Analysis of Optical Modes

A Finite Difference Approach

James Delaney
Supervised by: Prof. Frank Peters

Department of Physics
University College Cork

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- Study how the structure of a waveguide affects the distribution of the EM field within it

Introduction to Waveguides

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In a very simple sense, a waveguide is exactly that, a structure that guides waves.

Rather than allowing the wave energy to diffuse in all directions a waveguide confines the wave, and thus its energy, allowing the transfer of energy over distances not possible without a waveguide.

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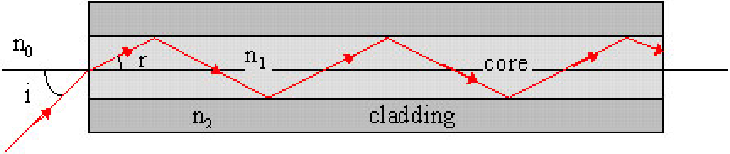
Optical fibres use the confinement property of waveguides to transfer information over very large distances at high speeds.
A simple way of visualising the operation of an optical fibre is through ray diagram.



However in order to fully understand the behaviour of an EM field in a waveguide we must think of it as a field rather than a simple ray

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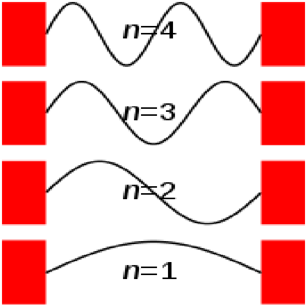
This necessity to satisfy boundary conditions means that the distribution of the field inside the structure is restricted to certain patterns.

These allowed patterns are called **modes** and they can be thought of as analogous to the discretisation of a particle wavepacket in potential.



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Why is this approach important?

- This approach is crucial in understanding the behaviour of an EM field in a waveguide mainly because there is no analytical modal solution for higher than 1D.
- Numerical techniques allow us to study this field in a very scalable way.

Finite Difference Methods

There are several different methods that can be used in the numerical analysis of this problem.
The most popular two methods are Finite Element Methods (FEM) and Finite Difference Methods (FDM).
The choice generally depends on the problem being studied

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This project focuses completely on FD methods.
This is due to there being no major benefit in using FE methods over FDMs for this particular system and FD methods tend to require less of a mathematical foundation to implement efficiently.

Wave Equation

From electrodynamics we know that interacting electric and magnetic fields oscillate and can be described by a wave equation.

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From Maxwell's equation's it can be shown that in a medium with a relative permittivity ϵ_r and a relative permeability $\mu_r = 1$, the electric and magentic fields are governed by the following equations.

$$\nabla^2 \boldsymbol{E} + \nabla \left(\frac{\nabla \epsilon_r}{\epsilon_r} \cdot \boldsymbol{E} \right) + k_0^2 \epsilon_r \boldsymbol{E} = 0 \tag{1}$$

$$\nabla^2 \boldsymbol{H} + \frac{\nabla \epsilon_r}{\epsilon_r} \times (\nabla \times \boldsymbol{H}) + k_0^2 \epsilon_r \boldsymbol{H} = 0 \tag{2}$$

1D Slab Waveguide

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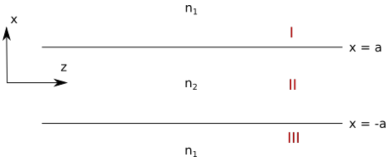

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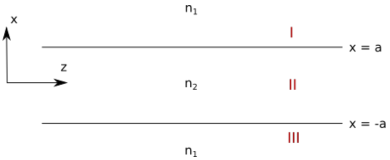

This waveguide is chosen as it has a continuous permittivity except at the boundaries between media. This allows for the great simplification of letting $\nabla \epsilon_r = 0$.
This waveguide also captures the fundamental properties of a waveguide with minimal complexity.

TE and TM Modes

When we talk about allowed modes there are two main types we are interested in; what are known as Transverse Electric (TE) and Transverse Magnetic (TM) modes. TE modes have no electric field in the direction of propagation $E_z = 0$ and TM modes have no magnetic field in the direction of propagation $H_z = 0$.

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Using Maxwell's equations for EM waves.

$$\nabla \times \boldsymbol{E} = -i\omega\mu_0\boldsymbol{H} \tag{3}$$

$$\nabla \times \boldsymbol{H} = i\omega\epsilon_0\epsilon_r\boldsymbol{E} \tag{4}$$

Allows the vector wave equations (1) and (2) to be simplified to a scalar form.

Scalar Wave Equations

$$\frac{d^2 E_y}{dx^2} + k_0^2 \left(\epsilon_r - n_{eff}^2\right) E_y = 0 \qquad (5)$$

$$\frac{d^2 H_y}{dx^2} + k_0^2 \left(\epsilon_r - n_{eff}^2\right) H_y = 0 \qquad (6)$$

The above equations can be solved quite easily analytically and using the boundary condition that the tangential component of both fields are continuous across the boundary between lossless dielectrics yields solutions that vary sinusoidally within guide and decay exponentially outside.

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The above equations can be solved quite easily analytically and using the boundary condition that the tangential component of both fields are continuous across the boundary between lossless dielectrics yields solutions that vary sinusoidally within guide and decay exponentially outside.

The parameter n_{eff} is called the 'effective index' and is very important in the confinement of waves. It will be discussed in the next slide.

Effective Index

The effective index n_{eff} is defined as the ratio between the wave vector in a vacuum and the component of the wave vector in the direction of propagation in the given medium, in this case $n_{eff} = \frac{k_0}{k_z}$ where $k_0 = nk$ is the wave vector in a vacuum of light being used.

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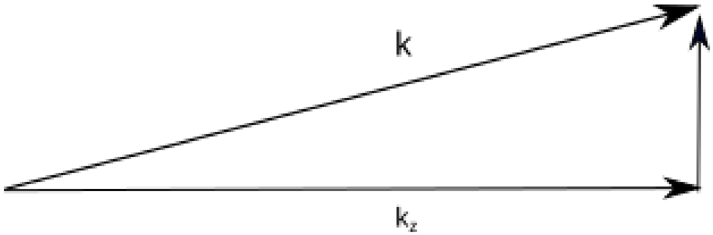

Analytical Solutions to 1D Case

Below are the analytical solutions to the first two TE modes using light of wavelength $\lambda = 1.55\mu m$ confined in guide of width $2a = 4\mu m$.

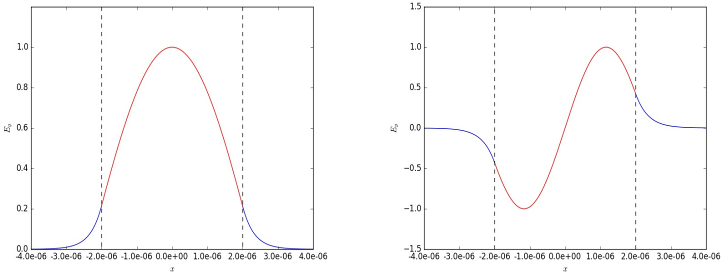

Figure: First two TE modes

Finite Difference Approximation

Now that we have an analytical solution we want to design an algorithm to numerical compute the solution and compare results.
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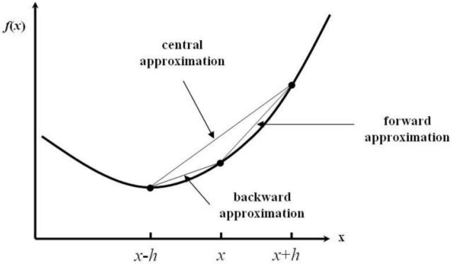
There are several ways to implement this method but all are based on using local Taylor Expansions to approximate derivatives at discrete points on the interval of interest.



Finite Difference Approximation

As we just need to approximate second order derivatives we are going to use the central difference method.
This is called 'central' as it works by taking Taylor expansions a step h either side of each discrete point on the interval.

$$E_y(x+h) = E_y(x) + E_y'(x)h + \frac{E_y''(x)}{2}h^2 + O(h^3)$$
$$E_y(x-h) = E_y(x) - E_y'(x)h + \frac{E_y''(x)}{2}h^2 - O(h^3)$$

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As we are dealing with discrete points, we will treat the field as discrete rather than continuous, labelling each point as $E_p = E_y(x_0 + ph)$, where x_0 is some starting point.

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$$E_p'' = \frac{1}{h^2}\left(E_{p+1} - 2E_p + E_{p-1}\right)$$

Finite Difference Approximation

If we make the substitution into our scalar wave equation and rearrange, we are left with a linear equation in E.

$$\frac{E_{p+1}}{(k_0 h)^2} + \frac{E_{p-1}}{(k_0 h)^2} + \left(n_p^2 - \frac{2}{(k_0 h)^2}\right) E_p = n_{eff}^2 E_p \tag{7}$$

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$$\tilde{\boldsymbol{A}}\boldsymbol{E} = n_{eff}^2 \boldsymbol{E}$$

Reduction to Eigenvalue Equation

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As the interval we are dealing with was quite a lot larger than the guide itself, boundary conditions were chosen such that the field went to zero at the interval boundary.

Therefore the finite difference matrix $\tilde{\boldsymbol{A}}$ has the form

$$\tilde{\boldsymbol{A}} = \begin{bmatrix} n_0^2 - \frac{2}{(k_0 h)^2} & \frac{1}{(k_0 h)^2} & 0 & \dots & 0 \\ \frac{1}{(k_0 h)^2} & n_1^2 - \frac{2}{(k_0 h)^2} & \frac{1}{(k_0 h)^2} & \dots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ \vdots & \vdots & \frac{1}{(k_0 h)^2} & n_{n-1}^2 - \frac{2}{(k_0 h)^2} & \frac{1}{(k_0 h)^2} \\ 0 & \dots & 0 & \frac{1}{(k_0 h)^2} & n_n^2 - \frac{2}{(k_0 h)^2} \end{bmatrix}$$

Solving Eigenvalue Problem

As this is a symmetric tridiagonal matrix it isn't very computationally complex and is easily solvable on a standard laptop.
If we compare the first two TE modes with the analytical results we can see that they agree very well.

Analytical vs Numerical Comparison

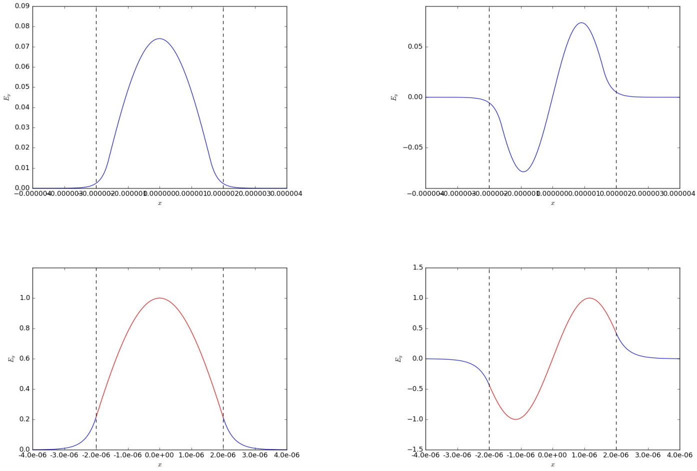

Figure: Comparison of first two TE modes for numerical (top) and analytical (bottom) solutions

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- The width of the waveguide
- The difference in refractive index across the boundary between media

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If we consider the case of a particle in a square well, these properties are analogous to:

- The energy of the particle
- The width of the well
- The height of the well walls (potential step)

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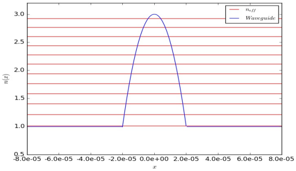

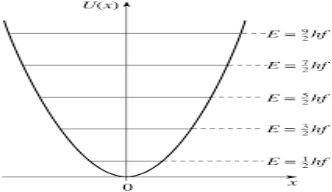

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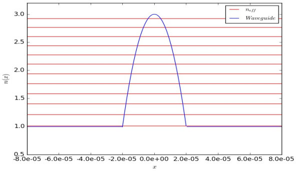

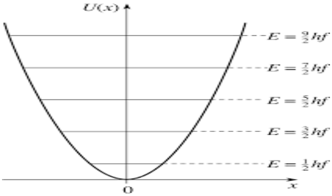

Note: here a wide waveguide was used to make the comparison clearer

Extension to 2D

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As mentioned previously, once we reach two dimensions there is no longer an analytical solution the field distribution in a waveguide. There are approximate methods, but these tend to be limited to certain geometries and so we are very reliant on numerical methods to understand the operation of waveguides in higher dimensions

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Before applying the finite difference method in 2D, we must first define the governing equations.
The equations used in 2D are very once again simplifications of the general vector wave equations (1) and (2) shown at the beginning of the presentation.
In the 2D case we don't allow $\nabla \epsilon_r = 0$, instead we assume that the components of the fields are decoupled.
This leads to what are know as the semivectorial wave equations

Semivectorial Wave Equations

In order to clarify this explanation, we will just discuss the semivectorial equation for the E_x field in was is now called the Quasi-TE mode (due to the decoupling assumption)

$$\frac{\partial^2 E_x}{\partial x^2} + \frac{\partial}{\partial x}\left(\frac{1}{\epsilon_r}\frac{\partial \epsilon_r}{\partial x}E_x\right) + \frac{\partial^2 E_y}{\partial y^2} + (k_0^2\epsilon_r - \beta^2)E_x = 0$$

Here the term β used by convention to represent the component of the wave vector in the direction of propagation k_z.

2D Finite Difference Method

The finite difference method in two dimensions works in the exact same way as in the 1D case but now more care is needed as we need to consider 4 points around each discrete field point (2 in either direction)

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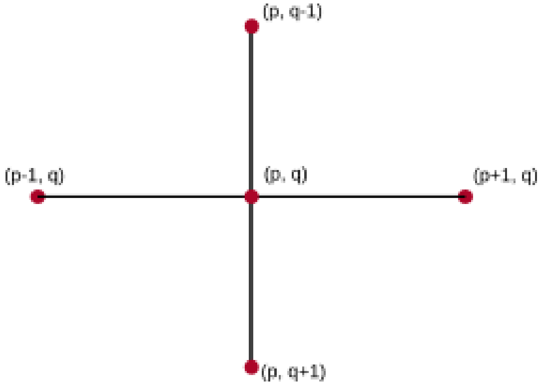

2D Finite Difference Method

Applying the same central difference approximations at each point adjacent to (p, q) we are left with an equation of the form.

$$\alpha_l E_{p-1,q} + \alpha_r E_{p+1,q} + \alpha_t E_{p,q-1} + \alpha_b E_{p,q+1} \\ + \alpha_c E_{p,q} + \left(k_0^2 \epsilon_r(p,q) - \beta^2\right) E_{p,q} = 0$$

2D Finite Difference Method

$$\alpha_l = \frac{1}{(\delta x)^2} \frac{2\epsilon_r(p-1,q)}{\epsilon_r(p,q)+\epsilon_r(p-1,q)}$$

$$\alpha_r = \frac{1}{(\delta x)^2} \frac{2\epsilon_r(p+1,q)}{\epsilon_r(p,q)+\epsilon_r(p+1,q)}$$

$$\alpha_t = \frac{1}{(\delta y)^2}$$

$$\alpha_b = \frac{1}{(\delta y)^2}$$

$$\alpha_c = -\frac{4}{(\delta x)^2} + \alpha_l + \alpha_r - \alpha_t - \alpha_b$$

Implementation of 2D FD Method

As we are now dealing with a 2D system, our matrix will be in four dimensions. In order implement this on a computer we need to be able to formulate the system in way that is compatible with our eigenvalue formulation.

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This is done by letting the vector **E** be of the form

$$\boldsymbol{E} = \begin{pmatrix} E_{00} & E_{01} & E_{02} & \dots & E_{10} & E_{11} & \dots & E_{nn} \end{pmatrix}^T$$

Implementation of 2D FD Method

The easiest way to think of this is by defining the node number $r = pN_y + q$, where (p, q) define a discrete point in our matrix and N_y is the number of points in the y mesh.

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Using this node formulation we can rewrite the multipliers α_i in terms of node points.

Our new multipliers are given by:

$$
\begin{aligned}
\alpha_l &\leftrightarrow a_{r,r-N_y} \\
\alpha_r &\leftrightarrow a_{r,r+N_y} \\
\alpha_t &\leftrightarrow a_{r,r-1} \\
\alpha_b &\leftrightarrow a_{r,r+1} \\
\alpha_c + k_0^2 \epsilon_r(p,q) &\leftrightarrow a_{r,r}
\end{aligned}
$$

2D Finite Difference Matrix

$$\tilde{\boldsymbol{A}} = \begin{bmatrix} a_{0,0} & a_{0,1} & 0 & \dots & 0 & a_{0,N_y} & 0 & \dots & 0 \\ a_{1,0} & a_{1,1} & a_{1,2} & \dots & \dots & 0 & a_{1,1+N_y} & 0 & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & a_{r,r-N_y} & \dots & a_{r,r-1} & a_{r,r} & a_{r,r+1} & \dots & a_{r,r+N_y} & \dots \\ \vdots & \vdots & \vdots & \ddots & \ddots & \ddots & \vdots & \vdots & \vdots \\ 0 & \dots & \dots & 0 & a_{N,N-N_y} & 0 & \dots & a_{N,N-N_y} & a_{N,N} \end{bmatrix}$$

2D solutions

The waveguide of interest is known as a ridge waveguide and it has the following form.

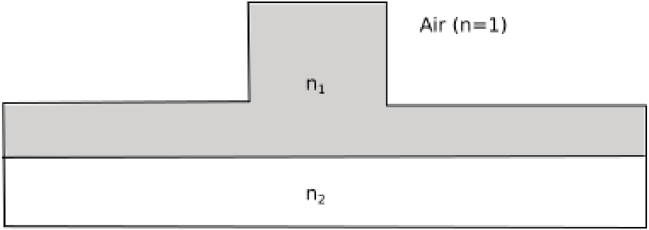
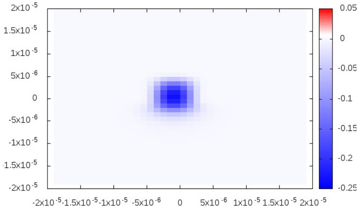
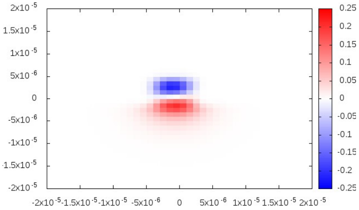
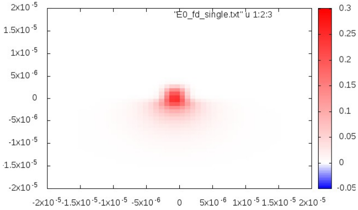


Figure: Rib waveguide 2D slice

Solutions to Modes in a Rib Waveguide

Level of Confinement with Waveguide Size

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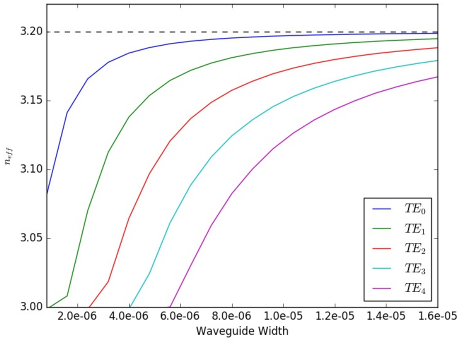
Visualising how the effective index changes with the 'size' of the waveguide is much clearer in 1D and so a plot of n_{eff} vs width (1D) is shown below.



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There is still plenty of interesting work that can be done on this topic, including:

- Incorporating a variable step size into the numerical algorithms (particularly the 2D case)
- Carry out an analysis of how the dimensions of the 2D waveguide affect the n_{eff} as we did for the 1D case
- Move the code to an object-orientated style to allow for easier modification

Questions