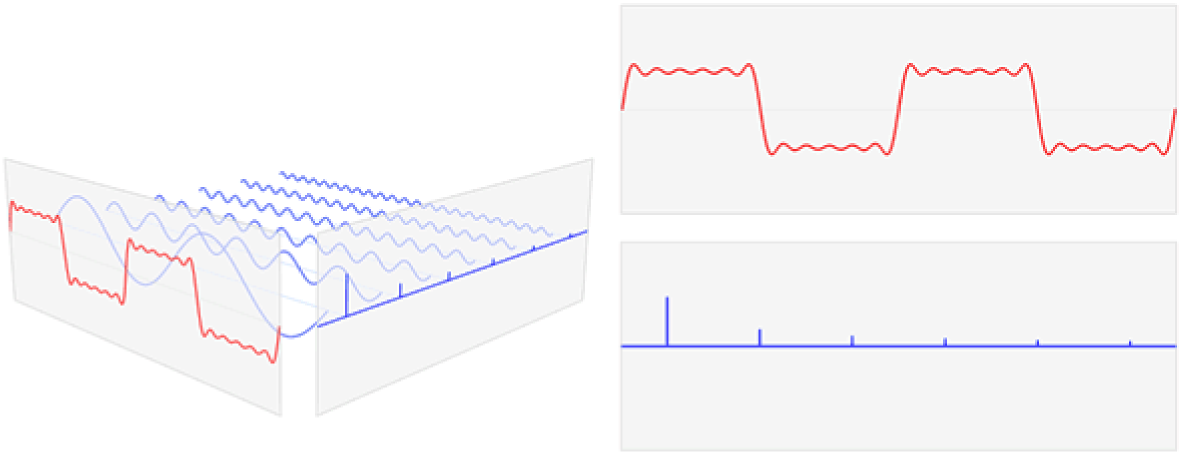
Obwody elektryczne, elektroniczne



Rys. Związek Serii Furiera z funkcją w dziedzinie częstotliwości [By Lucas V. Barbosa - File:Fourier transform time and frequency domains (small).gif, CC0, https://commons.wikimedia.org/w/index.php?curid=28399050]

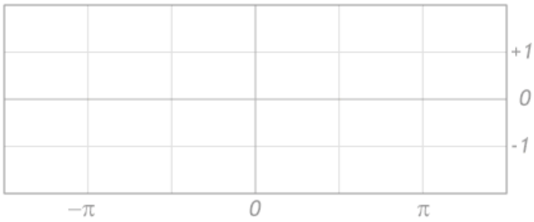
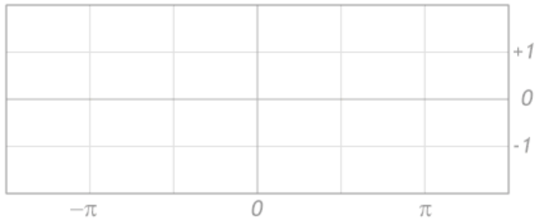
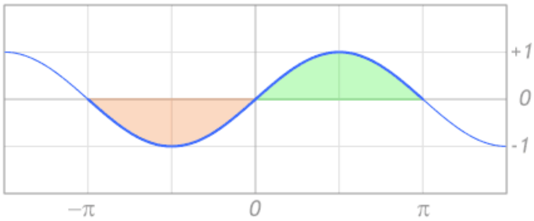
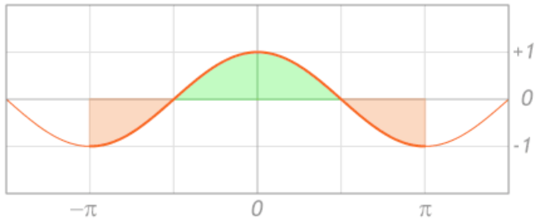
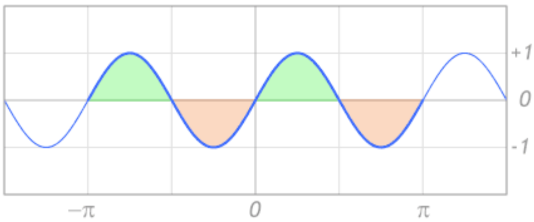
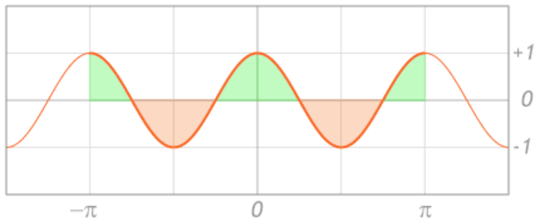
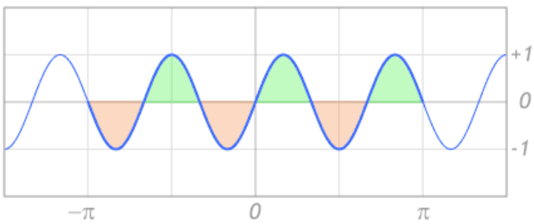
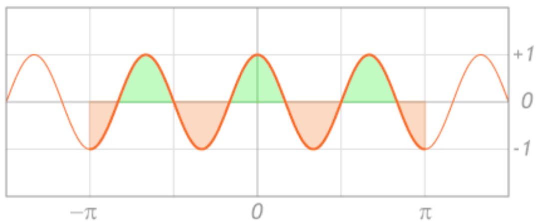
$$\langle f,g\rangle \triangleq \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\overline{g(x)}\,dx.$$

$$\begin{aligned} a_n &= \frac{2}{P}\int_P s(x)\cdot\cos\left(2\pi x\frac{n}{P}\right)dx \\ b_n &= \frac{2}{P}\int_P s(x)\cdot\sin\left(2\pi x\frac{n}{P}\right)dx. \end{aligned}$$

$$s_N(x)=\frac{a_0}{2}+\sum_{n=1}^{N}\left(a_n\cos\left(\frac{2\pi n x}{P}\right)+b_n\sin\left(\frac{2\pi n x}{P}\right)\right).$$

$$\{e_n=e^{inx}:n\in Z\}$$

$$f=\sum_{n=-\infty}^{\infty}\langle f,e_n\rangle e_n.$$

 $\int_{-\pi}^{+\pi} \sin(nx)\ dx = 0$	 $\int_{-\pi}^{+\pi} \cos(nx)\ dx = 0$
 $\int_{-\pi}^{+\pi} \sin(1x)\ dx = 0$	 $\int_{-\pi}^{+\pi} \cos(1x)\ dx = 0$
 $\int_{-\pi}^{+\pi} \sin(2x)\ dx = 0$	 $\int_{-\pi}^{+\pi} \cos(2x)\ dx = 0$
 $\int_{-\pi}^{+\pi} \sin(3x)\ dx = 0$	 $\int_{-\pi}^{+\pi} \cos(3x)\ dx = 0$
Sin	Cos

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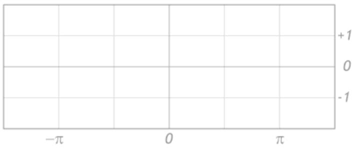
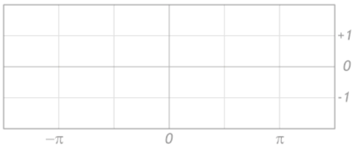
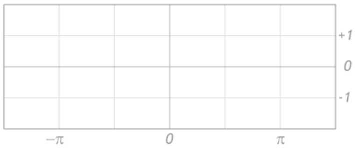
<td>
$\int_{-\pi}^{+\pi} \sin(nx)\sin(mx)\ dx = \begin{cases} \pi, & n=m \\ 0, & n\neq m \end{cases}$</td>
<td>
$\int_{-\pi}^{+\pi} \sin(nx)\cos(mx)\ dx = 0$</td>
<td>
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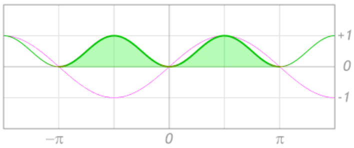
<td>
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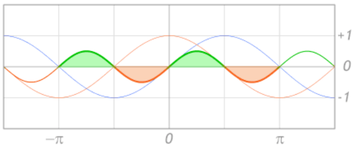
<td>
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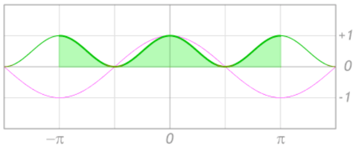
<td>
$\int_{-\pi}^{+\pi} \cos(1x)\cos(1x)\ dx = \pi$</td>
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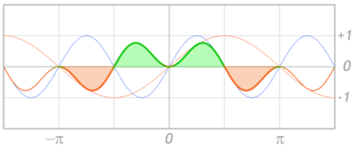
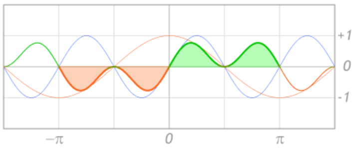
<td>
$\int_{-\pi}^{+\pi} \sin(2x)\sin(1x)\ dx = 0$</td>
<td>
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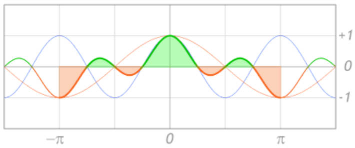
<td>
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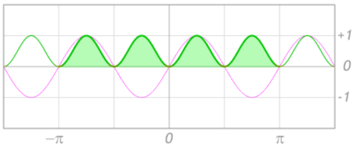
<td>
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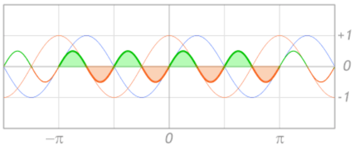
<td>
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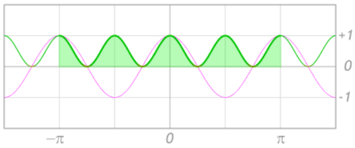
<td>
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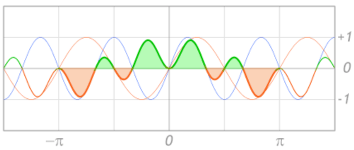
</tr>
<tr>
<td>
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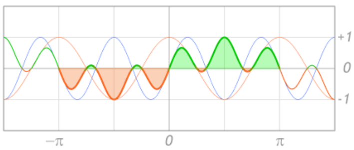
<td>
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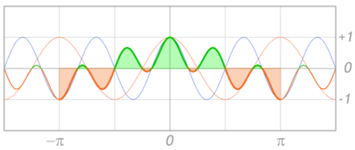
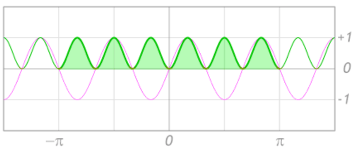
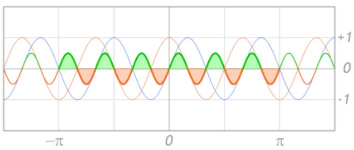
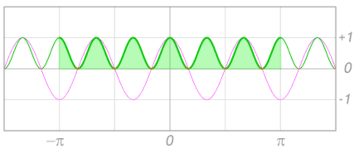
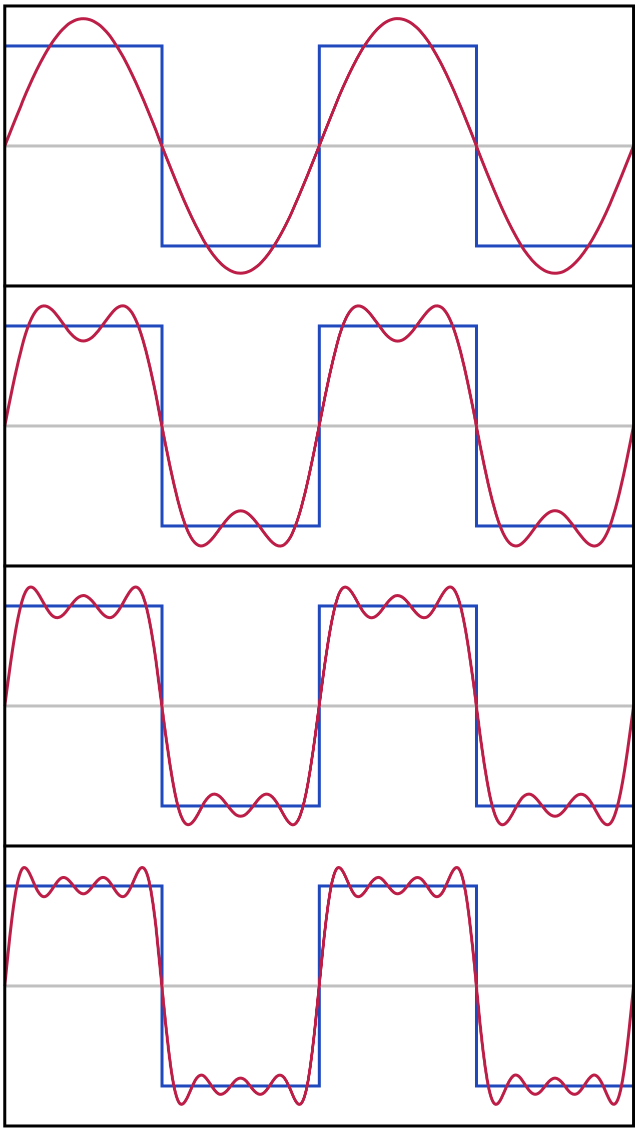
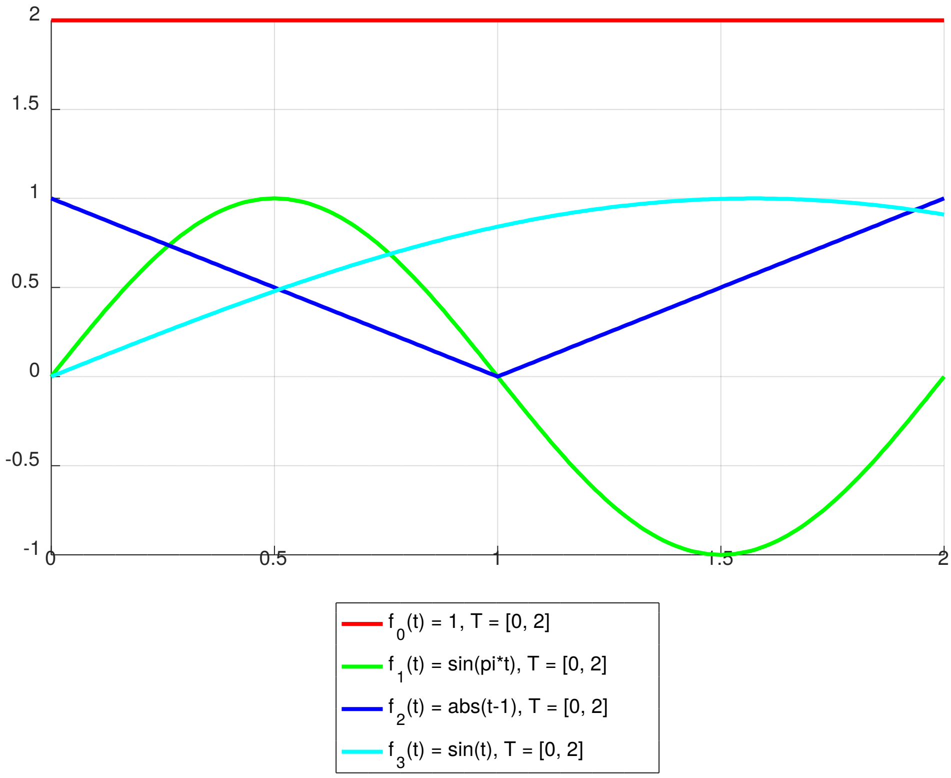
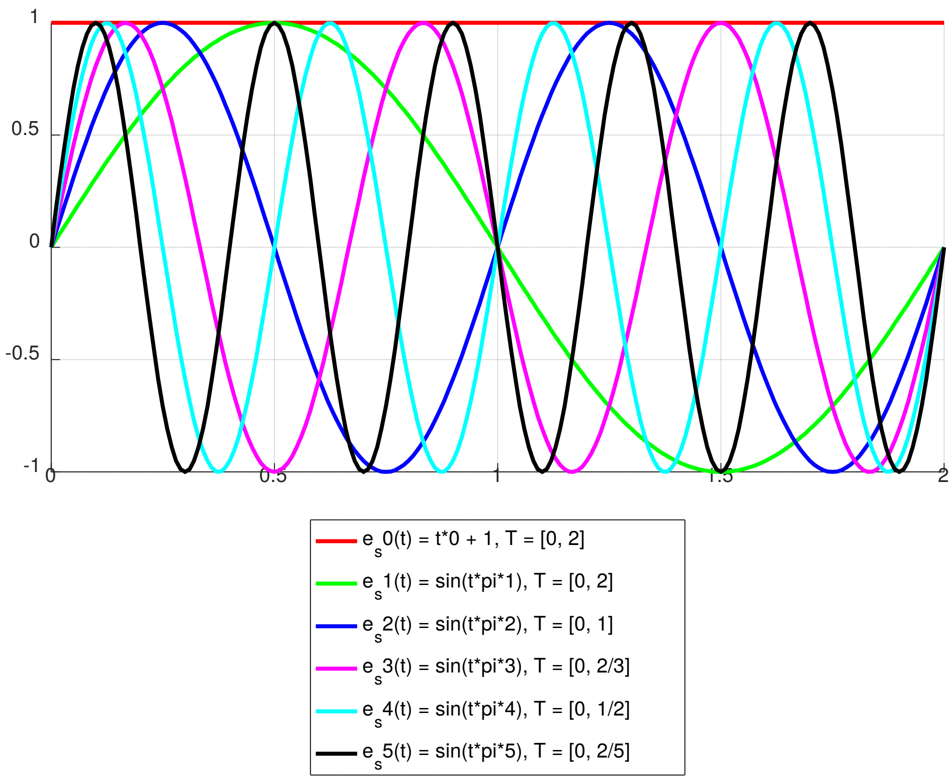
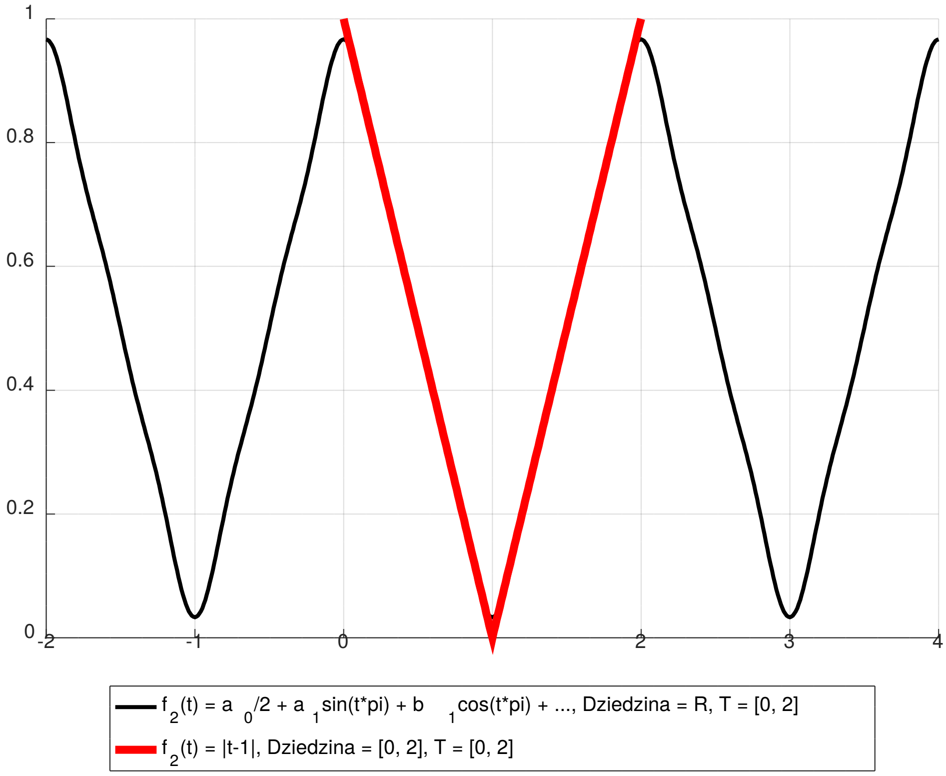
<td>
$\int_{-\pi}^{+\pi} \cos(3x)\cos(2x)\ dx = 0$</td>
</tr>
<tr>
<td>
$\int_{-\pi}^{+\pi} \sin(3x)\sin(3x)\ dx = \pi$</td>
<td>
$\int_{-\pi}^{+\pi} \sin(3x)\cos(3x)\ dx = 0$</td>
<td>
$\int_{-\pi}^{+\pi} \cos(3x)\cos(3x)\ dx = \pi$</td>
</tr>
<tr>
<td>Sin sin</td>
<td>Sin cos</td>
<td>Cos cos</td>
</tr>
</table>

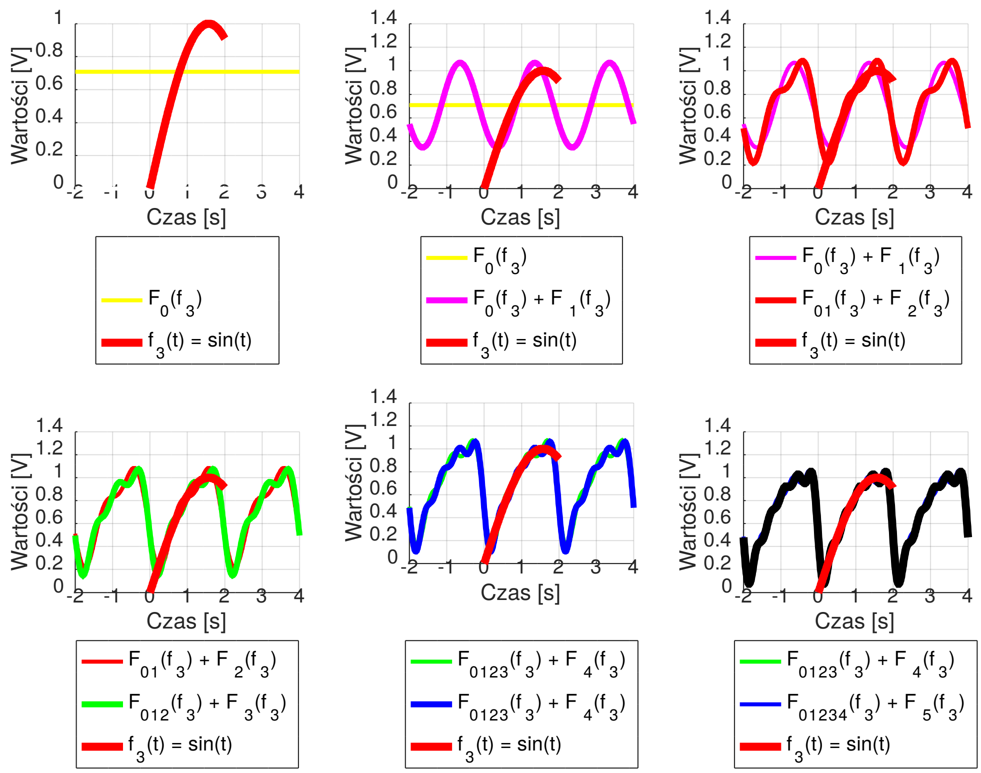


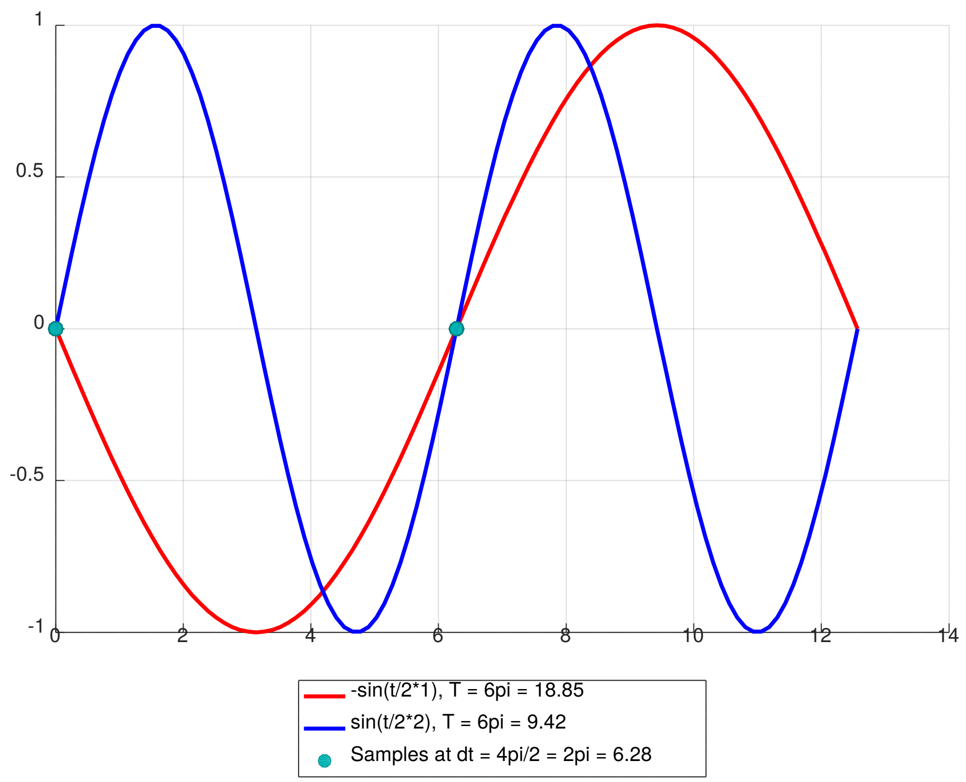
Rys. Sumy częściowe szeregu Fourier’a dla fali kwadratowej.

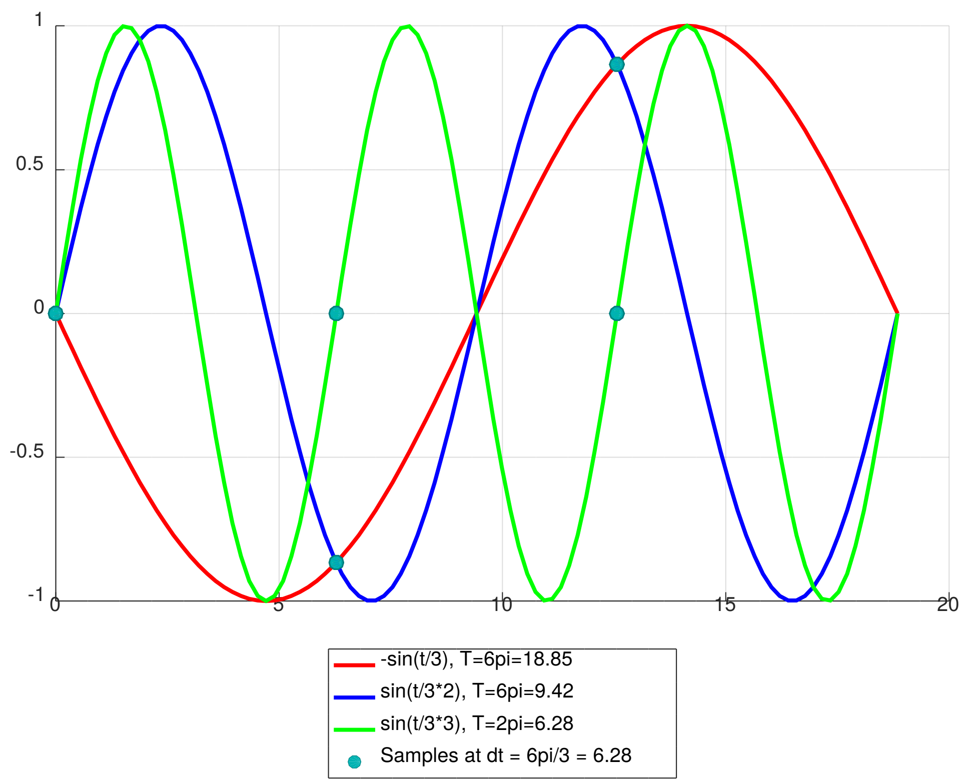

2
1.5
1
0.5
0
-0.5
-1
0
0.5
1
1.5
2
f_0(t) = 1, T = [0, 2]
f_1(t) = sin(pi*t), T = [0, 2]
f_2(t) = abs(t-1), T = [0, 2]
f_3(t) = sin(t), T = [0, 2]

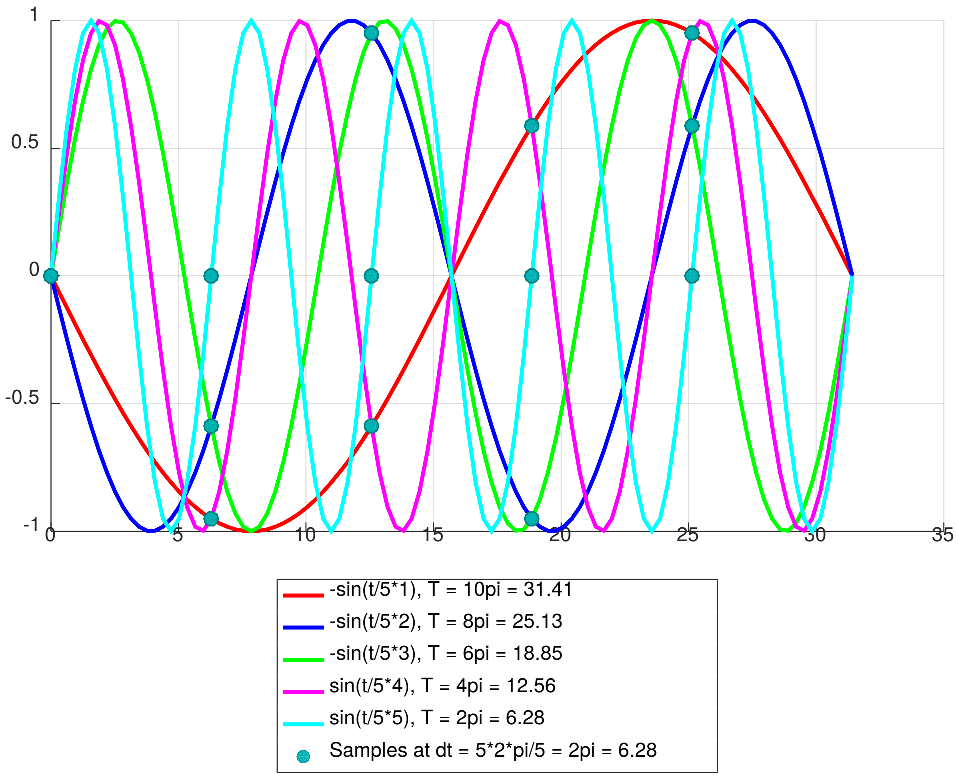

1
0.5
0
-0.5
-1
0
0.5
1
1.5
2
e_s0(t) = t*0 + 1, T = [0, 2]
e_s1(t) = sin(t*pi*1), T = [0, 2]
e_s2(t) = sin(t*pi*2), T = [0, 1]
e_s3(t) = sin(t*pi*3), T = [0, 2/3]
e_s4(t) = sin(t*pi*4), T = [0, 1/2]
e_s5(t) = sin(t*pi*5), T = [0, 2/5]

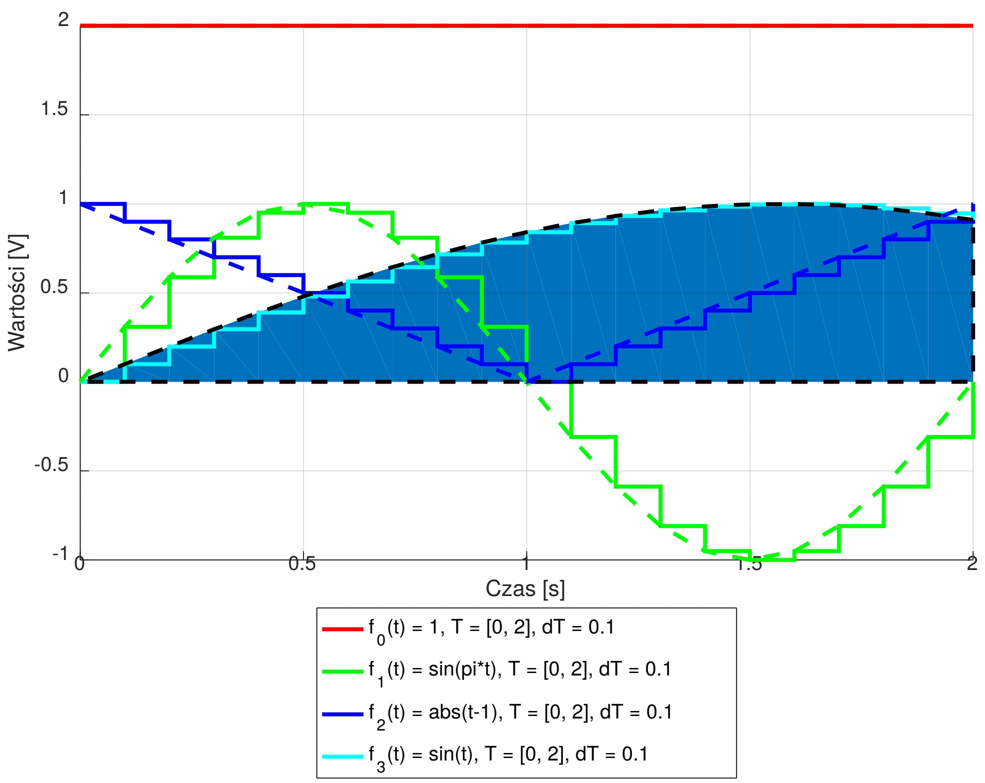

1
0.8
0.6
0.4
0.2
0
-2
-1
0
2
3
4
f_2(t) = a_0/2 + a_1 sin(t*pi) + b_1 cos(t*pi) + ..., Dziedzina = R, T = [0, 2]
f_2(t) = |t-1|, Dziedzina = [0, 2], T = [0, 2]

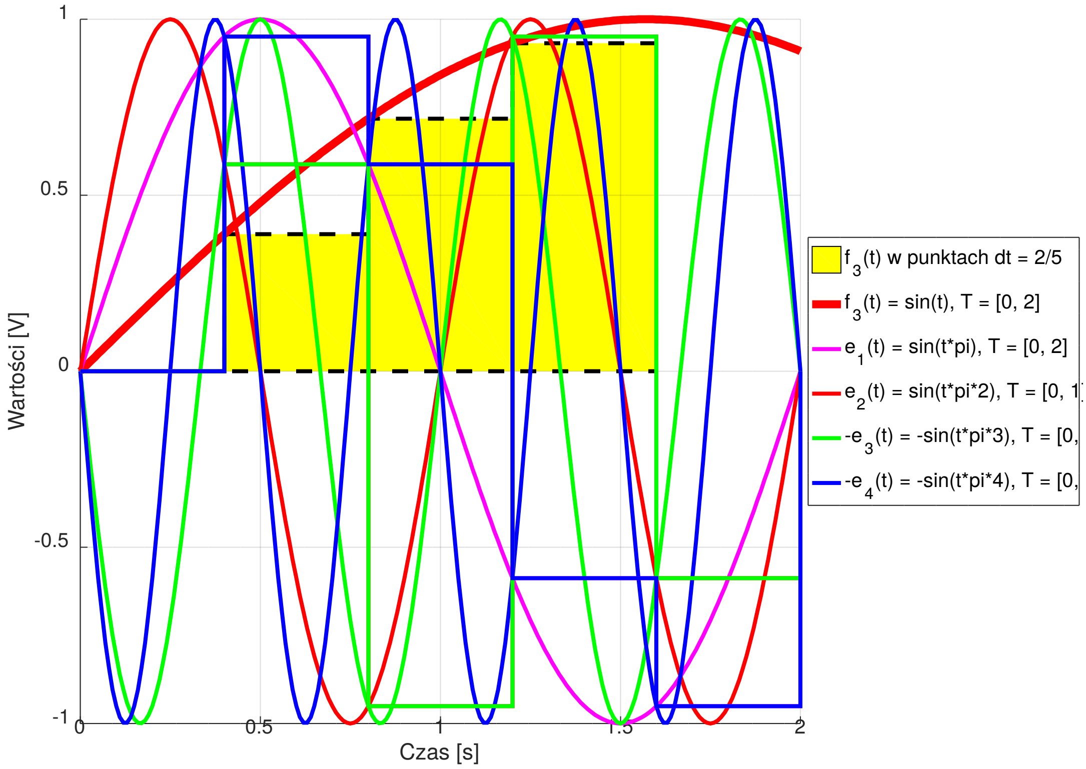

Wartości [V]
Czas [s]
F_0(f_3)
f_3(t) = sin(t)
F_0(f_3) + F_1(f_3)
F_01(f_3) + F_2(f_3)
F_012(f_3) + F_3(f_3)
F_0123(f_3) + F_4(f_3)
F_01234(f_3) + F_5(f_3)


1
0.5
0
-0.5
-1
0
2
4
6
8
10
12
14
-sin(t/2*1), T = 6pi = 18.85
sin(t/2*2), T = 6pi = 9.42
Samples at dt = 4pi/2 = 2pi = 6.28


1
0.5
0
-0.5
-1
0
5
10
15
20
-sin(t/3), T=6pi=18.85
sin(t/3*2), T=6pi=9.42
sin(t/3*3), T=2pi=6.28
Samples at dt = 6pi/3 = 6.28


1
0.5
0
-0.5
-1
0
5
10
15
20
25
30
35
-sin(t/5*1), T = 10pi = 31.41
-sin(t/5*2), T = 8pi = 25.13
-sin(t/5*3), T = 6pi = 18.85
sin(t/5*4), T = 4pi = 12.56
sin(t/5*5), T = 2pi = 6.28
Samples at dt = 5*2*pi/5 = 2pi = 6.28


2
1.5
1
0.5
0
-0.5
-1
Wartości [V]
0
0.5
1
1.5
2
Czas [s]
f_0(t) = 1, T = [0, 2], dT = 0.1
f_1(t) = sin(pi*t), T = [0, 2], dT = 0.1
f_2(t) = abs(t-1), T = [0, 2], dT = 0.1
f_3(t) = sin(t), T = [0, 2], dT = 0.1



Przykład Matlab/Octave

```
% problem
% jak wygląda funkcja f(t) przedstawiona w postaci szeregu Fourier'a?
% do 5-tego elementu

f0 = @(t) 2+0*t; % w przedziale [0, 2]
f1 = @(t) sin(pi*t); % w przedziale [0, 2]
f2 = @(t) abs(t-1); % w przedziale [0, 2]
f3 = @(t) sin(t); % w przedziale [0, 2]
ts = 0:0.01:2;

%figure;
%hold 'on';
%grid 'on';
%p0 = plot(ts, f0(ts), 'linewidth', 2, 'color', 'red');
%p1 = plot(ts, f1(ts), 'linewidth', 2, 'color', 'green');
%p2 = plot(ts, f2(ts), 'linewidth', 2, 'color', 'blue');
%p3 = plot(ts, f3(ts), 'linewidth', 2, 'color', 'cyan');
%legend([p0,p1,p2,p3],
%"f_0(t) = 1,         T = [0, 2]",
%"f_1(t) = sin(pi*t), T = [0, 2]",
%"f_2(t) = abs(t-1),  T = [0, 2]",
%"f_3(t) = sin(t),    T = [0, 2]",
%"location", "southoutside");
%%print -dsvg ExampleFunctionsOn0to2.svg; %%% zapisanie okna do pliku


% e1(t) potrzeba aby miało okres T1 = [0,2] natomiast wiadomo że ma [0, 2*pi]
% jak należy wyskalować argument t żeby tak było? (a później wprowadzić to
% skalowanie do wnętrza e1)
% no więc efekt ma być taki: 0 -> 0, 2 -> 2*pi
% czyli jak t pomnożymy przez pi to tak będzie.
% w związku z tym funkcje bazowe będą postaci:
```

```
es0 = @(t) t*0 + 1; % sin(t*pi*0) == 1, T0 jest dowolny
ec0 = @(t) t*0;     % cos(t*pi*0) == 0, T0 jest dowolny

es1 = @(t) sin(t*pi*1); % T1 == [0, 2*pi/pi] == [0, 2]
ec1 = @(t) cos(t*pi*1); % T1 == [0, 2*pi/pi] == [0, 2]

es2 = @(t) sin(t*pi*2); % T2 == [0, 2*pi/2/pi] == [0, 1] == 1/2 * T1
ec2 = @(t) cos(t*pi*2); % T2 == [0, 2*pi/2/pi] == [0, 1] == 1/2 * T1

es3 = @(t) sin(t*pi*3); % T3 == [0, 2*pi/3/pi] == [0, 2/3] == 1/3 * T1
ec3 = @(t) cos(t*pi*3); % T3 == [0, 2*pi/3/pi] == [0, 2/3] == 1/3 * T1

es4 = @(t) sin(t*pi*4); % T4 == [0, 2*pi/4/pi] == [0, 1/2] == 1/4 * T1
ec4 = @(t) cos(t*pi*4); % T4 == [0, 2*pi/4/pi] == [0, 1/2] == 1/4 * T1

es5 = @(t) sin(t*pi*5); % T5 == [0, 2*pi/5/pi] == [0, 2/5] == 1/5 * T1
ec5 = @(t) cos(t*pi*5); % T5 == [0, 2*pi/5/pi] == [0, 2/5] == 1/5 * T1

ts = 0:0.01:2;

%figure;
%hold 'on';
%grid 'on';
%p0 = plot(ts, es0(ts), 'linewidth', 2, 'color', 'red');
%p1 = plot(ts, es1(ts), 'linewidth', 2, 'color', 'green');
%p2 = plot(ts, es2(ts), 'linewidth', 2, 'color', 'blue');
%p3 = plot(ts, es3(ts), 'linewidth', 2, 'color', 'magenta');
%p4 = plot(ts, es4(ts), 'linewidth', 2, 'color', 'cyan');
%p5 = plot(ts, es5(ts), 'linewidth', 2, 'color', 'black');
%legend([p0,p1,p2,p3,p4,p5],
%"e_s0(t) = t*0 + 1,     T = [0, 2]",
%"e_s1(t) = sin(t*pi*1), T = [0, 2]",
%"e_s2(t) = sin(t*pi*2), T = [0, 1]",
%"e_s3(t) = sin(t*pi*3), T = [0, 2/3]",
%"e_s4(t) = sin(t*pi*4), T = [0, 1/2]",
%"e_s5(t) = sin(t*pi*5), T = [0, 2/5]",
%"location", "southoutside");
%%print -dsvg FurierBasesOn0to2.svg; %%% zapisanie okna do pliku


% teraz obliczymy współczynniki szeregu Fourier'a dla f2:

f2a0 = quad(@(t) f2(t).*es0(t), 0, 2)/2;
f2b0 = quad(@(t) f2(t).*ec0(t), 0, 2)/2;
f2a1 = quad(@(t) f2(t).*es1(t), 0, 2);
f2b1 = quad(@(t) f2(t).*ec1(t), 0, 2);
f2a2 = quad(@(t) f2(t).*es2(t), 0, 2);
f2b2 = quad(@(t) f2(t).*ec2(t), 0, 2);
f2a3 = quad(@(t) f2(t).*es3(t), 0, 2);
f2b3 = quad(@(t) f2(t).*ec3(t), 0, 2);
f2a4 = quad(@(t) f2(t).*es4(t), 0, 2);
f2b4 = quad(@(t) f2(t).*ec4(t), 0, 2);
f2a5 = quad(@(t) f2(t).*es5(t), 0, 2);
f2b5 = quad(@(t) f2(t).*ec5(t), 0, 2);


% teraz przedstawmy funkcję w postaci szeregu Furier'a:

f2Furier0 = @(t) f2a0*es0(t) + f2b0*ec0(t);
f2Furier1 = @(t) f2a1*es1(t) + f2b1*ec1(t);
f2Furier2 = @(t) f2a2*es2(t) + f2b2*ec2(t);
f2Furier3 = @(t) f2a3*es3(t) + f2b3*ec3(t);
f2Furier4 = @(t) f2a4*es4(t) + f2b4*ec4(t);
f2Furier5 = @(t) f2a5*es5(t) + f2b5*ec5(t);


f2Furier01 = @(t) f2Furier0(t) + f2Furier1(t);
f2Furier012 = @(t) f2Furier01(t) + f2Furier2(t);
f2Furier0123 = @(t) f2Furier012(t) + f2Furier3(t);
f2Furier01234 = @(t) f2Furier0123(t) + f2Furier4(t);

f2Furier = @(t) f2Furier0(t) + f2Furier1(t) + f2Furier2(t) + f2Furier3(t) + f2Furier4(t) +
f2Furier5(t);
tsts = -2:0.01:4;



%figure;
%hold 'on';
```

```
%grid 'on';
%p0 = plot(tsts, f2Furier0(tsts), 'linewidth', 2, 'color', 'yellow');
%p1 = plot(tsts, f2Furier01(tsts), 'linewidth', 2, 'color', 'magenta');
%p2 = plot(tsts, f2Furier012(tsts), 'linewidth', 2, 'color', 'red');
%p3 = plot(tsts, f2Furier0123(tsts), 'linewidth', 2, 'color', 'green');
%p4 = plot(tsts, f2Furier01234(tsts), 'linewidth', 2, 'color', 'blue');
%p5 = plot(tsts, f2Furier(tsts), 'linewidth', 2, 'color', 'black');
%p = plot(ts, f2(ts), 'linewidth', 4, 'color', 'red');
%legend([p,p0,p1,p2,p3,p4,p5],
%%legend([p,p1,p3,p5],
%"f_2(t) = |t-1|, Dziedzina = [0, 2], T = [0, 2]",
%"f_2(t) = a_0/2, Dziedzina = R, T = [0, 2]",
%"f_2(t) = a_0/2 + a_1 sin(t*pi) + b_1 cos(t*pi), Dziedzina = R, T = [0, 2]",
%"f_2(t) = ... + a_2 sin(t*pi*2) + b_2 cos(t*pi*2), Dziedzina = R, T = [0, 2]",
%"f_2(t) = ... + a_3 sin(t*pi*3) + b_3 cos(t*pi*3), Dziedzina = R, T = [0, 2]",
%"f_2(t) = ... + a_4 sin(t*pi*4) + b_4 cos(t*pi*4), Dziedzina = R, T = [0, 2]",
%"f_2(t) = a_0/2 + a_1 sin(t*pi) + b_1 cos(t*pi) + ... + a_5 sin(t*pi*5) + b_5 cos(t*pi*5),
Dziedzina = R, T = [0, 2]",
%"location", "southoutside");
%xlabel('Czas [s]');
%ylabel('Wartości [V]');
%%print -dsvg FurierSeries.svg; %%% zapisanie okna do pliku
```