Computer Science 203
Programming Languages

Bindings, Procedures, Functions, Functional Programming, and the Lambda Calculus

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Plan

- Informal discussion of procedures and bindings
- Introduction to the lambda calculus
  - Syntax and operational semantics
  - Technicalities
  - Evaluation strategies
- Relationship to programming languages
- Study of types and type systems (later)

Beyond IMP: Procedures and Bindings

Bindings

A binding associates a value or attribute with a name.

It can occur at various times:

- language definition or implementation time (e.g., defining the meaning of +),
- compile time or link time (e.g., constant inlining),
- run time (e.g., passing parameters).

with a trade-off between efficiency and flexibility

Static vs Dynamic Scoping

- Should this program print 0 or 1?

- Dynamic Scoping
  - use most recent value of Y when print(X × Y) is reached
  - based on chain of activations
  - cute, concise, confusing code

- Static Scoping
  - use "nearest" binding of Y that encloses print(X × Y)
  - based on structure of the program
  - easier to understand

```
declare Y;

procedure P(X);
   begin
      print(X * Y);
   end;

procedure Q(Y);
   begin
      P(Y);
   end;

begin
   Y := 0;
   Q(1);
end;
```

Higher-Order Languages and Scoping

- A language is higher-order if procedures can be passed as arguments or returned as results.
- Scoping issues are important for free variables in procedure parameters and results.

- In part because of scoping difficulties, some higher-order languages are not fully general.
(E.g., Pascal does not allow procedure results.)

The “downward funarg” Problem

```
declare Y;

procedure P( );
   begin print(Y); end;

procedure Q(R);
   declare Y;
   begin Y := 0;  R( ); end;

begin Y := 1;  Q(P); end;
```

- Q should be given a closure for P (including Y).

The “upward funarg” Problem

```
procedure R( );
  declare Y;

  procedure Q( );
    begin print(Y); end;

  begin Y := 0; return Q; end;

begin T := R( ); T( ); end;
```

- R should return a closure for Q (including Y).

Parameter Passing

- There are many parameter-passing modes, such as:
  - By value: the formal is bound to a variable with an unused location, set to the actual's value.
  - By name (in the ALGOL sense): the actual is not evaluated until the point of use.
  - By reference: the formal is bound to the variable designated by the actual ("aliased").
  - In-only: by reference, but the procedure is not allowed to modify the parameter.
  - Out-only: on termination, the value of the formal is assigned to the actual.
  - By value-result: like by value, plus the copying of out-only.

Lambda Calculus and Functional Programming

Background

- Developed in 1930's by Alonzo Church.
- Subsequently studied (and still studied) by many people in logic and computer science.
- Considered the "testbed" for procedural and functional languages.
  - Simple.
  - Powerful.
  - Easy to extend with features of interest.

"*Whatever the next 700 languages turn out to be, they will surely be variants of lambda calculus.*"

(Landin '66)

Syntax

- The lambda calculus has three kinds of expressions (terms):

$$
\begin{array}{lll}
e ::= x & & \text{Variables} \\
\quad\;\; \mid & \lambda x.e & \text{Functions (abstraction)} \\
\quad\;\; \mid & e_1\ e_2 & \text{Application}
\end{array}
$$

- $\lambda x.e$ is a one-argument function with body e.
- $e_1\ e_2$ is a function application.

- Application associates to the left:

  $x\ y\ z$ means $(x\ y)\ z$

- Abstraction extends to the right as far as possible:

  $\lambda x.\ x\ \lambda y.\ x\ y\ z$ means $\lambda x.(x\ (\lambda y.\ ((x\ y)\ z)))$

Examples of Lambda Expressions

- The identity function:

$$I =_{def} \lambda x.\ x$$

- A function that *given* an argument y discards it and computes the identity function:

$$\lambda y.\ (\lambda x.\ x)$$

- A function that *given* a function f invokes it on the identity function:

$$\lambda f.\ f\ (\lambda x.\ x)$$

Scoping, Free and Bound Variables

- Scope of an identifier
  - the portion of a program where the identifier is accessible
- An abstraction $\lambda x.\ E$ binds the variable x in E:
  - x is the newly introduced variable.
  - E is the scope of x.
  - We say x is bound in $\lambda x.\ E$.
- y a free variables of E
  - if it has occurrences that are not bound in E.
  - defined recursively as follows:

$$\mathrm{Free}(x) = \{ x \}$$
$$\mathrm{Free}(E_1\ E_2) = \mathrm{Free}(E_1) \cup \mathrm{Free}(E_2)$$
$$\mathrm{Free}(\lambda x.\ E) = \mathrm{Free}(E) - \{ x \}$$

- Example: $\mathrm{Free}(\lambda x.\ x\ (\lambda y.\ x\ y\ z)) = \{ z \}$

Free and Bound Variables (Cont.)

- Just like in any language with statically nested scoping we have to worry about variable shadowing.
  - An occurrence of a variable might refer to different things in different contexts.

- E.g., in IMP with locals: let x ← E in x + (let x ← E′ in x) + x

- In lambda calculus: $\lambda x.\ x\ (\lambda x.\ x)\ x$

Renaming Bound Variables

- λ-terms that can be obtained from one another by renaming bound variable occurrences are considered identical.
- Example: $\lambda x.\ x$ is identical to $\lambda y.\ y$ and to $\lambda z.\ z$
- Convention: we will often try to rename bound variables so that they are all unique
  - e.g., write $\lambda x.\ x\ (\lambda y.y)\ x$ instead of $\lambda x.\ x\ (\lambda x.x)\ x$
- This makes it easy to see the scope of bindings.

Substitution

- The substitution of E′ for x in E (written [E′/x]E)
  - Step 1. Rename bound variables in E and E′ so they are unique.
  - Step 2. Perform the textual substitution of E′ for x in E.

- Example: $[y\ (\lambda x.\ x)\ /\ x]\ \lambda y.\ (\lambda x.\ x)\ y\ x$
  - After renaming: $[y\ (\lambda v.\ v)/x]\ \lambda z.\ (\lambda u.\ u)\ z\ x$
  - After substitution: $\lambda z.\ (\lambda u.\ u)\ z\ (y\ (\lambda v.\ v))$

The deBruijn Notation

- An alternative syntax avoids naming of bound variables (and the subsequent confusions).
- The deBruijn index of a variable *occurrence* is the number of lambda's that separate the occurrence from its binding lambda in the abstract syntax tree.
- The deBruijn notation replaces names of occurrences with their deBuijn index
- Examples:
  - $\lambda x.x$ — $\lambda.0$
  - $\lambda x.\lambda x.x$ — $\lambda.\lambda.0$
  - $\lambda x.\lambda y.y$ — $\lambda.\lambda.0$
  - $(\lambda\ x.\ x\ x)\ (\lambda\ x.\ x\ x)$ — $(\lambda.0\ 0)\ (\lambda.0\ 0)$
  - $\lambda x.\ (\lambda x.\ \lambda y.x)\ x$ — $\lambda.(\lambda.\lambda.1)\ 0$

Identical terms have identical representations !

Informal Semantics

- The evaluation of

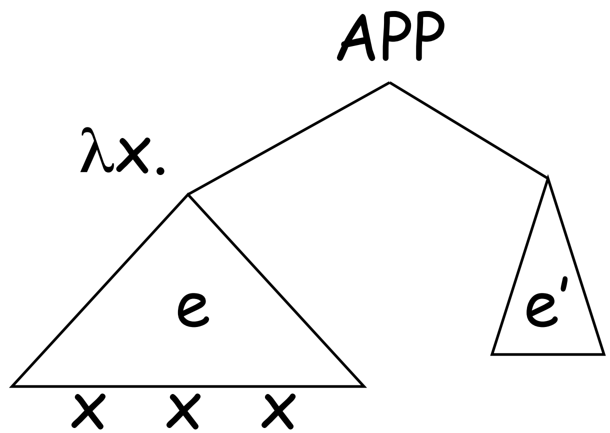
$$(\lambda x.\ e)\ e'$$

  1. binds x to e',
  2. evaluates e with the new binding,
  3. yields the result of this evaluation.
- Like "let x = e' in e".
- Example:

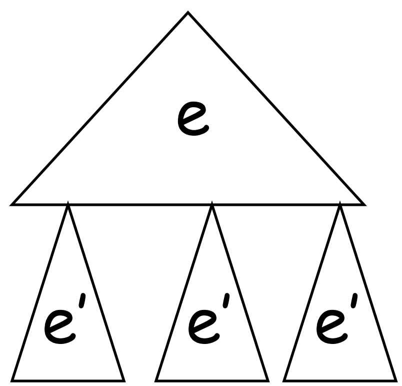
  $(\lambda f.\ f\ (f\ e))\ g$ evaluates to $g\ (g\ e)$

Another View of Reduction

- The application



- becomes:



Terms can "grow" substantially through reduction!

Operational Semantics

- We formalize this semantics with the β-reduction rule:

$$(\lambda x.\ e)\ e' \ \to_\beta\ [e'/x]e$$

- A term $(\lambda x.\ e)\ e'$ is a β-redex.

- We write $e \to_\beta e'$ if e β-reduces to e' in one step.
- We write $e \to_\beta^* e'$ if e β-reduces to e' in many steps.

Examples of Evaluation

- The identity function:

  $(\lambda x.\ x)\ E$

  $\rightarrow [E\ /\ x]\ x$

  $=\ \ E$

- Another example with the identity:

  $(\lambda f.\ f\ (\lambda x.\ x))\ (\lambda x.\ x)$

  $\rightarrow [\lambda x.\ x\ /\ f]\ f\ (\lambda x.\ x)$

  $=\ \ [(\lambda x.\ x)\ /\ f]\ f\ (\lambda y.\ y)$

  $=\ \ (\lambda x.\ x)\ (\lambda y.\ y)$

  $\rightarrow [\lambda y.\ y\ /x]\ x$

  $=\ \ \lambda y.\ y$

Examples of Evaluation (Cont.)

- A non-terminating evaluation:

$$
\begin{aligned}
& (\lambda x.\ xx)(\lambda y.\ yy) \\
\to\ & [\lambda y.\ yy\ /\ x]xx \\
=\ & (\lambda y.\ yy)(\lambda y.\ yy) \\
=\ & (\lambda x.\ xx)(\lambda y.\ yy) \\
\to\ & \ldots
\end{aligned}
$$

Evaluation and Static Scoping

- The definition of substitution guarantees that evaluation respects static scoping:

  $$(\lambda\, x.\ (\lambda y.\ y\ x))\ (y\ (\lambda x.\ x)) \to_\beta \lambda z.\ z\ (y\ (\lambda v.\ v))$$

  (y remains free, i.e., defined externally)

- If we forget to rename the bound y:

  $$(\lambda\, x.\ (\lambda y.\ y\ x))\ (y\ (\lambda x.\ x)) \to^*_\beta \lambda y.\ y\ (y\ (\lambda v.\ v))$$

  (y was free before but is bound now)

Nondeterministic Evaluation

- We define a small-step reduction relation:

$$\frac{}{(\lambda x.\ e)\ e' \to [e'/x]e}$$

$$\frac{e_1 \to e_1'}{e_1\ e_2 \to e_1'\ e_2} \qquad \frac{e_2 \to e_2'}{e_1\ e_2 \to e_1\ e_2'}$$

$$\frac{e \to e'}{\lambda x.\ e \to \lambda x.\ e'}$$

- This is a nondeterministic set of rules.
- Three *congruence* rule saying where to evaluate
  - e.g. under λ

Contexts

- Define contexts with one hole

$$H ::= \bullet \mid \lambda x.\, H \mid H\ e \mid e\ H$$

- $H[e]$ fills the hole in H with the expression e.

- Example:

$$H = \lambda x.\ x\ \bullet \qquad\qquad H[\lambda y.y] = \lambda x.\ x\ (\lambda y.\ y)$$

- Filling the hole allows variable capture!

$$H = \lambda x.\ x\ \bullet \qquad\qquad H[x] = \lambda x.x\ x$$

Context-Based Version of Operational Semantics

- Contexts simplify writing congruence rules.

$$\frac{}{(\lambda x.\ e)\ e' \rightarrow [e'/x]e} \qquad\qquad \frac{e \rightarrow e'}{H[e] \rightarrow H[e']}$$

- Reduction occurs at a β-redex that can be anywhere inside the expression.
- The above rules do not specify which redex must be reduced first.
- The second rule is called a congruence or structural rule.

The Order of Evaluation

- In a λ-term there could be many β-redexes $(\lambda x.\ E)\ E'$
- $(\lambda y.\ (\lambda x.\ x)\ y)\ E$
  - We could reduce the inner or the outer application.
  - Which one should we pick?

$$\begin{tikzcd}[column sep=small] & (\lambda y.\ (\lambda x.\ x)\ y)\ E \arrow[dl, "\text{inner}"'] \arrow[dr, "\text{outer}"] & \\ (\lambda y.\ [y/x]\ x)\ E = (\lambda y.\ y)\ E \arrow[dr] & & [E/y]\ (\lambda x.\ x)\ y = (\lambda x.\ x)\ E \arrow[dl] \\ & E & \end{tikzcd}$$

Normal Forms

- A term without redexes is in normal form.
- A reduction sequence stops at a normal form.

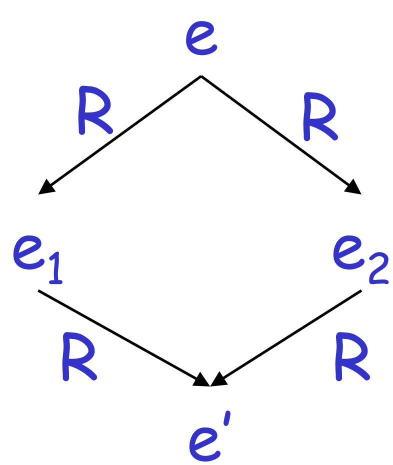
- If e is in normal form and $e \to^*_\beta e'$ then e is identical to e' .

- $K = \lambda x.\lambda y.\ x$ is in normal form.

- $K\ \lambda z.\ z$ is not in normal form.

The Diamond Property

- A relation R has the diamond property if whenever $e\ R\ e_1$ and $e\ R\ e_2$ then there exists e' such that $e_1\ R\ e'$ and $e_2\ R\ e'$.



- $\rightarrow_\beta$ does not have the diamond property.
  - For example, consider $(\lambda x.\ x\ x\ x)(\lambda y.\ (\lambda x.\ x)\ y)$.
- $\rightarrow_\beta{}^*$ has the diamond property.
  - The proof is quite technical.

The Diamond Property

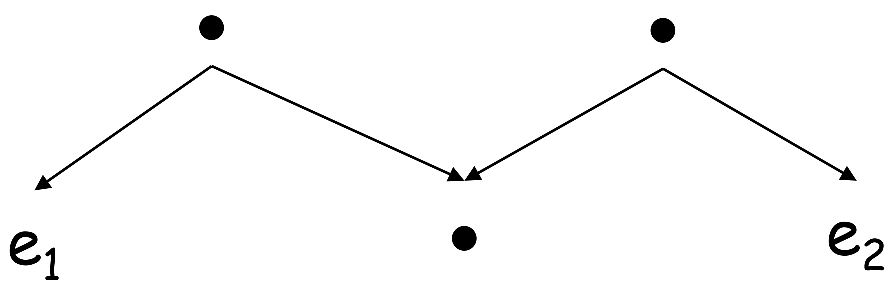
- Languages defined by nondeterministic sets of rules are common:
  - Logic programming languages.
  - Expert systems.
  - Constraint satisfaction systems.
  - Make.
- It is useful to know whether such systems have the diamond property.

Equality

- Let $=_\beta$ be the reflexive, transitive and symmetric closure of $\to_\beta$:

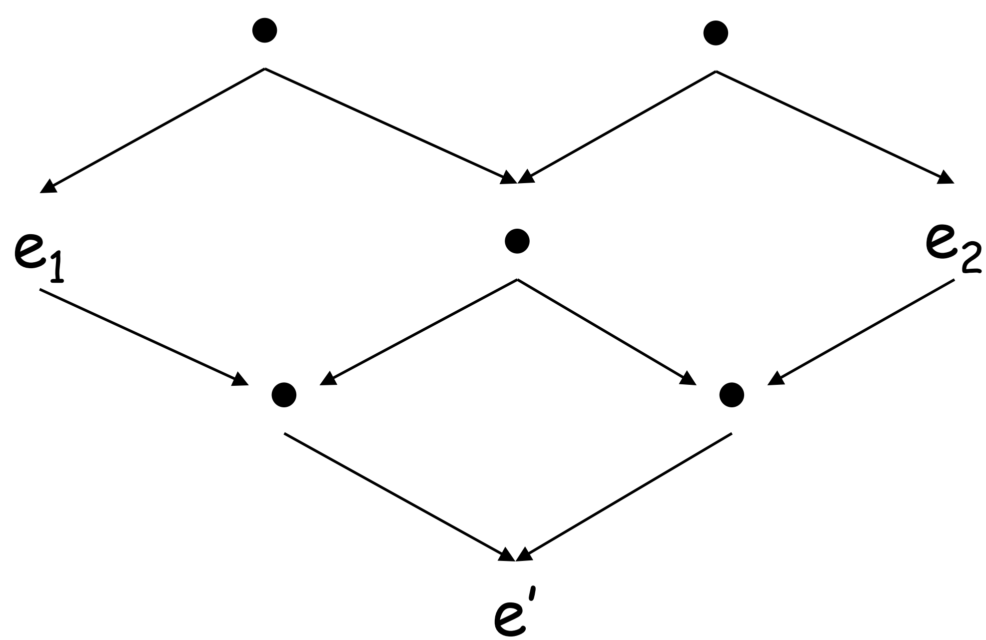
$$=_\beta \text{ is } (\to_\beta \cup \leftarrow_\beta)^*$$

- That is, $e_1 =_\beta e_2$ if e_1 converts to e_2 via a sequence of forward and backward $\to_\beta$:



The Church-Rosser Theorem

- If $e_1 =_\beta e_2$ then there exists e' such that $e_1 \to_\beta^* e'$ and $e_2 \to_\beta^* e'$:



- Proof (informal): apply the diamond property as many times as necessary.

Corollaries

- If $e_1 =_\beta e_2$ and e_1 and e_2 are normal forms then e_1 is identical to e_2.
  - From CR we have $\exists e'.\ e_1 \rightarrow^*_\beta e'$ and $e_2 \rightarrow^*_\beta e'$.
  - Since e_1 and e_2 are normal forms they are identical to e'.

- If $e \rightarrow^*_\beta e_1$ and $e \rightarrow^*_\beta e_2$ and e_1 and e_2 are normal forms then e_1 is identical to e_2.
  - Every term has a unique normal form
    (if it has a normal form at all).

Combinators

- A λ-term without free variables is a closed term or a combinator.
  - Some interesting examples:

    $I = \lambda x.\ x$

    $K = \lambda x.\lambda y.x$

    $S = \lambda f.\lambda g.\lambda x.f\ x\ (g\ x)$

    $D = \lambda x.\ x\ x$

    $Y = \lambda f.(\lambda x.\ f\ (x\ x))\ (\lambda x.\ f\ (x\ x))$

- Theorem: Any closed term is equivalent to one written with just S, K, I.
  - Example: $D =_{\beta} S\ I\ I$

    (we will discuss this form of equivalence)

Evaluation Strategies

- Church-Rosser theorem says that independently of the reduction strategy we will not find more than one normal form.

- Some reduction strategies might fail to find a normal form:
  - $(\lambda x.\ y)\ \underline{((\lambda y.y\ y)\ (\lambda y.y\ y))} \to (\lambda x.\ y)\ \underline{((\lambda y.y\ y)\ (\lambda y.y\ y))} \to \ldots$
  - $\underline{(\lambda x.\ y)\ ((\lambda y.y\ y)\ (\lambda y.y\ y))} \to y$

- We will consider three strategies:
  - normal order
  - call-by-name
  - call-by-value

Normal-Order Reduction

- A redex is outermost if it is not contained inside another redex.

- Example:

  S (K x y) (K u v)

- K x, K u and S (K x y) are all redexes.
- Both K u and S (K x y) are outermost.
- Normal order always reduces the leftmost outermost redex first.

- Theorem: If e has a normal form e′ then normal order reduction will reduce e to e′.

Why Not Normal Order? (Weak vs. Strong Reduction)

- In most (all?) programming languages, functions are considered values (fully evaluated).
- Thus, no reduction is done under lambdas. Reduction is “weak”.
- Reduction under lambdas (“strong” reduction) can play a role in partial evaluation and other optimizations.

Call-by-Name

- Don't reduce under λ.
- Don't evaluate the argument to a function call.

- Call-by-name is demand-driven
  - an expression is not evaluated unless needed.
- It is normalizing
  - it converges whenever normal order converges.
- Call-by-name does not necessarily evaluate to a normal form.

Call-by-Name

- Example:

$$(\lambda y.\ (\lambda x.\ x)\ y)\ ((\lambda u.\ u)\ (\lambda v.\ v))$$
$$\rightarrow_\beta (\lambda x.\ x)\ ((\lambda u.\ u)\ (\lambda v.\ v))$$
$$\rightarrow_\beta (\lambda u.\ u)\ (\lambda v.\ v)$$
$$\rightarrow_\beta \lambda v.\ v$$

Call-by-Value Evaluation

- Don't reduce under lambda.
- Do evaluate the argument to a function call.

- Most languages are primarily call-by-value.
- But CBV is not normalizing
  - $(\lambda x.\ I)\ (D\ D)$
  - CBV may diverge even if normal order (or CBN) converges.

Considerations

- Call-by-value:
  - Easy to implement.
  - Predictable evaluation order
    - well-behaved with respect to side-effects

- Call-by-name:
  - More difficult to implement
    - must pass unevaluated exprs
  - Order of evaluation is less predictable
    - side-effects are problematic
  - Has a simpler theory than call-by-value.
  - Terminates more often than call-by-value.

CBV vs. CBN

- The debate about whether languages should be strict (CBV) or lazy (CBN) is decades old.
- This debate is confined to the functional programming community (where it is sometimes intense).
- CBV appears to be winning at the moment.
- Outside the functional community CBN is rarely considered (though it arises in special cases).

Review

- The lambda calculus is a calculus of functions:

$$e := x \mid \lambda x.\ e \mid e_1\ e_2$$

- Several evaluation strategies exist based on β-reduction:

$$(\lambda x.e)\ e' \rightarrow_\beta [e'/x]\ e$$

- How does this simple calculus relate to real programming languages?

Functional Programming

- The lambda calculus is a prototypical functional language with:
  - no side effects,
  - several evaluation strategies,
  - lots of functions,
  - nothing but functions
    (pure lambda calculus does not have any other data type).

- How can we program with functions?
- How can we program with only functions?

Programming With Functions

- Functional programming style is a programming style that relies on lots of functions.
- A typical functional paradigm is using functions as arguments or results of other functions.
  - Higher-order programming.
- Some “impure” functional languages permit side-effects (e.g., Lisp, Scheme, ML, OCaml):
  - references (pointers), arrays, exceptions.

Variables in Functional Languages

- We can introduce new variables:

  let $x = e_1$ in e_2

  - x is bound by let.
  - x is statically scoped in e_2.

- This is much like $(\lambda x.\ e_2)\ e_1$.

- In a functional language, variables are never updated.
  - They are just names for expressions.
  - E.g., x is a name for the value denoted by e_1 in e_2.

- This models the meaning of "let" in mathematics.

Referential Transparency

- In "pure" functional programs, we can reason equationally, by substitution:

  $\text{let } x = e_1 \text{ in } e_2 \quad \equiv [e_1/x]e_2$

- In an imperative language a "side-effect" in e_1 might invalidate this equation.

- The behavior of a function in a "pure" functional language depends only on the actual arguments.
  - Just like a function in mathematics.
  - This makes it easier to understand and to reason about functional programs.

Expressiveness of Lambda Calculus

- The lambda calculus is a minimal system but can express:
  - data types (integers, booleans, pairs, lists, trees, etc.),
  - branching,
  - recursion.
- This is enough to encode Turing machines.

- Corollary: $e =_\beta e'$ is undecidable.

- Still, how do we encode all these constructs using only functions?
- Idea: encode the "behavior" of values and not their structure.

Encoding Booleans in Lambda Calculus

- What can we do with a boolean?
  - We can make a binary choice.

- A boolean is a function that *given two choices* selects one of them:
  - true $=_{def}$ $\lambda x.\ \lambda y.\ x$
  - false $=_{def}$ $\lambda x.\ \lambda y.\ y$
  - if E_1 then E_2 else E_3 $=_{def}$ $E_1\ E_2\ E_3$

- Example: "if true then u else v" is

  $(\lambda x.\ \lambda y.\ x)\ u\ v \rightarrow_\beta (\lambda y.\ u)\ v \rightarrow_\beta u$

Encoding Pairs in Lambda Calculus

- What can we do with a pair?
  - We can select one of its elements.

- A pair is a function that given a boolean returns the left or the right element:

  mkpair x y $=_{def}$ λb. b x y

  fst p $=_{def}$ p true

  snd p $=_{def}$ p false

- Example:

  fst (mkpair x y) → (mkpair x y) true → true x y → x

Encoding Natural Numbers in Lambda Calculus

- What can we do with a natural number?
  - We can iterate a number of times over some function.
- A natural number is a function that given an operation f and a starting value s, applies f to s a number of times:

  $0 =_{def} \lambda f.\ \lambda s.\ s$

  $1 =_{def} \lambda f.\ \lambda s.\ f\ s$

  $2 =_{def} \lambda f.\ \lambda s.\ f\ (f\ s)$

  and so on.

- These are numerals in unary representation, or Church numerals. There are others (e.g., Scott's).

Computing with Natural Numbers

- The successor function

  succ n $=_{def}$ $\lambda f.\ \lambda s.\ f\ (n\ f\ s)$

  or succ n $=_{def}$ $\lambda f.\lambda s.n\ f\ (f\ s)$

- Addition

  add n_1 n_2 $=_{def}$ n_1 succ n_2

- Multiplication

  mult n_1 n_2 $=_{def}$ n_1 (add n_2) 0

- Testing equality with 0

  iszero n $=_{def}$ n (λb. false) true

Computing with Natural Numbers: Example

$\text{mult } 2\ 2 \to$

$2\ (\text{add } 2)\ 0 \to$

$(\text{add } 2)\ ((\text{add } 2)\ 0) \to$

$2\ \text{succ}\ (\text{add } 2\ 0) \to$

$2\ \text{succ}\ (2\ \text{succ}\ 0) \to$

$\text{succ}\ (\text{succ}\ (\text{succ}\ (\text{succ}\ 0))) \to$

$\text{succ}\ (\text{succ}\ (\text{succ}\ (\lambda f.\ \lambda s.\ f\ (0\ f\ s)))) \to$

$\text{succ}\ (\text{succ}\ (\text{succ}\ (\lambda f.\ \lambda s.\ f\ s))) \to$

$\text{succ}\ (\text{succ}\ (\lambda g.\ \lambda y.\ g\ ((\lambda f.\ \lambda s.\ f\ s)\ g\ y)))$

$\text{succ}\ (\text{succ}\ (\lambda g.\ \lambda y.\ g\ (g\ y))) \to^{*} \lambda g.\ \lambda y.\ g\ (g\ (g\ (g\ y))) = 4$

Computing with Natural Numbers: Example

- What is the result of the application add 0 ?

  $(\lambda n_1.\ \lambda n_2.\ n_1\ succ\ n_2)\ 0\ \to_\beta$

  $\lambda n_2.\ 0\ succ\ n_2 =$

  $\lambda n_2.\ (\lambda f.\ \lambda s.\ s)\ succ\ n_2 \to_\beta$

  $\lambda n_2.\ n_2 =$

  $\lambda x.\ x$

- By computing with functions we can express some optimizations.
  - But we need to reduce under lambdas.

Encoding Recursion

- Given a predicate P encode the function "find" such that "find P n" is the smallest natural number which is larger than n and satisfies P.
  - With find we can encode all recursion
- "find" satisfies the equation:

  find p n = if p n then n else find p (succ n)

- Define

  $F = \lambda f.\lambda p.\lambda n.(p\ n)\ n\ (f\ p\ (succ\ n))$

- We need a fixed point of F:

  find = F find

  or

  find p n = F find p n

The Y Fixed-Point Combinator

- Let $Y = \lambda F.\ (\lambda y.F(y\ y))\ (\lambda x.\ F(x\ x))$
  - This is called the (or a) fixed-point combinator.
  - Verify that $Y\ F$ is a fixed point of F
    $Y\ F \to_\beta (\lambda y.F\ (y\ y))\ (\lambda x.\ F\ (x\ x)) \to_\beta F((\lambda y.F\ (y\ y))(\lambda x.\ F\ (x\ x)))$
    $F(YF) \to_\beta F((\lambda y.F\ (y\ y))\ (\lambda x.\ F\ (x\ x)))$
  - Thus $Y\ F =_\beta F\ (Y\ F)$
- Given any function in lambda calculus we can compute its fixed-point (if it has one).
  - We may also let $\mathrm{rec}\ x.\ b = Y(\lambda x.\ b)$
- Thus we can define "find" as the fixed-point of the function from the previous slide.

- The essence of recursion is the self-application "$y\ y$".

Expressiveness of Lambda Calculus

- Encodings are fun.

- But programming in pure lambda calculus is painful.
- Encodings complicate static analysis.

- We will add constants (0, 1, 2, …, true, false, if-then-else, etc.).

- And we will add types.

Lisp, Briefly

Lisp (from ca. 1960)

- Not Fortran or C (a chance to think differently).
- A fairly elegant, minimal language.
- Representing many general themes in language design.
- By now, with many dialects and a wide influence.
- Emphasis on artificial intelligence and symbolic computation.

Syntax

- Simple, regular syntax:
  - (+ (* 1 2 3 4) 5)
  - (f x y z)
  - (cond (p1 e1) … (pn en))
- No explicit typing.

Atoms, S-expressions, Lists

- Atoms include numbers and indivisible strings.
- Symbolic expressions (s-expressions) are atoms and pairs.
- Lists are built up from the atom nil and pairing.

```
<atom> ::= <symbol>
         | <number>

<s-exp> ::= <atom>
          | (<s-exp> . <s-exp>)
```

Primitives

- Basic functions on numbers and pairs:
  cons car cdr eq atom
- Control:
  cond
- Declaration and evaluation:
  lambda quote eval
- Some functions with side-effects (for efficiency):
  rplaca rplacd set setq

Example:

```
(lambda (x) (cond ((atom x) x) (T (cons 'A x))))
```

Evaluation

- Interactive evaluation, often with an interpreter: read-eval-print loop.
- Also compilation (though with some historical complications).

- Function calls evaluate all their arguments.
- Special forms do not evaluate all their arguments.
  - E.g., (cond …).
  - E.g., (quote A).

Some Contributions of Lisp

- Expression-oriented language.
  - Lots of parentheses!
- Abstract view of memory:
  - Cells (rather than concrete addresses).
  - Garbage collection.
- Programs as data.
  - Higher-order functions.
  - "Metacircular" interpreters.

Reading

- Read Cardelli's paper "Type Systems".