Philosophy of Language and Logic

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Chapter 1

Introduction to conditionals

Conditionals in the English language exist in various constructions and operate in equally various ways. Categorizing these constructions and unifying them under one theory is a seemingly transparent question, though it turns out more complicated than one might anticipate. Given a number of conditional statements, one might conclude the connecting factor to be the word *"if"*, or an equivalent term in other languages. This word will turn out to play a major part in the analysis of conditionals, though it is not a proper indicator, as it is neither necessary nor necessarily indicative.

While the topic of conditionals is not simple, it is definitely worthwhile studying, as conditionals are used in much of our thinking, argumentation, and by-extension: philosophising. Within philosophy, conditionals are used to express various concepts such as dispositions and causation as well as being used for basic philosophical Logic. Due to this wide use, it is nearly impossible to study philosophy of language without encountering the problem of conditionals.

1.1 Kinds of conditionals

We can divide conditionals broadly into at least two categories. There are those sentences that indicate a state of affairs and those that indicate a possibility. These are respectively called *indicative* and *subjunctive*[1].

Indicative conditionals relate to the material conditional ($\supset$) of classical logic. Whereas the subjunctive conditionals do not. In fact – they do not

[1] These are also occasionally called *counterfactual*

relate to any concept in classical logic, and therefore lack a straightforward method of analysis, they are not truth-functional.

Despite the obvious connection between the indicative conditional and material conditional, they are not necessarily the same, and a large body of literature is written on the topic. For this reason, we cannot express the material conditional in English by using the typical construction "if ... then" as it would confuse the material conditional with the indicative. Therefore, we express the material conditional instead using a different operator, namely *or* ($\vee$). Thus, $A \supset C$ becomes $\neg A \vee C$.

Subjunctive conditionals can further be separated into *would subjunctives* and *might subjunctives*. These indicate the words used in the respective sentences which relate to whether they express a possible consequent or a definitive one, though in either case the antecedent in negated (thus the term "counterfactual"). This course will hardly ever mention the might subjunctive, and any reference to "the subjunctive", unless otherwise noted, can be taken to refer to the would subjunctive.

These various conditionals will be formalized using the following symbols:

Conditional	symbol
Material	$\supset$
Indicative	$\rightarrow$
Subjunctive	$>$
Might	$>_m$
Subjunctive (alternative)	$\Box\!\!\rightarrow$
Might (alternative)	$\Diamond\!\!\rightarrow$

Part I

Indicatives

Chapter 2

The material analysis

"The material analysis" is a general term for analysing the indicative conditional ($\rightarrow$) as being logically equivalent to the material conditional ($\supset$)[1]. The difficulty in proving the material analysis comes in the fact that the indicative conditional cannot be assumed to be truth-functional, thus not allowing a mere truth-functional analysis.

If the indicative conditional is truth-functional, it must be equivalent to the material conditional. This belief stems from the following proof:

1. $P \wedge Q \rightarrow P$ is assumed to be a logical truth
2. $A \rightarrow C$ is assumed not to be a logical truth

Given these assumptions, if $\rightarrow$ is truth functional, it must function in the same way in every case. Assuming that (1) is a logical truth allows us to derive the following three truths:

- $F \rightarrow T$
- $T \rightarrow T$
- $F \rightarrow F$

Given (2), and the derived truths, we can also know derive one falsehood:

- $T \rightarrow F$

[1] This is also sometimes called "the horseshoe analysis", referring to the shape of the material conditional symbol ($\supset$).

Which leaves us with the truth table for the material conditional.

The reason this discussion remains interesting is because (1) and (2) are assumed, rather than argued for.

2.1 Argument for the material analysis

One reason for accepting the material analysis is the following: Given two mutually exclusive things, the indicative cannot be true. Similarly, the material conditional cannot be true. Meaning that the material conditional *entails* the indicative.

More interesting is the other direction (the indicative conditional entailing the material conditional). This is called the "or to if" analysis which goes as follows: Given two options, if we know that one option is not the case, it implies that the other is. This is a widely held assumption which leads us to accept that the indicative *entails* the material conditional.

Given that the material conditional and indicative entail each other, they are logically equivalent.

2.2 Paradoxes

Despite the aforementioned "proof", there are cases where we encounter paradoxes due to the fact that any false antecedent leads to a true statement in the medial conditional. Or the fact that a true consequent, leads to a true statement. While these are logically coherent truths, the ability to put *any* consequent or antecedent, whether nonsensical, related or otherwise, allows us to prove anything we wish.

2.3 Implicatures

If something is "implied", it is not stated directly, but nevertheless conveyed. It may be said that $A \to C$ *states* only $A \supset C$, while *implying* more. Such non-direct statements make problems for the paradoxes listed above.

Grice distinguishes between conversational-, and conventional-implicates, which together remove the paradoxes from the material analysis.

2.3.1 Conversational Implicatures

In a conversation, we implicitly follow a certain principle know as "the cooperative principle" and we are allowed to assume that our interlocutors follow this principle. The principle can be generalized as a set of rules such as:

1. Be appropriate in amount
2. Make true contributions
3. Be relevant to the topic being discussed
4. etc.

There are various – intentional and unintentional – ways in which we violate these rues where we imply certain information while staying cooperative.

With the cooperative principle, we can argue against some of the paradoxes listed in the earlier section, and – by extension – argue for the material analysis. This would be because, while an implication is logically true, falsehood is implied by the context of the conversation.

2.3.2 Conventional Implicatures

Conventional implicatures are tied to the manner in which we use particular words, where a certain word in a given context implies a different meaning than the apparent one.

As such, we can say that $A \to C$ implicates *a connection between* A and C. Thus, given examples wherein the antecedent and consequent are not in any way related to each other, we go against the conventional use of $\to$.

2.3.3 Criticism

While conventional-, and conversational- implicatures solves the paradoxes in the material analysis. It may be said that the implied information of a given word (in the case of conventional implicatures) is simply truth-functional. Meaning that the difference between $\supset$ and $\to$ is a logical one rather than a implied difference, thus causing a problem for the material analysis.

Furthermore, conventional implicatures may be said to contribute something to the *tone* of a statement, changing not the meaning, but the way of stating the meaning. This does not apply to $\to$ however, where the paradoxes are not problems in tone, but distinctly in meaning.

Chapter 3

Non-material analyses

3.1 Probability theory

In probability theory, there exist two main notions of the mathematics of probability, and by extension philosophical probability. These two notions are the subjective-, and objective- notions, referring respectively to the epistemic perspective of a given agent, and the agent-independent features of the world. Subjective probability is something we often encounter, for instance due to ignorance of all the facts. Object probability can be seen for instance in quantum events, and prior to the discovery of quantum theory, it was doubted that objective probabilities existed[1].

Axioms of probability theory handed out in class

$P(A|B) = \frac{P(A) \cdot P(B|A)}{P(B)}$, all the rest is commentary

3.1.1 Notation

Symbol	Meaning
$P(Q)$	Probability of Q
$P(R\|Q)$	The probability of Q *given* R
P_A	The updated function given new information A

[1] For instance Einstein: "God does not play dice"

3.2 Conditional probability

According to the Ramsey test, $P(A \to C) = P_A(C) andthrereforP(C|A)$. In other words, the probability of an indicative is the same as the probability of the consequent given the antecedent is true. The proof for this is given by entering $P(Q \to R)$ in AT, followed by the ratio formula, and the if-and conversion, simplifying by T1 and T3[2].

However, if There is a chance of that C is true or false, given A, then $P(A \to C) = P(R)$, according to the axioms. This result is obviously not true though, as various counterexamples exist. The same problem holds for combining the Ramsey test and the ratio formula, which gets us $P(RQ) = P(R) \cdot P(Q)$, which allows for more mathematical counterexamples.

Both of these formulae from problems for the Ramsey test, as the axioms of probability theory are not to be doubted.

3.3 Appiah

Appiah argues against Jackson's material analysis as expounded in the previous chapter. He does this though formulating an embedded conditional which is acceptable due to various consequences of probability theory. If Jackson goes along with the fact that robustness can be expressed in terms of robustness, and furthermore allows for embedded conditionals, then Appiah's argument is convincing. Furthermore, Appiah requires probability to be dependent only on what one is saying, not on what one is implying[3]. A further consideration is the definition of the term "high probability", as one may claim this to also be context dependent.

See slides "Rebutting Robustness I"

[2]See handout for reference numbers

[3]potential paper topic

Part II

Subjunctives

Chapter 4

Similarity semantics

TO talk about subjunctives, propositional logic needs to be expanded with modal operators. Modal logic speaks about necessary and possible things, represented respectively by $\Box$ and $\Diamond$, either of which can be taken as a primitive for modal logic. Crucially, $\Box$ and $\Diamond$ are not truth functional, as for any p, $\Box p$ is unsolvable if p is true, and false if p is false.

p	$\Box P$
T	
F	F

To analyse modality then, we use the method of possible worlds. Something is necessary if it is the case in every possible world, whereas it is possible if true in at least one world. In doing this, we limit ourselves to certain worlds as described by an accessibility relation.

Strict conditionals are a type of material conditional where the implication is taken as necessary $\Box(A \supset C)$. This is true if $A \supset C$ is true in every accessible world.

The subjunctive $A \Box\!\!\rightarrow C$ is sometimes analysed as being equivalent to $\Box(A \supset C)$ This makes the subjunctive a strict conditional. Lewis argues against this idea.

He does this through Antecedent Strengthening. This is a structure where, in any given world, $\Box(A \supset C)$ is true and $\Box A \wedge B \supset C$ is also true by extension. However, for subjunctives, there are many examples where $B \supset \neg C$.

Lewis thus concludes that $\Box\!\!\rightarrow$ is not a strict conditional, it is a variably strict conditional, where the accessibility of worlds id dependent on the antecedent. This is called similarity semantics, as only those worlds are considered that are more similar to the world of evaluation.

To check the truth value of a subjunctive conditional under similarity semantics, we must look at the closest worlds where the antecedent is true. The closest here means that we do not look at worlds that are any more different from the actual world than necessary. Of course, any subjunctive can be made false or true depending on the accessibility relations and the truth values of the consequent in every accessible antecedent-world.

One matter under discussion is weather multiple worlds can be equally similar or dissimilar to a given reference world. According to Lewis: w, where $A \wedge B$, is dissimilar in equal amounts to w', where $A \wedge \neg B$ and w^*, where $\neg A \wedge B$. According to Stalnaker, there is a priority of propositions, where – for instance – A being different is more substantial than B being different, thus placing w' and w^* exist in different spheres. Stalnaker's theory leaves the question of *why* this priority exists.

Stalnaker appeals to the vague nature of subjunctives, thus making it that there is not a single correct function which selects the closest antecedent world. A subjunctive in Stalnaker's theory then is true according to every possible selection function. This is called supervaluation, and it allows for the benefits of Lewis' theory while still only allowing one most similar world *per selection function.* In problematic cases, one may then speak of truth as being truth in all possible worlds under a specific selection function, while we speak of super-truth if the proposition is true under every relevant selection function.

Lewis believes that $\Box\!\!\rightarrow$ and $\Diamond\!\!\rightarrow$ stand in a simmilar relationship as $\Box$ and $\Diamond$ in that $\Diamond p \equiv \neg\Box\neg p$ and $a \Diamond\!\!\rightarrow c \equiv \neg(a \Box\!\!\rightarrow \neg c)$. This allows us to get truth values for "might" sentences, as we can now translate them as "would sentences".

This is not possible in Stalnaker's account, as it requires a singular closest world, making $\Box\!\!\rightarrow$ and $\Diamond\!\!\rightarrow$ the same. Thus, Stalnaker gives a different account of $\Diamond\!\!\rightarrow$

Chapter 5

Non-similarity semantics

Lewis's system of possible words, wherein other worlds exists but are inaccessible to us. A world in this system is a maximally complete space-time continuum, just like the actual world. Nearly everything in logic can be reduced to possible world semantics. Propositions for instance are merely the set of possible worlds wherein that proposition holds true. This however requires us to accept that things which do not exist in the actual world truly exist in other worlds, as long as those things are possible. Thus, flying donkeys exist, they are just inaccessible to us. This system is called extreme realism and any philosophers are not willing to accept it.

Am alternative system is abstract realism, where a world is viewed as a set of sentences. A world is thus identified as a proposition which describes completely a way for things to be. This proposal is usually not reductive, as extreme realism tends to be.

5.1 Sobel sequences

Lewis considered cases wherein an implicative is true, though false in light of new information $A \Box\!\!\rightarrow C$ and $A \wedge B \Box\!\!\rightarrow \neg C$. These sequences are called Sobel sequences, and are explained in a previous chapter. Lewis uses this problem to argue for his similarity semantics. Alternatively, other philosophers argue that both "necessities" in the two sentences, rely on context to receive their meaning. In other words, the epistemice use of modal logic depends on the epistemic state of the agent.

In allowing this caveat, it allows one to analise $A \Box\!\!\rightarrow C$ as $\Box(A \supset C)$ and

$A \wedge B \Box\!\!\rightarrow \neg C$ as $\Box(A \wedge B \supset \neg C)$ while granting that the $\Box$ in the former is not equivalent to the $\Box$ in the latter, thus avoiding the seeming logical inconsistency. Ultimately, the result is the same as Lewis' theory. However, Lewis' theory allows for a more systematic analysis than in possible for mere context dependant proposals. One can hover give a systematic account of the context dependence of conditionals.

One such systematic account is given by Fintel and Gillies. They argue that the context dependent approach is more powerful than Lewis' account, through using reverse Sobel sequences. It is claimed by Fintel and Gillies that these sequences are logically incoherent. This incoherence cannot be explained by Lewis, as his theory does not allow the context (such as order) to influence truth-value. The context for Fintel and Gillies determines the accessibility relation, and is therefore both truth-functional and systematic.

In the Sobel sequences listed above then, B is not so much influence the truth, but rather the context. Uttering B shifts the context so as to change the evaluation of $A \Box\!\!\rightarrow C$. Of course, A is also context function, as the utterance $A \rightarrow$ already establishes the context which mks the accessibility relation such that there are at least some A-worlds accessible.