01.11.2021

Лабораторная работа №4

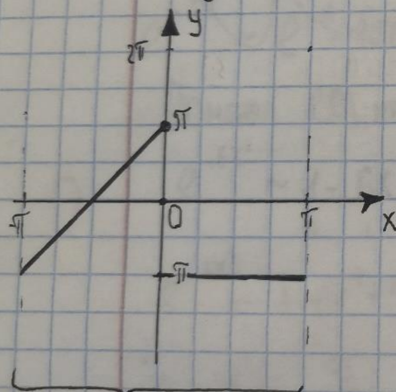
„Ряды Фурье"

Цель: научиться раскладывать функцию в ряд Фурье по тригонометрической системе функций и по ортогональным полиномам, определять область сходимости полученного ряда к порождающей его функции, контролировать результаты с помощью средств системы Maple.

Номер в журнале – 21

Вариант – 1.

$\boxed{\text{Задание 1}}$ $f(x) = \begin{cases} \pi + 2x, & -\pi \le x < 0 \\ -\pi, & 0 \le x < \pi \end{cases}$



$T = 2\pi = 2l \Rightarrow l = \pi.$

$$\underbrace{f(x) \sim \frac{a_0}{2} + \sum_{k=1}^{\infty} (a_k \cos kx + b_k \sin kx)}$$

[это для 2π-периодической, т.к. $\sin\frac{k\pi x}{l} = \sin\frac{k\pi x}{\pi} = \sin kx$].

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)dx = \frac{1}{\pi}\left(\int_{-\pi}^{0}(\pi+2x)dx + \int_{0}^{\pi}-\pi dx\right) =$$

$$= \frac{1}{\pi}\left((\pi x + x^2)\Big|_{-\pi}^{0} + \pi x\Big|_{0}^{\pi}\right) = \frac{1}{\pi}\left(0 - \underbrace{(-\pi^2+\pi^2)}_{0} + \pi^2\right) = \frac{\pi^2}{\pi} = -\pi.$$

$$\to a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx = \frac{1}{\pi}\left(\int_{-\pi}^{0} f(x)\cos nx\,dx + \int_{0}^{\pi} f(x)\cos nx\,dx\right) =$$

$$= \frac{1}{\pi}\left(\overset{①}{\int_{-\pi}^{0}(\pi+2x)\cos nx\,dx} + \overset{②}{\int_{0}^{\pi}(-\pi)\cos nx\,dx}\right) = \circledast$$

// прекращаю вычисление этого примера.

сти
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ю

$$① \int(\pi+2x)\cos nx\,dx = \begin{bmatrix} \pi+2x = u & du = 2\,dx \\ v = \dfrac{\sin nx}{n} & dv = \cos nx\,dx \end{bmatrix} =$$

$$= \frac{(\pi+2x)\sin nx}{n} - \int \frac{2\sin nx}{n}\,dx = \frac{(\pi+2x)}{n}\sin nx - \frac{2}{n}\left(\frac{-\cos nx}{n}\right) =$$

$$= \frac{(\pi+2x)\sin nx}{n} + \frac{2\cos nx}{n^2} + C.$$

$$\left(\frac{(\pi+2x)\sin nx}{n} + \frac{2\cos nx}{n^2}\right)\Bigg|_{-\pi}^{0} = \left(0 + \underbrace{\frac{2\cos 0}{n^2}}_{\frac{2}{n^2}}\right) - \left(\underbrace{\frac{(-\pi)\sin \pi n}{n}}_{0} + \frac{2\cos \pi n}{n^2}\right) =$$

$$= \frac{2}{n^2} - \frac{2\cos \pi n}{n^2} = \underline{\frac{2}{n^2}\left(1-(-1)^n\right)}$$

$$② \int_{0}^{\pi}(-\pi)\cos nx\,dx = \left(\frac{-\pi\sin nx}{n}\right)\Bigg|_{0}^{\pi} = 0$$

$$\circledast = \frac{2}{\pi n^2}\left(1-(-1)^n\right) \qquad // a_n.$$

$$\to b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx = \cancel{\frac{1}{\pi}} \; \frac{1}{\pi}\left(\int_{-\pi}^{0}(\pi+2x)\sin nx\,dx + \int_{0}^{\pi}(-\pi)\sin nx\,dx\right) =$$

$$=$$

$$= \frac{1}{\pi}\left(\int_{-\pi}^{0} \overset{①}{(\pi+2x)} \sin nx\,dx + \int_{0}^{\pi} (-\pi) \overset{②}{\sin} nx\,dx\right) = ⊛$$

$$① \int (\pi+2x)\sin nx\,dx = \int (\pi+2x)\sin nx\,dx = \left[\begin{array}{ll} u=\pi+2x & du = 2dx \\ v = -\frac{\cos nx}{n} & dv = \sin nx\,dx \end{array}\right] =$$

$$= -\frac{(\pi+2x)\cos nx}{n} - \int -\frac{2\cos nx}{n}dx = -\frac{(\pi+2x)\cos nx}{n} + \frac{2}{n}\int \cos nx\,dx =$$

$$= -\frac{(\pi+2x)\cos nx}{n} + \frac{2}{n^2}\sin nx + C.$$

$$\left(-\frac{(\pi+2x)\cos nx}{n} + \frac{2}{n^2}\sin nx\right)\Bigg|_{-\pi}^{0} = \left(-\frac{\pi}{n}\right) - \left(-\frac{(-1)^n(-\pi)}{n} + 0\right) =$$

$$= -\frac{\pi}{n} - \frac{\pi(-1)^n}{n} = -\frac{\pi}{n}\left(1+(-1)^n\right)$$

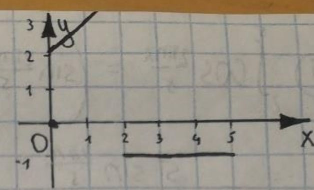
$$② \int_0^{\pi} (-\pi)\sin nx\,dx = \frac{(-\pi)\cdot(-\cos nx)}{n}\Bigg|_0^{\pi} = \left(\frac{\pi\cdot\cos nx}{n}\right)\Bigg|_0^{\pi} =$$

$$= \frac{\pi\cdot(-1)^n}{n} - \frac{\pi}{n} = \frac{\pi}{n}\left((-1)^n - 1\right)$$

итого $b_n = \frac{1}{\pi}\left(-\frac{\pi}{n}\left(1+(-1)^n + (-1)^n - 1\right)\right) = -\frac{2(-1)^n}{n}$

Итого: $f(x) \sim -\frac{\pi}{2} + \sum_{n=1}^{\infty}\left(\frac{2}{\pi n^2}\left(1-(-1)^n\right)\cos nx - \frac{2(-1)^n}{n}\sin nx\right)$

$\boxed{\text{Задание 2}}$ $f(x)=\begin{cases} x+2, & 0<x<2 \\ -1, & 2\le x\le 5 \end{cases}$



① Период $T=5-0=5.$
Полупериод $l=T/2=2.5=5/2.$

② $a_0=\frac{1}{l}\int_a^b f(x)dx=\left(\frac{2}{5}\int_0^5 f(x)dx\right)=\frac{2}{5}\left(\int_0^2 (x+2)dx+\int_2^5(-1)dx\right)=\frac{2}{5}\left(\left(\frac{x^2}{2}+2x\right)\Big|_0^2+(-x)\Big|_2^5\right)=$

$=\frac{2}{5}\left(2+4-5+2\right)=\frac{6}{5}=1.2.$ ✓ by Maple

$n\omega x=n\cdot\frac{2\pi}{T}x=\left(\frac{n\pi x}{l}\right)$

③ $a_n=\frac{1}{l}\int_a^b f(x)\cos n\omega x\,dx=\frac{1}{l}\int_a^b f(x)\cos\frac{2n\pi x}{5}dx=$

$=\frac{2}{5}\left(\int_0^2(x+2)\cos\frac{2n\pi x}{5}dx+\int_2^5(-1)\cos\frac{2n\pi x}{5}dx\right)=\frac{2}{5}\left(\overset{\text{I}}{\int_0^2(x+2)\cos\frac{2n\pi x}{5}dx}-\overset{\text{II}}{\int_2^5\cos\frac{2n\pi x}{5}dx}\right)$

Ⅰ $\int(x+2)\cos\frac{2n\pi x}{5}dx=\left|\begin{matrix} x+2=u & du=dx \\ v=\frac{5\sin\frac{2n\pi x}{5}}{2n\pi} & dv=\cos\frac{2n\pi x}{5}dx\end{matrix}\right|=\frac{(x+2)\cdot5\sin\frac{2n\pi x}{5}}{2n\pi}-\int\frac{5}{2n\pi}\sin\frac{2n\pi x}{5}=$

$=\frac{5(x+2)\sin\frac{2n\pi x}{5}}{2n\pi}+\frac{5}{2\pi n}\cdot\cos\frac{2\pi nx}{5}\cdot\frac{5}{2\pi n}=\underbrace{\frac{5(x+2)\sin\frac{2\pi nx}{5}}{2n\pi}+\frac{25}{4\pi^2n^2}\cos\frac{2n\pi x}{5}}_{\text{Неопределённый интеграл для } a_n \text{ 1го интеграл}}+C$

(ну и плюс на $\frac{1}{l}=\frac{2}{5}$ надо умножить)

подтверждено Photomath.

$\left(\left(\frac{5(x+2)\sin\frac{2\pi nx}{5}}{2\pi n}\right)\Big|_0^{x=2}+\left(\frac{25}{4\pi^2n^2}\cdot\cos\frac{2n\pi x}{5}\right)\Big|_0^{x=2}\right)=$

$=\frac{5}{2\pi n}\left(4\sin\frac{4\pi n}{5}-2\cdot\overset{\sin 0}{0}\right)+\frac{25}{4\pi^2n^2}\left(\cos\frac{4\pi n}{5}-1\right)=$

$=\frac{20\sin\frac{4\pi n}{5}}{2\pi n}+\frac{25\cos\frac{4\pi n}{5}}{4\pi^2n^2}-\frac{25}{4\pi^2n^2}$ ◄ подтверждено Photomath

$\underline{a_n}=\frac{2}{5}\left(\frac{20\sin\frac{4\pi n}{5}}{2\pi n}+\frac{25\cos\frac{4\pi n}{5}}{4\pi^2n^2}-\frac{25}{4\pi^2n^2}\right)=\underbrace{\frac{4\sin\frac{4\pi n}{5}}{\pi n}+\frac{5\cos\frac{4\pi n}{5}}{2\pi^2n^2}-\frac{5}{2\pi^2n^2}}$

первая часть для a_n.

$$\text{II} \quad \int_2^5 \cos\frac{2\pi n x}{5} = \left(\sin\frac{2\pi n x}{5}\cdot\frac{5}{2\pi n}\right)\Bigg|_{2}^{x=5} = \frac{5}{2\pi n}\left(\sin 2\pi n - \sin\frac{4\pi n}{5}\right) =$$

$$= -\frac{5\sin\frac{4\pi n}{5}}{2\pi n.} \quad \Big/ \cdot \frac{2}{5} =$$

$$= -\frac{\sin\frac{4\pi n}{5}}{\pi n} \leftarrow \text{проверено Photomath.}$$

Итого $a_n = \dfrac{3\sin\frac{4\pi n}{5}}{\pi n} + \dfrac{5\cos\frac{4\pi n}{5}}{2\pi^2 n^2} - \dfrac{5}{2\pi^2 n^2}$

$$④ \; b_n = \frac{1}{l}\int_a^b f(x)\sin\frac{\pi n x}{l}\,dx = \frac{2}{5}\left(\int_0^2 (x+2)\sin\frac{2n\pi x}{5}\,dx + \int_2^5 (-1)\sin\frac{2n\pi x}{5}\,dx\right) =$$

$$\text{I} \quad \int (x+2)\sin\frac{2n\pi x}{5}\,dx = \left| \begin{matrix} u = x+2 & du = dx \\ v = \dfrac{-5\cos\frac{2n\pi x}{5}}{2\pi n} & dv = \sin\frac{2\pi n x}{5}\,dx \end{matrix} \right| =$$

$$= \frac{-5(x+2)\cos\frac{2\pi n x}{5}}{2n\pi} + \int\frac{5\cos\frac{2n\pi x}{5}}{2\pi n}\,dx = -\frac{5(x+2)\cos\frac{2n\pi x}{5}}{2n\pi} + \frac{5}{2\pi n}\cdot\frac{5}{2\pi n}\sin\frac{2\pi n x}{5} =$$

$$= -\frac{5(x+2)\cos\frac{2n\pi x}{5}}{2n\pi} + \frac{25}{4\pi^2 n^2}\sin\frac{2n\pi x}{5} + C \leftarrow \text{проверено Photomath.}$$

$$\left(-\frac{5(x+2)\cos\frac{2n\pi x}{5}}{2n\pi}\right)\Bigg|_0^{x=2} + \left(\frac{25}{4\pi^2 n^2}\sin\frac{2n\pi x}{5}\right)\Bigg|_0^{x=2} = \cancel{\left(-\frac{5\cdot 7}{2\pi n} + \frac{5\cdot 4\cdot\cos\frac{4\pi n}{5}}{2\pi n}\right)} +$$

$$+ \cancel{\left(\frac{25}{4\pi^2 n^2}\cdot\left(0 - \sin\frac{4\pi n}{5}\right)\right) = -\frac{35}{2\pi n} + \frac{20\cos\frac{4\pi n}{5}}{2\pi n} - \frac{25\sin\frac{4\pi n}{5}}{4\pi^2 n^2}}$$

$$= \left(-\frac{5\cdot 4\cdot\cos\frac{4\pi n}{5}}{2\pi n} + \frac{+5\cdot 2}{2\pi n}\right) + \left(\frac{25}{4\pi^2 n^2}\sin\frac{4\pi n}{5}\right) = -\frac{20\cos\frac{4\pi n}{5}}{2\pi n} + \frac{10}{2\pi n} + \frac{25\sin}{4\pi^2 n}$$

проверено в Photomath.

$$\text{(II)}\ \int(-1)\sin\frac{2\pi n x}{5}\,dx = \frac{5\cos\frac{2n\pi x}{5}}{2\pi n} + C.$$

$$\left.\frac{5\cos\frac{2n\pi x}{5}}{2\pi n}\right|_2^5 = \frac{5}{2\pi n} - \frac{5\cos\frac{4\pi n}{5}}{2\pi n}.$$ ← проверено в Photomath.

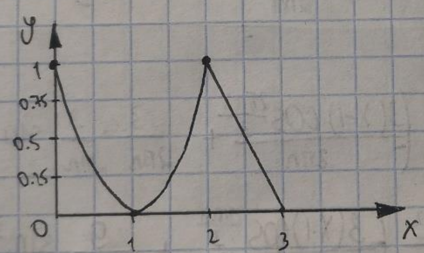
Итого: $$b_n = \frac{2}{5}\left(-\frac{20\cos\frac{4\pi n}{5}}{2\pi n} + \frac{10}{2\pi n} + \frac{25\sin\frac{4\pi n}{5}}{4\pi^2 n^2} + \frac{5}{2\pi n} - \frac{5\cos\frac{4\pi n}{5}}{2\pi n}\right) =$$

$$= \frac{2}{5}\left(-\frac{25\cos\frac{4\pi n}{5}}{2\pi n} + \frac{15}{2\pi n} + \frac{25\sin\frac{4\pi n}{5}}{4\pi^2 n^2}\right) = -\frac{5\cos\frac{4\pi n}{5}}{\pi n} + \frac{3}{\pi n} + \frac{5\sin\frac{4\pi n}{5}}{2\pi^2 n^2}$$

В итоге: $$f(x) \sim \frac{1.2}{2} + \sum_{n=1}^{\infty}\left(\frac{3\sin\frac{4\pi n}{5}}{\pi n} + \frac{5\cos\frac{4\pi n}{5}}{2\pi^2 n^2} - \frac{5}{2\pi^2 n^2}\right)\cos\frac{2n\pi x}{5} + \sum_{n=1}^{\infty}\left(-\frac{5\cos\frac{4\pi n}{5}}{\pi n} + \frac{3}{\pi n} + \frac{5\sin\frac{4\pi n}{5}}{2\pi^2 n^2}\right)\sin\frac{2n\pi x}{5}$$

← ответ →

Задание 3.



$$f(x) = \begin{cases} -x+3, & 2 \le x \le 3 \\ (x-1)^2, & 0 \le x < 2. \end{cases}$$

$T_{3.1} = 3. \quad L_{3.1} = 3/2.$

Проанализирую график кусочно-заданной функции, чтобы представить её формулу.

- На участке [2...3] это: $y = -x+3$
- На участке [0...2] это:

$y = ax^2 + bx + c$

$$\begin{cases} \frac{-b}{2a} = 1 \\ c = 1 \\ 4a + 2b + 1 = 1. \end{cases} \qquad \begin{cases} -b = 2a. \\ c = 1 \\ 2a + b = 0 \end{cases}$$

$a + b + 1 = 0$

$$\begin{cases} a + b = -1 \\ 2a + b = 0 \end{cases}$$

$a = 1$

$b = -2$

$$y = x^2 - 2x + 1 = (x-1)^2.$$

3.1) На полном периоде.

① $a_0 = \frac{1}{l}\int_a^b f(x)dx = \frac{2}{3}\left(\int_0^2 (x^2-2x+1)dx + \int_2^3 (-x+3)dx\right) =$

$= \frac{2}{3}\left(\left(\frac{x^3}{3} - x^2 + x\right)\Big|_0^2 + \left(-\frac{x^2}{2} + 3x\right)\Big|_2^3\right) = \frac{2}{3}\left(\frac{2}{3} + \frac{1}{2}\right) = \frac{7}{9}$

② $a_n = \frac{1}{l}\int_a^b f(x)\cos\frac{\pi n x}{l}dx = \frac{2}{3}\int_0^2 (x^2-2x+1)\cos\frac{2n\pi x}{3}dx + \frac{2}{3}\int_2^3 (-x+3)\cos\frac{2\pi n x}{3}dx$

Ⓘ $\int (x^2-2x+1)\cos\frac{2\pi n x}{3}dx = \left| \begin{array}{ll} u = x^2-2x+1 & du = (2x-2)dx \\ v = \frac{3}{2n\pi}\sin\frac{2n\pi x}{3} & dv = \cos\frac{2\pi n x}{3}dx \end{array} \right| = ⊛$

$= \frac{3(x^2-2x+1)\sin\frac{2\pi n x}{3}}{2\pi n} - \int (2x-2)\cdot\frac{3\sin\frac{2\pi n x}{3}}{2n\pi}dx = ⊛$

I.I $\int \frac{3(2x-2)\sin\frac{2\pi n x}{3}}{2n\pi}dx = \frac{6}{2n\pi}\int (x-1)\sin\frac{2n\pi x}{3}dx = \left| \begin{array}{ll} u = x-1 & du = dx \\ v = -\frac{3\cos\frac{2\pi n x}{3}}{2\pi n} & dv = \sin\frac{2\pi n x}{3}dx \end{array} \right| =$

$= \frac{6}{2\pi n}\left(-\frac{3(x-1)\cos\frac{2\pi n x}{3}}{2\pi n} + \frac{3}{2\pi n}\cdot\frac{3}{2\pi n}\sin\frac{2\pi n x}{3}\right) =$

$= \frac{3}{\pi n}\left(-\frac{3(x-1)\cos\frac{2\pi n x}{3}}{2\pi n} + \frac{9}{4\pi^2 n^2}\sin\frac{2\pi n x}{3}\right)$

$⊛ = \frac{3(x^2-2x+1)\sin\frac{2n\pi x}{3}}{2\pi n} + \frac{9(x-1)\cos\frac{2\pi n x}{3}}{2\pi^2 n^2} - \frac{27\sin\frac{2\pi n x}{3}}{4\pi^3 n^3}$

$\frac{2}{3}\cdot ⊛ = \frac{(x^2-2x+1)\cdot\sin\frac{2n\pi x}{3}}{\pi n} + \frac{3(x-1)\cos\frac{2n\pi x}{3}}{\pi^2 n^2} - \frac{9\sin\frac{2\pi n x}{3}}{2\pi^3 n^3}$

Ⓘ $\Big|_0^2 = \frac{\sin\frac{4\pi n}{3}}{\pi n} + \frac{3\cos\frac{4\pi n}{3}}{\pi^2 n^2} - \frac{9\sin\frac{4\pi n}{3}}{2\pi^3 n^3} - 0 + \frac{3}{\pi^2 n^2} - 0 =$

$= \frac{\sin\frac{4\pi n}{3}}{\pi n} + \frac{3\cos\frac{4\pi n}{3}}{\pi^2 n^2} - \frac{9\sin\frac{4\pi n}{3}}{2\pi^3 n^3} + \frac{3}{\pi^2 n^2}$. ← проверено в Photomath

$$(II)\ \int(-x+3)\cos\frac{2n\pi x}{3}dx = \left|\begin{matrix} u=-x+3 & du=-dx \\ v=\frac{3}{2n\pi}\sin\frac{2n\pi x}{3} & dv=\cos\frac{2n\pi x}{3}dx\end{matrix}\right| =$$

$$=\frac{3(-x+3)}{2\pi n}\sin\frac{2n\pi x}{3}+\int\frac{3}{2n\pi}\sin\frac{2n\pi x}{3}dx = \frac{3(-x+3)}{2\pi n}\sin\frac{2\pi n x}{3}+\frac{3}{2\pi n}\int\sin\frac{2\pi n x}{3}dx =$$

$$=\frac{3}{2\pi n}(-x+3)\sin\frac{2\pi n x}{3}+\frac{3}{2\pi n}\cdot 3\,\frac{\left(-\cos\frac{2\pi n x}{3}\right)}{2\pi n} = \frac{3(-x+3)\sin\frac{2n\pi x}{3}}{2\pi n}-\frac{9\cos\frac{2n\pi x}{3}}{4\pi^2 n^2}.$$

$$\frac{2}{3}\cdot(II) = \frac{(-x+3)\sin\frac{2n\pi x}{3}}{\pi n}-\frac{3\cos\frac{2n\pi x}{3}}{2\pi^2 n^2}.$$

$$(II)\Big|_2^3 = \cancel{+\frac{\sin\frac{4n\pi}{3}}{\pi n}-\frac{3\cos\frac{4n\pi}{3}}{2\pi^2 n^2}-\left(0-\frac{3}{2\pi^2 n^2}\right)} =$$

$$=\cancel{+\frac{\sin\frac{4n\pi}{3}}{\pi n}-\frac{3\cos\frac{4n\pi}{3}}{2\pi^2 n^2}+\frac{3}{2\pi^2 n^2}}.$$

итого $a_n = \cancel{\frac{\sin\frac{4\pi n}{3}}{\pi n}+\frac{3\cos\frac{4\pi n}{3}}{\pi^2 n^2}-\frac{9\sin\frac{4\pi n}{3}}{2\pi^3 n^3}+\frac{3}{\pi^2 n^2}+\frac{\sin\frac{4\pi n}{3}}{\pi n}-\frac{3\cos\frac{4\pi n}{3}}{2\pi^2 n^2}+\frac{3}{2\pi^2 n^2}} =$

$$=\cancel{\frac{2\sin\frac{4\pi n}{3}}{\pi n}+\frac{3\cos\frac{4\pi n}{3}}{2\pi^2 n^2}+\frac{9}{2\pi^2 n^2}-\frac{9\sin\frac{4\pi n}{3}}{2\pi^3 n^3}}$$

НА интервале [2...3], а не [0...2] !!!

(3) $\cancel{a_n = \frac{2}{3}}$

$$(II)\Big|_2^3 = \left(0-\frac{3}{2\pi^2 n^2}\right)-\left(\frac{\sin\frac{4\pi n}{3}}{\pi n}-\frac{3\cos\frac{4\pi n}{3}}{2\pi^2 n^2}\right) = -\frac{3}{2\pi^2 n^2}-\frac{\sin\frac{4\pi n}{3}}{\pi n}+\frac{3\cos\frac{4\pi n}{3}}{2\pi^2 n^2}.$$

итого $a_n = \frac{\sin\frac{4\pi n}{3}}{\pi n}+\frac{3\cos\frac{4\pi n}{3}}{2\pi^2 n^2}-\frac{9\sin\frac{4\pi n}{3}}{2\pi^3 n^3}+\frac{3}{2\pi^2 n^2}-\frac{3}{2\pi^2 n^2}-\frac{\sin\frac{4\pi n}{3}}{\pi n}+\frac{3\cos\frac{4\pi n}{3}}{2\pi^2 n^2} =$

$$=\boxed{\frac{9\cos\frac{4\pi n}{3}}{2\pi^2 n^2}+\frac{3}{2\pi^2 n^2}-\frac{9\sin\frac{4\pi n}{3}}{2\pi^3 n^3}}\qquad a_n -$$

проверено в Maple ! :)

③ b_n. В связи с трудоёмкостью вычислений коэффициент b_n посчитан при поддержке калькулятора на телефоне. (Умение считать продемонстрировано ранее)

$$b_n = \frac{2}{3}\int_0^2 (x^2+2x+1)\sin\frac{2n\pi x}{3}dx + \frac{2}{3}\int_2^3 (-x+3)\sin\frac{2n\pi x}{3}dx =$$

$$= \frac{3\sin\frac{4n\pi}{3}}{2n^2\pi^2} + \frac{9\cos\frac{4\pi n}{3}}{2n^3\pi^3} - \frac{\cos\frac{4n\pi}{3}}{n\pi} - \frac{9}{2n^3\pi^3} + \frac{21}{n\pi} +$$

$$+ \frac{\cos\frac{4n\pi}{3}}{n\pi} + \frac{3\sin\frac{4n\pi}{3}}{2n^2\pi^2} = \frac{6\sin\frac{4\pi n}{3}}{2n^2\pi^2} + \frac{9\cos\frac{4\pi n}{3}}{2n^3\pi^3} - \frac{9}{2\pi^3 n^3} + \frac{1}{\pi n}.$$

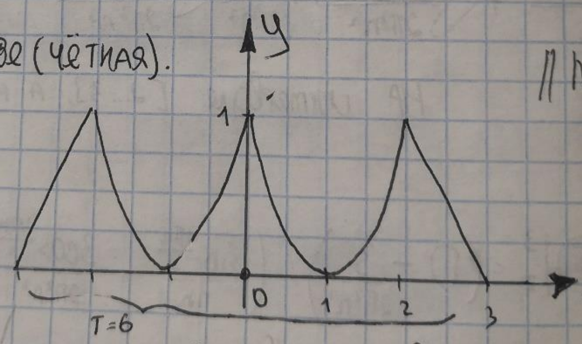
Итого:

$$f(x) \sim \frac{7}{6\cdot 2} + \sum_{n=1}^{\infty}\left(\frac{9\cos\frac{4\pi n}{3}}{2\pi^2 n^2} + \frac{3}{2\pi^2 n^2} - \frac{9\sin\frac{4\pi n}{3}}{2\pi^3 n^3}\right)\cos\frac{2n\pi x}{3} + \sum_{n=1}^{\infty}\left(\frac{6\sin\frac{4\pi n}{3}}{2\pi^2 n^2} + \frac{9\left(\cos\frac{4\pi n}{3}-1\right)}{2\pi^3 n^3} + \frac{1}{\pi n}\right)\sin\frac{2\pi n x}{3}$$

ответ для полного периода.

③.2 На полупериоде (чётная).

$\frac{1}{l} \xrightarrow{1.5} \frac{1}{2l} \Big| \cdot 2 \Rightarrow \frac{1}{l}$



// посчитал a_0 здесь и увидел, что $a_0^{new} = 1.5 a_0^{old}$.

Для четной функции вид: $f(x) \sim \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{l}$

$a_0 = 1.5 a_0$ из первого примера.

$a_0 = \frac{7\cdot 1.5}{6}$ // т.к. четная

$a_n = 1.5 a_n$ из первого примера $= \frac{9\cos\frac{4\pi n}{3}}{\pi^2 n^2} + \frac{3}{\pi^2 n^2} - \frac{9\sin\frac{4\pi n}{3}}{\pi^3 n^3}$.

$f(x)\cos$ — четная!!!

$f(x)\sin$ — нечет

Итого: $f(x) = \frac{7}{4\cdot 2} + 1.5\sum^{\infty}\left(\frac{9\cos\frac{4\pi n}{3} + 3}{2\pi^2 n^2} - \frac{9\sin\frac{4\pi n}{3}}{2\pi^3 n^3}\right)$

$$a_0=\frac{1}{l}\int_{-3}^{3} f(x)dx = \frac{1}{3}\cdot 2\int_0^3 f(x)\,dx = \frac{2}{3}\left(\int_0^2 (x^2-2x+1)dx + \int_2^3 (-x+3)\,dx\right) =$$

$$= \frac{7}{6}$$

$$a_n = \frac{1}{3}\cdot 2\cdot\int_0^3 f(x)\cos\frac{n\pi x}{3}dx =$$

// на калькуляторе

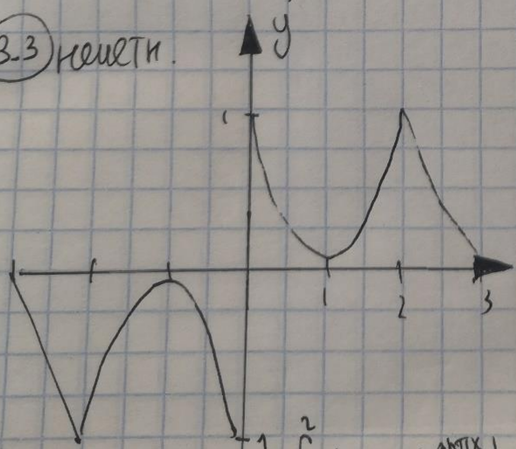
$$= \frac{2}{3}\left(\int_0^2 (x^2-2x+1)\overset{\cos\frac{n\pi x}{3}}{}dx + \int_2^3 (-x+3)\overset{\cos\frac{n\pi x}{3}}{}dx\right) = \frac{2\sin\frac{2n\pi}{3}}{n\pi} + \frac{12\cos\frac{2n\pi}{3}}{n^2\pi^2} - \frac{36\sin\frac{2\pi n}{3}}{n^3\pi^3} + \frac{12\pi n}{n^2\pi^2}$$

$$+ \frac{2\sin\frac{2n\pi}{3}}{n\pi} - \frac{6\cos\frac{2n\pi}{3}}{n^2\pi^2} + \frac{6}{n^2\pi^2} =$$

$$= \frac{4\sin\frac{2\pi n}{3}}{n\pi} + \frac{6\cos\frac{2\pi n}{3}}{\pi^2 n^2} + \frac{18}{n^2\pi^2} - \frac{36\sin\frac{2\pi n}{3}}{\pi^3\pi^3}$$

$$\overset{\text{чет}}{f(x)} = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n\cos\frac{n\pi x}{3}$$

(3.3) нечетн.



тут $\overset{\text{неч}}{f(x)} \sim \sum_{n=1}^{\infty} b_n \cdot \sin\frac{n\pi x}{l}$

почему? т.к.

$f\cdot\sin$ — четная

$f\cdot\cos$ — нечетн.

$$b_n = \frac{1}{3}\int_{-3}^{3} f(x)\overset{\sin\frac{n\pi x}{3}}{}dx = \frac{2}{3}\int_0^3 f(x)\sin\frac{n\pi x}{3}dx =$$

$$\frac{2}{3}\int_0^2 (x^2-2x+1)\sin\frac{n\pi x}{3}dx + \int_2^3 (-x+3)\sin\frac{n\pi x}{3}dx \quad \text{четная}$$

$$=$$

$$= -\frac{2\cos\frac{2n\pi}{3}}{n\pi} + \frac{12\sin\left(\frac{2n\pi}{3}\right)}{n^2\pi^2} + \frac{36\cos\left(\frac{2n\pi}{3}\right)}{n^3\pi^3} - \frac{36}{n^3\pi^3} + \frac{2}{n\pi} \cdot 2$$

$$\cancel{-16\sin n\pi}\ \frac{2}{n\pi}\cos\left(\frac{2n\pi}{3}\right) + \frac{6\sin\left(\frac{2n\pi}{3}\right)}{n^2\pi^2} - 2$$

$$= \boxed{\frac{4\cos\frac{2\pi n}{3}}{\pi n} + \frac{18\sin\frac{2\pi n}{3}}{\pi^2 n^2} - \frac{36}{n^3\pi^3} + \frac{2}{\pi n} + \frac{36\cos\frac{2\pi n}{3}}{n^3\pi^3}}$$

$$f(x) \sim \sum_{n=1}^{\infty} b_n \cdot \sin\frac{n\pi x}{3}$$