall

Issues with Fuzzy Logic

Heavy human-intervention is required. Converting real sentences into their canonical form often requires inputs from subject matter experts

Biswas & Ghose

Friday 14 November 2025 17:34

This paper discusses **Fuzzy Expert Systems** in the context of grading silk cocoons which are usually graded by subjective human visual inspection.

Why is Fuzzy Logic Appropriate

- The silk cocoons can be described by a number of vague notions which influence the quality of the silk e.g. the size of the cocoon could be *small, medium, large*. These words have no exact numerical definition
- A crisp set would have definitive cutoffs e.g. size < 25 mm is small
- But in reality, size is fuzzy and gradually changes. It cannot simply be defined with 24.9mm being small and 25.1mm being medium.
- Fuzzy logic allows partial memberships e.g. a cocoon of size 25mm might be 0.7 small and 0.4 medium

Fuzzy Expert Systems are able to handle judgement from subject matter experts.

Fuzzification is conducted on the available information.

- The values of cocoon size (small, medium, large)
- And the values of cocoon length (short, average, long)
- Are converted from crisp values into fuzzy values.
- Each category has a membership function that maps the crisp inputs into fuzzy degrees of inclusion.
- Therefore, for each input into the FES, it should be evaluated against 6 membership functions, as there are two fuzzy notions (size, length) and each having three **overlapping** membership functions.

For example, if we had a cocoon of size 18mm in size and 26mm in length:

- 18 would be compared into the membership function for small
- 18 would be compared into the membership function for medium
- 18 would be compared into the membership function for large
- 26 would be compared into the membership function for short
- 26 would be compared into the membership function for average
- 26 would be compared into the membership function for long

Fuzzy Expert Systems are also able to utilise fuzzy operations

Fuzzy AND = the minimum of two membership functions
Fuzzy OR = the maximum of two membership functions
Fuzzy NOT = $1 - \mu$

Fuzzy sets and their complements do not behave like crisp sets, there is no such law of excluded middle

$$\neg(p \wedge \neg p)$$

The law of excluded middle states that either some proposition (p) or its complement (¬p) MUST be true. In crisp sets, there is no such thing as "Neither true nor false". Any statement that you make which is "partially true" must be False because it does not obey the Truth.

Fuzzy logic allows us to have "neither true nor false" and "partially true"

Linguistic term	Fuzzy number	Crisp score
Low	M_1	0.115
Below average	M_2	0.295
Average	M_3	0.495
Above average	M_4	0.695
High	M_5	0.895

An example of turning a human expert's judgement regarding the size of a cocoon into an objective number.

The actual inputs into this FES were: shell ratio (SR%), defective cocoon percentage (DC%), size (linguistic term)
The output of this FES gives a cocoon score with fuzzy levels 1-9

Input	**Fuzzy Set**
SR%	{low, med, high}
DC%	{low, med, high}
Size	{low, med, high}

Three inputs with three linguistic variables (membership functions) each, thus each cocoon must be evaluated against 3^3 = 27 membership functions / 27 rules in the fuzzy rule base.

Fuzzy Rule Base
Each rule in the fuzzy rule base is in the form of: *IF (SR% = X) AND (DC% = Y) AND (Size = Z) THEN Cocoon Score = N*
Example Fuzzy Rule Base - fuzzy rule bases are defined by the subject matter expert (so for the sake of notes, these grade values are just made up based on assumptions based on the fuzzy variables because I'm not an expert cocoon grader)

Rule	**SHELL RATIO % (fuzzy/linguistic)**	**DEFECTIVE COCOON % (fuzzy/linguistic)**	**SIZE (fuzzy/linguistic)**	**GRADE (crisp score)**
1	Low	Low	Low	4
2	Low	Low	Med	4
3	Low	Low	High	5
4	Low	Med	Low	3
5	Low	Med	Med	3
6	Low	Med	High	4
7	Low	High	Low	1
8	Low	High	Med	2
9	Low	High	High	2
10	Med	Low	Low	5
11	Med	Low	Med	8
12	Med	Low	High	8
13	Med	Med	Low	7
14	Med	Med	Med	7
15	Med	Med	High	8
16	Med	High	Low	5
17	Med	High	Med	5
18	Med	High	High	6
19	High	Low	Low	7
20	High	Low	Med	9

21	High	Low	High	9
22	High	Med	Low	7
23	High	Med	Med	8
24	High	Med	High	8
25	High	High	Low	5
26	High	High	Med	6
27	High	High	High	6

Good features like a high shell ratio, large size and minimal defects push the score up.
Bad features push the score downwards.

The fuzzy memberships of the inputs (from a cocoon) are combined using fuzzy and to produce the fuzzy values for that particular input e.g.

	SR	DC	Size
Cocoon Source Input	50%	15%	25mm
Fuzzified Low	0.4	0.8	0.3
Fuzzified Med	0.7	0.3	0.6
Fuzzified High	0.1	0	0.2

(I did not actually get these fuzzy values from a membership function, I made them up)
SR% is mostly medium (0.7)
DC% is mostly low (0.8)
Size is mostly medium (0.6)

This corresponds to rule 11 in the fuzzy rule base which gives a grade of 8 to the cocoon

But we must also take into consideration the other rules in the FES given the input

Rule	SR%	DC%	SIZE	How strong is the rule with the given input?	Grade
1	Low - 0.4	Low - 0.8	Low - 0.3	min(0.4. 0.8, 0.3) = 0.3	4
2	Low - 0.4	Low - 0.8	Med - 0.6	min(0.4, 0.8, 0.6) = 0.4	4
3	Low - 0.4	Low - 0.8	High - 0.2	min(0.4, 0.8, 0.2) = 0.2	5
4	Low - 0.4	Med - 0.3	Low - 0.3	min(0.4, 0.3, 0.3) = 0.3	3
5	Low - 0.4	Med - 0.3	Med - 0.6	min(0.4, 0.3, 0.6) = 0.3	3
6	Low - 0.4	Med - 0.3	High - 0.2	min(0.4, 0.3, 0.2) = 0.2	4
7	Low - 0.4	High - 0	Low - 0.3	min(0.4, 0, 0.3) = 0	1
8	Low - 0.4	High - 0	Med - 0.6	min(0.4, 0, 0.6) = 0	2
9	Low - 0.4	High - 0	High - 0.2	min(0.4, 0, 0.2) = 0	2
10	Med - 0.7	Low - 0.8	Low - 0.3	min(0.7, 0.8, 0.3) = 0.3	5
11	Med - 0.7	Low - 0.8	Med - 0.6	min(0.7, 0.8, 0.6) = 0.6	8
12	Med - 0.7	Low - 0.8	High - 0.2	min(0.7, 0.8, 0.2) = 0.2	8
13	Med - 0.7	Med - 0.3	Low - 0.3	min(0.7, 0.3, 0.3) = 0.3	7
14	Med - 0.7	Med - 0.3	Med - 0.6	min(0.7, 0.3, 0.6) = 0.3	7
15	Med - 0.7	Med - 0.3	High - 0.2	min(0.7, 0.3, 0.2) = 0.2	7
16	Med - 0.7	High - 0	Low - 0.3	min(0.7, 0, 0.3) = 0	5
17	Med - 0.7	High - 0	Med - 0.6	min(0.7, 0, 0.6) = 0	5
18	Med - 0.7	High - 0	High - 0.2	min(0.7, 0, 0.2) = 0	6
19	High - 0.1	Low - 0.8	Low - 0.3	min(0.1, 0.8, 0.3) = 0.1	7

20	High - 0.1	Low - 0.8	Med - 0.6	min(0.1, 0.8, 0.6) = 0.1	9
21	High - 0.1	Low - 0.8	High - 0.2	min(0.1, 0.8, 0.2) = 0.1	9
22	High - 0.1	Med - 0.3	Low - 0.3	min(0.1, 0.3, 0.3) = 0.1	7
23	High - 0.1	Med - 0.3	Med - 0.6	min(0.1, 0.3, 0.6) = 0.1	8
24	High - 0.1	Med - 0.3	High - 0.2	min(0.1, 0.3, 0.2) = 0.1	8
25	High - 0.1	High - 0	Low - 0.3	min(0.1, 0, 0.3) = 0	5
26	High - 0.1	High - 0	Med - 0.6	min(0.1, 0, 0.6) = 0	6
27	High - 0.1	High - 0	High - 0.2	min(0.1, 0, 0.2) = 0	6

Any rule that is non-zero fires
Many rules may point to the same grade in the Fuzzy Expert System

Rules which fire (non-zero firing strength)

Rule	**SR%**	**DC%**	**SIZE**	**How strong is the rule with the given input?**	**Grade**
1	Low - 0.4	Low - 0.8	Low - 0.3	min(0.4. 0.8, 0.3) = 0.3	4
2	Low - 0.4	Low - 0.8	Med - 0.6	min(0.4, 0.8, 0.6) = 0.4	4
3	Low - 0.4	Low - 0.8	High - 0.2	min(0.4, 0.8, 0.2) = 0.2	5
4	Low - 0.4	Med - 0.3	Low - 0.3	min(0.4, 0.3, 0.3) = 0.3	3
5	Low - 0.4	Med - 0.3	Med - 0.6	min(0.4, 0.3, 0.6) = 0.3	3
6	Low - 0.4	Med - 0.3	High - 0.2	min(0.4, 0.3, 0.2) = 0.2	4
10	Med - 0.7	Low - 0.8	Low - 0.3	min(0.7, 0.8, 0.3) = 0.3	5
11	Med - 0.7	Low - 0.8	Med - 0.6	min(0.7, 0.8, 0.6) = 0.6	8
12	Med - 0.7	Low - 0.8	High - 0.2	min(0.7, 0.8, 0.2) = 0.2	8
13	Med - 0.7	Med - 0.3	Low - 0.3	min(0.7, 0.3, 0.3) = 0.3	7
14	Med - 0.7	Med - 0.3	Med - 0.6	min(0.7, 0.3, 0.6) = 0.3	7
15	Med - 0.7	Med - 0.3	High - 0.2	min(0.7, 0.3, 0.2) = 0.2	7
19	High - 0.1	Low - 0.8	Low - 0.3	min(0.1, 0.8, 0.3) = 0.1	7
20	High - 0.1	Low - 0.8	Med - 0.6	min(0.1, 0.8, 0.6) = 0.1	9
21	High - 0.1	Low - 0.8	High - 0.2	min(0.1, 0.8, 0.2) = 0.1	9
22	High - 0.1	Med - 0.3	Low - 0.3	min(0.1, 0.3, 0.3) = 0.1	7
23	High - 0.1	Med - 0.3	Med - 0.6	min(0.1, 0.3, 0.6) = 0.1	8
24	High - 0.1	Med - 0.3	High - 0.2	min(0.1, 0.3, 0.2) = 0.1	8

For each grade that is considered (3,4,5,7,8,9), take the maximum of all the firing strengths that produce it

Grade	**Rules**	**Maximum of Firing**	**Result (μ)**
3	4,5	max(0.3, 0.3) = 0.3	0.3
4	1,2,6	max(0.3, 0.4, 0.2) = 0.4	0.4
5	3,10	max(0.2, 0.3) = 0.3	0.3
7	13,14,15,19,22	max(0.3, 0.3, 0.2, 0.1, 0.1) = 0.3	0.3
8	11,12,23,24	max(0.6, 0.2, 0.1, 0.1) = 0.6	0.6
9	20,21	max(0.1, 0.1) = 0.1	0.1

Now we can construct a membership function for which grade should be given to our input cocoon that had SR% = 50, DC% = 15 and Size = 25

To fuzzily this membership function, I'm just going to use a centroid/average defuzzification for simplicity

(3*0.3 + 4*0.4 + 5*0.3 + 7*0.3 + 8*0.6 + 9*0.1) / (0.3 + 0.4 + 0.3 + 0.3 + 0.6 + 0.1)

11.8 / 2 = 5.9

Therefore the defuzzified grade of this cocoon with a Shell Ratio of 50%, a Defection Cocoon Percentage of 15% and a Size of 25mm is 5.9

Using a Fuzzy Expert System

- Incorporates both objective data (size, percentages) and subjective, linguistic variables (low, high)
- It produces reliable, crisp value for grading the cocoons
- It matches expert judgement closely
- It can be applied to other cocoon inputs not just (50, 15, 25)

TAKAGI SUGENO

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Properties & Relationships

Wednesday, October 08, 2025 11:01 AM

Pipe of len 20cm cut incisions about 2-3cm. What is the result? You don't cut it to 100% precision. Length is always a fuzzy number.

What is a fuzzy number? How can I describe large sets of numbers which are fuzzy?

- Ordinary set theory require the maintenance of a crisp set.
- Fuzzy set theory is composed of an organised body of mathematical tools and this approach can help in dealing with incomplete information, the lack of sharp boundaries
- Fuzzy sets can be used to 'mathematically represent uncertainty and vagueness and provide formalized tools for dealing with the imprecision intrinsic to many problems.'

	Example sentence	Elaboration	Status
ESTIMATIVE PROBABILITY	"And at this location there is a new airfield. [He could have located it to the second on a larger map.] Its longest runway is 10,000 feet."	**A statement of indisputable fact.** It describes something knowable and **known** with a high degree of certainty.	Known
ESTIMATIVE CERTAINTY	"It is almost certainly a military airfield."	**A judgment or estimate**. It describes something which is knowable in terms of the human understanding but not precisely known by the man [or woman] who is talking about it.	Knowable or inferable
ESTIMATIVE CERTAINTY	"The terrain is such that the Blanks could easily lengthen the runways, otherwise improve the facilities, and incorporate this field into their system of strategic staging bases. It is *possible* that they will." Or, more daringly, "It would be logical for them to do this and *sooner or later they probably will.*"	A [weaker] judgment or estimate, this [was] made almost without any evidence direct or indirect. It may be an estimate of something that no man alive can know	Indirectly inferable/ An assertion

Middle section is fuzzy. The transitions from known to unknown

100% Certainty		
The General Area of Possibility		
93%	give or take about 6%	Almost certain
75%	give or take about 12%	Probable
50%	give or take about 10%	Chances about even
30%	give or take about 10%	Probably not
7%	give or take about 5%	Almost certainly not
0% Impossibility		

'A fuzzy set is a class of objects with a continuum of grades of membership. Such a set is characterized by a membership (characteristic) function which assigns to each object a grade of membership ranging between zero and one

Membership function

Consider the *tallness* membership function. In the Western world, men who are 1.9 meters tall is regarded as definitely *tall*. And, men of height 1.2 meters are regarded as *not-tall;* men of height around 1.5 to 1.7 meters are regarded of *medium* height.

So

$$tall\,(h) = 1\ if\ h \geq 1.9;$$

$$tall\,(h) = 0, if\ h \leq 1.2$$

Let us fit a straight line to the above set of conditions

$$tall\,(h) = m * h + c$$

Where *m* & *c*, are constants that can be computed by using the conditions

$$tall\,(1.9) = m * 1.9 + c = 1$$

$$tall\,(1.2) = m * 1.2 + c = 0$$

Membership function

We have two simultaneous equations, which give us:

$$m = \frac{1}{0.7} = 1.428$$

$$c = -\frac{0.2}{0.7} = -0.285$$

We can express the truth of the statement *tall* as:

$$tall\,(h) = \frac{h}{0.7} - \frac{0.2}{0.7}$$

Or

$$tall\,(h) = \frac{\quad}{0.7} - \frac{\quad}{0.7}$$

Or

$$tall\,(h) = 1.4285 * h - 0.285; 0 \leq h \leq 1;$$

$$tall\,(h) = 1\ for\ h \geq 1.9;$$

$$tall(h) = 0\ for\ h \leq 1.2$$

Short is simply 1-tall

FUZZY NUMBER DEFINES AN UNCERTAIN REAL NUMBER BY EXTENDING THE CONCEPT OF A PRECISE NUMBER TO A FUZZY SUBSET OF REAL NUMBERS, REPRESENTED BY A MEMBERSHIP FUNCTION THAT ASSIGNS AS DEGREE OF MEMBERSHIP (0-1) TO EACH POSSIBLE VALUE.
Fuzzy number example: your height with shoes on, your height with a defective ruler
A fuzzy number is a type of number that represents a range of possible values rather than a single value, with each possible value assigned a weight between 0 and 1, known as the membership function.

Dispersion of data in crisp sets is defined as the degree to which the data approaches to an average value.
For real data, a dispersion measure is performed by determining a central position measurement (usually the arithmetic mean) and calculating the average distance (or semi-distances) from the data to such value. For instance, the variance is worked out as the average squared distance to the mean.

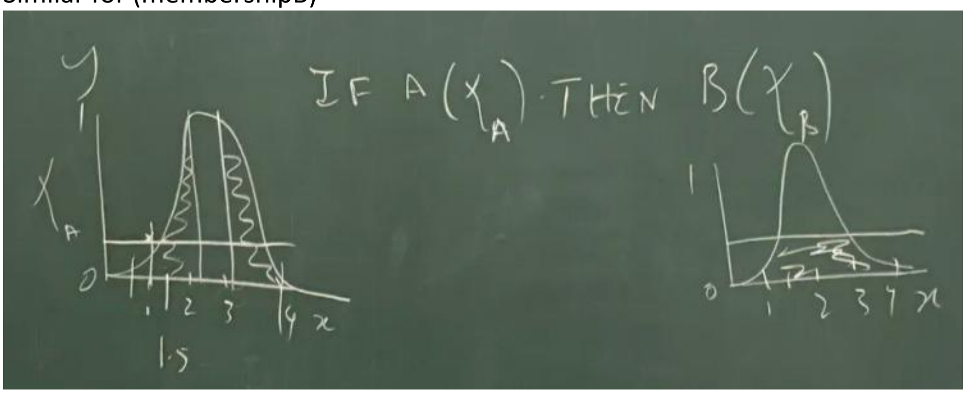
If something is belonging to a set with 0.3 and 0.6 members. Have to draw a line. Make the fuzzy membership function a crisp set

Fuzzy set variables can be numerical *and linguistic*. Crisp sets just have numerical variables
e.g. X weighs 5KG vs. X weighs about 5KG. Cannot trust inference more than evidence.

Linguistic variable - variable whose values are sentences in a natural/artificial language: tall, not tall, very tall, very very tall. Hence height is a linguistic variable
Fuzzy condition statements expressions of if A then B where A and B have fuzzy meaning
Fuzzy algorithm sequence of instructions which may contain fuzzy assignment
All notions of classical logic but fuzzified

Fuzzy relation extension of crisp relation that allows expressions involving ambiguity e.g. x and y are also the same. Relates elements from 2 sets of interest by assigning degrees of membership to pairs of elements – it is an elastic relationship

If A (membershipA) then B (membershipB) - CHOP THE MEMBERSHIP FUNCTION
(membershipA) takes various values – depending on the value of X it can definitely be in A or fuzzy in A.
Similar for (membershipB)



What do you think we do with fuzzy numbers?
Are the statistical techniques designed to deal with uncertainty? Why/why not?

We model at 1-mew(A)

DILATON & CONCENTRATION ALLOWS US TO GET CLOSE TO A CRISP SET
The alpha level cut allows you to turn a fuzzy set into a crisp set. Chops the fuzzy

TWO FUZZY SETS ARE EQUAL IF THEIR MEMBERSHIP FUNCTIONS ARE EQUAL FOR ALL POINTS

Fuzzy Patch – Fuzzy Relationships:
A mapping of two membership functions e.g. if warm then humid
The size of the fuzzy patch is the measure of uncertainty
These are patches:

EXAMPLE:
The rules governing the air-conditioner are as follows:

RULE#1:	IF	TEMP *is* COLD	THEN	SPEED *is* MINIMAL
RULE#2:	IF	TEMP *is* COOL	THEN	SPEED *is* SLOW
RULE#3:	IF	TEMP *is* PLEASENT	THEN	SPEED *is* MEDIUM
RULE#4:	IF	TEMP *is* WARM	THEN	SPEED *is* FAST
RULE#5:	IF	TEMP *is* HOT	THEN	SPEED *is* BLAST

Cross product: CONTROL = TEMP X SPEED
Minimum of temperature and speed (**AND**)
A implies B is an AND condition, take min val of two sets

Mamdani

Thursday, November 06, 2025 3:12 PM

Walkthrough of Mamdani and Takagi Sugero inteference systems [**WRITE THIS OUT**]
https://chatgpt.com/share/690cd54e-a310-8001-aafa-66dd7ded9729

Example of Fuzzy Logic Controller using Mamdani Approach- Part 1

solved Example of mamdani approach part 2

Mamdani Control System Walkthrough:

Step 1 – Problem Statement
We want a control system for how long to set the washing machine, given how dirty the clothes are and how sweaty the clothes are.
Both dirt and sweat are fuzzy, subjective measures.

Step 2 – Define Input & Output Variables

Variable	**I/O**	**Linguistic Variables**	**Range**
Dirt level	Input	{Low, Mid, High}	0-10
Sweat level	Input	{Low, Mid, High}	0-10
Washing time	Output	{Very Short, Short, Mid, Long, Very Long}	0-60 minutes

Step 3 – Fuzzification – Defining Membership Functions Based on the Linguistic Variables:
I am defining triangular membership functions for each aspect.
Dirt Level
Low Dirt: μ_dLow(x): 1 for x </ 2, decreases to 0 at x=5

Mid Dirt: μ_dMid(x): 1 at x=5, 0 at x=2 and x=8
High Dirt: μ_dHigh(x): 0 until x=5, 1 at x >/ 8

Sweat Level
Low Sweat: μ_sLow(x): 1 for x </ 2, decreases to 0 at x=5
Mid Sweat: μ_sMid(x): 1 at x=5, 0 at x=2 and x=8
High Sweat: μ_sHigh(x): 0 until x=5, 1 at x >/ 8

Washing Time
Very Short Wash: Peak at 10 minutes, μ_vsWash(x)
Short Wash: Peak at 15 minutes, μ_sWash(x)
Mid Wash: Peak at 30 minutes, μ_mWash(x)
Long Wash: Peak at 45 minutes, μ_lWashx)
Very Long Wash: Peak at 60 minutes, μ_vlWash(x)

Step 4 – Fuzzy Rule Base:

Rule	IF Dirt	AND Sweat	THEN Wash Time
1	Low	Low	Very Short
2	Low	Mid	Short
3	Low	High	Mid
4	Mid	Low	Mid
5	Mid	Mid	Mid
6	Mid	High	Long
7	High	Low	Mid
8	High	Mid	Long
9	High	High	Very Long

Step 5 – Fuzzy Inference (Mamdani Method):
Suppose that Dirt = 7 and Sweat = 6
Fuzzify the inputs according to their membership functions:

$$\mu_{low}(x) = \begin{cases} 1 & x \leq 0 \\ \frac{5-x}{5} & 0 < x < 5 \\ 0 & x \geq 5 \end{cases}$$

$$\mu_{med}(x) = \begin{cases} 0 & x \leq 2 \text{ or } x \geq 8 \\ \frac{x-2}{3} & 2 < x < 5 \\ \frac{8-x}{3} & 5 \leq x < 8 \end{cases}$$

$$\mu_{high}(x) = \begin{cases} 0 & x \leq 5 \\ \frac{x-5}{5} & 5 < x < 10 \\ 1 & x \geq 10 \end{cases}$$

Dirt(7): μ_dLow(7) = 0, μ_dMid(7) = 0.3, μ_dHigh(7) = 0.4
Sweat(6): μ_sLow(6) = 0, μ_sMid(6) = 0.6, μ_sHigh(6) = 0.2

Step 6 – Get the Firing Rates for Each of the Rules:

Rule	Input	Firing Strength	Output
1	Low, Low	Min(0,0) = 0	Very Short
2	Low, Mid	Min(0,0.6) = 0	Short
3	Low, High	Min(0, 0.2) = 0	Mid
4	Mid, Low	Min(0.3,0) = 0	Mid
5	Mid, Mid	Min(0.3,0.6) = 0.3	Mid
6	Mid, High	Min(0.3,0.2) = 0.2	Long
7	High, Low	Min(0.4,0) = 0	Mid
8	High, Mid	Min(0.4,0.6) = 0.4	Long
9	High, High	Min(0.4,0.2) = 0.2	Very Long

Mid = 0.3
Long has multiple firing rates, take the max(0.2, 0.4) = 0.4
Very Long = 0.2

Step 7 – Defuzzification:

Wash Time	Peak of Membership Function	Range
Mid	30	15-45
Long	45	30-60
Very Long	60	45-70

Wash Time: ((0.3*30) + (0.4*45) + (0.2*60)) / (0.3 + 0.4 + 0.2)
The wash time when dirt = 7 and sweat = 6 is 43.3 minutes