

Module - 1

CO1 - Perform number System conversion, binary arithmetic Operation and binary Coding.

Number systems

Number System is an ordered set of symbols called digits with rules defined for addition, subtraction etc.

Base / Radix

Base / radix of a number system specifies the actual number of digits in its set.

Positional weight

It is the position of a digit with reference to decimal points.

In this module we are going discuss four types of number systems.

- 1) Decimal number System.
- 2) Binary numbered System.

3) Octal number system.

4) Hexadecimal number system.

1) Decimal number system.

Symbols - 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

Base / radix = 10

Positional weight = 10^x ($x = 0, 1, 2, 3, \dots$)

2) Binary number system

Symbols - 0, 1

Base / radix = 2

Positional weight = 2^x

3) Octal number system

Symbols = 0, 1, 2, 3, 4, 5, 6, 7

Base / radix = 8

Positional weight = 8^x

4) Hexadecimal number system

Symbols = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, a, b, c, d, e, f

Base / radix = 16

Number System Conversion

1) Decimal to binary

$$(46 \cdot 25)_{10} \rightarrow (?)_2$$

2	4	6	0
2	2	3	1
2	1	1	1
2	5	1	1
2	2	0	1

$$0 \cdot 25 \times 2 = 0.5 \quad 0$$

$$0.5 \times 2 = 1.0$$

$$(101110.01)_2 \cdot 2^{3P+Q} = S \times 2^{3P+Q}$$

2)

$$(25 \cdot 75)_{10} \rightarrow (100 \cdot 0010010)_2$$

$$\begin{array}{r}
 2 \overline{)25} & 0 \\
 2 \overline{)12} & 1 \\
 2 \overline{)6} & 0 \\
 2 \overline{)3} & 0 \\
 1 & 1
 \end{array}$$

$$\begin{array}{r} 15639 \quad 11 \quad 1000000 \\ \hline 2 \quad 125 \\ \hline 12 \quad 12 \\ \hline 6 \end{array}$$

$$0.75 \times 2 = 1.5$$

$$8.5 \times 2 = 1.0 \text{ kg} \quad \text{at } 21^{\circ}\text{C}$$

$$(11001.11)_2$$

$$3) (420.24)_{10} \rightarrow (\text{?})_2$$

$$\begin{array}{r} 2 | 420 \\ 2 | 210 \quad 0 \\ 2 | 105 \quad 0 \\ 2 | 52 \quad 1 \\ 2 | 26 \quad 0 \\ 2 | 13 \quad 0 \\ 2 | 6 \quad 1 \\ 2 | 3 \quad 0 \end{array}$$

$$0.24 \times 2 = 0.48 \quad 0$$

$$\begin{aligned} 0.48 \times 2 &= 0.96 \quad 0 \\ 0.96 \times 2 &= 1.92 \quad 1 \\ 0.92 \times 2 &= 1.84 \quad 1 \\ (110100100.00)_2 & \end{aligned}$$

2) Decimal to octal

$$1) (86.45)_{10} \rightarrow (\text{?})_8$$

$$\begin{array}{r} 8 | 86 \\ 8 | 10 \quad 6 \\ \quad \quad \quad 1 \quad 2 \end{array}$$

$$45 \times 8 = 36 \quad 3$$

$$6 \times 8 = 48 \quad 4$$

$$48 \times 8 = 64 \quad 6$$

$$64 \times 8 = 32 \quad 3$$

$$(126 \cdot 3463) \overline{8}$$

$$2) (2142.53)_{10} \rightarrow (1000)_8$$

$$\begin{array}{r}
 8 \overline{)2142} \\
 8 \overline{)267} \quad 6 \\
 8 \overline{)33} \quad 3 \\
 \hline
 4 \quad 1 \quad 8 \quad 01
 \end{array}$$

$$0.\underline{5}3 \times 8 = 4.24$$

$$24 \times 8 = 192$$

$$-9.2 \times 8 = 7.36$$

$$36 \times 8 = 2.88 \quad 2$$

$$(4136 \cdot 4172)_8$$

3) Decimal to Hexadecimal

$$1) (214.35)_{10} \rightarrow (-)_{16}$$

$$\begin{array}{r}
 16 \\
 \overline{)214} \\
 16 \\
 \hline
 54 \\
 48 \\
 \hline
 6
 \end{array}$$

$$0.35 \times 16 = 5.6$$

$$0.6 \times 16 = 9.6 \quad 9$$

$$0.6 \times 16 = 9.6 \quad 9$$

$(D6.599)_{16}$

2) $(1731.56)_{10} \rightarrow (\underline{\hspace{2cm}} D_{16})_8$

$$\begin{array}{r} 16 | 1731 \\ 16 | 108 \quad 3 \end{array}$$

6 12

$$0.56 \times 16 = 8.96 \quad 8$$

$$0.96 \times 16 = 15.36 \quad 15$$

$$0.36 \times 16 = 5.76 \quad 5$$

$$0.76 \times 16 = 12.16 \quad 12$$

$(6C83.8F5)_{16}$

Any base number system to decimal

(Binary to decimal)

1) $(1101.11)_2 \rightarrow (\underline{\hspace{2cm}})_{10}$

$$\begin{aligned} &= (1 \times 2^3) + (1 \times 2^2) + (0 \times 2^1) + (1 \times 2^6) \\ &\quad + (1 \times 2^5) + (1 \times 2^2) \end{aligned}$$

$$= 8 + 2_1 + 0 + 1 + 0 \cdot 5 + 0 \cdot 25 = (13.75)_{10}$$

$$2) \left(\begin{smallmatrix} 5 & 4 & 3 & 2 & 1 & 0 \\ 1 & 0 & 1 & 0 & 11 & 101 \end{smallmatrix} \right)_2 \rightarrow \left(\text{D} \right)_{10}$$

$$\begin{aligned}
 &= (5 \times 2^5) + (0 \times 2^4) + (1 \times 2^3) + (0 \times 2^2) \\
 &\quad + (1 \times 2^1) + (1 \times 2^0) + (1 \times 2^{-1}) + \\
 &\quad (0 \times 2^{-2}) + (1 \times 2^{-3}) \\
 &= (43.625)
 \end{aligned}$$

$$\begin{aligned}
 & 3) \quad (110110 \cdot 1101)_2 \stackrel{+}{=} (1x2^5 + 1x2^4 + 0x2^3 + 1x2^2 + 1x2^1 + 0x2^0)_{10} \\
 & = (1 \times 2^5) + (1 \times 2^4) + (0 \times 2^3) + (1 \times 2^2) + (1 \times 2^1) \\
 & + (0 \times 2^0) + (1 \times 2^5) + (1 \times 2^4 - 2^3) + (0 \times 2^3) \\
 & + (1 \times 2^5 - 4) \\
 & + ((1 \times 2^5) + (1 \times 2^4) + (0 \times 2^3) + (1 \times 2^2) + (1 \times 2^1) + (0 \times 2^0)) \\
 & = (54 \cdot 81)_{10} + (1 \cdot 81) + (1 \cdot 81) + (1 \cdot 81) + (1 \cdot 81) + (0 \cdot 81)
 \end{aligned}$$

Octal to decimal

$$i) \begin{pmatrix} 2 & 1 & 0 \\ 4 & 2 & 1 \\ 7 & 3 & 5 \end{pmatrix} \rightarrow (\quad)_10$$

$$(4 \times 8^2) + (2 \times 8^1) + (7 \times 8^0) + (3 \times 8^{-1}) + (5 \times 8^{-2})$$

$$= \cancel{256} = (279 \cdot 45) \text{ } 10$$

$$(4136.4172)_8 \rightarrow (?)_{10}$$

$$\begin{aligned} & (4 \times 8^3) + (1 \times 8^2) + (3 \times 8^1) + (6 \times 8^0) + (4 \times 8^{-1}) \\ & + (1 \times 8^{-2}) + (7 \times 8^{-3}) + (2 \times 8^{-4}) \\ & = (2142.5397)_{10} \end{aligned}$$

Hexadecimal to Decimal

$$1) (6A13C.2A)_{16} \rightarrow (?)_{10}$$

$$\begin{aligned} & (6 \times 16^3) + (10 \times 16^2) + (11 \times 16^1) + (12 \times 16^0) \\ & + (2 \times 16^{-1}) + (10 \times 16^{-2}) \\ & = (27324.1640) \end{aligned}$$

$$2) (1A13C.7C)_{16} \rightarrow (?)_{10}$$

$$\begin{aligned} & (1 \times 16^3) + (10 \times 16^2) + (11 \times 16^1) + (12 \times 16^0) \\ & + (-7 \times 16^{-1}) + (13 \times 16^{-2}) \end{aligned}$$

$$= (6866.(-6845.4843))$$

10

Binary to co octal

Octal	Binary
0	0101010110110010
1	001
2	010
3	011
4	101001101001000110
5	101
6	110
7	111

1) $(\underline{00001} \underline{1011010} \cdot \underline{101101100})_2 \rightarrow (\text{ })_8$

$(0332, 554)_8$

2) $(\underline{01100} \underline{101110} \cdot \underline{101111})_2 \rightarrow (\text{ })_8$

$(1456.057)_8$

Octal to binary

$$1) (155.52)_8 \rightarrow ()_2$$

$$(001101101.101010)_2$$

$$2) (3047.62)_8 \rightarrow ()_2$$

$$(011000100111.110010)_2$$

Binary to Hexadecimal

Hexadecimal	Binary
0	0 0 0 0
1	0 0 0 1
2	0 0 1 0
3	0 0 1 1
4	0 1 0 0
5	0 1 0 1
6	0 1 1 0
7	0 1 1 1
8	1 0 0 0
9	1 0 0 1
A	1 0 1 0
B	1 0 1 1
C	1 1 0 0
D	1 1 0 1
E	1 1 1 0
F	1 1 1 1

$$1) \left(\underline{0001} \underline{00110111} \cdot \underline{101} \right)_{16} \rightarrow \left(\underline{01110110} \right)_2$$

$$(137. AE8)_{16} \rightarrow (01110110111)_{2}$$

$$2) \left(\underline{11110111010} \cdot \underline{11111011} \right)_{16} \rightarrow \left(\underline{01110110} \right)_2$$

$$(FBA \cdot FB)_{16} \rightarrow (01110110)_{2}$$

I-hexdecimal to binary

$$1) (FBA \cdot F13)_{16} \rightarrow (10011010)_{2}$$

$$(11110111010 \cdot 1111011)_{16} \rightarrow (10011010)_{2}$$

$$2) (1C12E \cdot 213)_{16} \rightarrow (100110101011)_{2}$$

$$(0001110011101101011001101011)_{2}$$

$$(FF0E \cdot A18FH)_{2}$$

Octal to Hexadecimal

1) $(7324.456)_8$, $(87A.581)_2$

$(\underline{111011010100} \cdot \underline{00101110000})_2$

$(\underline{\underline{B1D4.970}})_8$

$(87.A870)_2$

2) $(5137.26)_8$

$(\underline{10100101111} \cdot \underline{0101100})_2$

$(\underline{\underline{A5F.58}})_8$

Hexadecimal to Octal

1) $(4ECE.43F)_{16}$

$(\underline{0010011101001110} \cdot \underline{0100001111})_2$

$(047316.2077)_2$

~~16~~ 8

Binary Addition

$$0 + 0 = 0$$

$$0 = 0 \times 0$$

$$0 + 1 = 1$$

0.5-1.0

$$l + o = l$$

—

$$1 + 1 = 10$$

11

~~10~~

1 1 1 1 +

101

Lotto

11000

110

—

101

! !

1101

X011011 . W

1000

11011

1203

1103

013610

$$\begin{array}{r}
 111 \cdot 10 \\
 100 \cdot 1 \\
 \hline
 10010 \cdot 00
 \end{array}$$

(101100111010101010)

$$\begin{array}{r}
 111 \\
 111 \cdot 101 \\
 1011 \cdot 11 \\
 \hline
 10011 \cdot 011
 \end{array}$$

(101100111010101010)

Binary multiplication

$$0 \times 0 = 0$$

$$0 = 0 + 0$$

$$0 \times 1 = 0$$

$$1 = 1 + 0$$

$$1 \times 0 = 0$$

$$1 = 0 + 1$$

$$1 \times 1 = 1$$

$$01 = 1 + 1$$

i)

$$\begin{array}{r}
 1011 \cdot x \\
 101 \\
 \hline
 11011
 \end{array}$$

$$\begin{array}{r}
 + 1111 \\
 0101 \\
 \hline
 0011
 \end{array}$$

ii)

$$\begin{array}{r}
 110110 \cdot x \\
 + 111111 \\
 \hline
 1110110
 \end{array}$$

$$\begin{array}{r}
 + 1111 \\
 0111 \\
 \hline
 0011
 \end{array}$$

III

~~111.101~~

111.101

111.101

the message is

10 (doublet)

~~00110111.010~~

111.101

not formable 01

~~1101111010~~

0000000000 is not formable 01

~~1101111010~~

0000000000 is not formable 01

~~1101111010~~

0000000000 is not formable 01

the message is not restorable 01

undermost 110111.010 is not restorable

0010-1101 is not restorable 01

0100-1101 is not restorable 01

minimum 11011101 is not restorable 01

not formable 11011101

not formable 11011101

11010010101001

is not formable 01

but not made up to 00 and each of
several of digits differs at all the
number present before 10 formable 01

Signed number representations

1's complement
form

2's complement
form

1's complement form

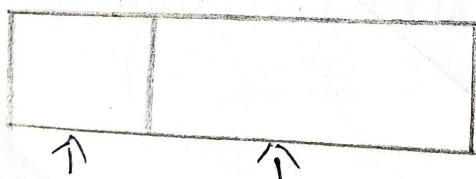
1's complement of a binary number is obtained by changing each 0 to 1 and each 1 to 0.

The complemented value represents the negative of the binary numbers.

e.g. 1's complement of 1011 = 0100

1's complement of 1101 = 0010

Hence a positive or negative number is represented in the 1's complement form as follows.



Signbit 1's complement of binary

In this form as shown above first bit will be the Signbit which is followed by 1's complement of binary number.

If Signbit = 0 the number is positive.
 And if Signbit = 1 binary number is negative. $(1110)_{10} = -10$

- represent -7 1's complement form.

$$2 \overline{) 7}$$

$$2 \overline{) 13} \quad 1$$

$$\underline{01}$$

Binary of 7 with 1 prepended is 10000000000000000000000000000000

$(7)_{10} \rightarrow (111)_2$ Therefore $01000000000000000000000000000000$

1's complement of 00000000000000000000000000000000 = 10000000000000000000000000000000

$$\begin{array}{|c|c|} \hline & 000 \\ \hline 1 & | \\ \hline 0 & 0 & 0 \\ \hline \end{array}$$

$$50 - 7 = (1000)_2$$

Binary of 7 with 1 prepended is 10000000000000000000000000000000
 Complement form.

- represent -16 in 1's complement form

$$2 \overline{) 16}$$

$$2 \overline{) 8} \quad 0$$

$$2 \overline{) 4} \quad 0$$

$$2 \overline{) 2} \quad 0$$

$$1 \quad 0$$

$(16)_{10} \rightarrow (100000)_2$ plus 10000000000000000000000000000000

1's of 7 $\rightarrow 00111000000000000000000000000000$

$\boxed{1} \boxed{0111}$

$$-16 = (10111)_2 \text{ in } 1's \text{ complement}$$

2's complement form

2's complement of a binary number is obtained by adding 1 to the 1's complement of a number. That is, 2's complement = 1's complement + 1.

$$\text{eg: } 2's \text{ complement} = 1's \text{ complement} + 1$$

$$\begin{array}{r} 000 \\ + 1 \\ \hline 0101 \end{array}$$

Hence positive or negative numbers is represented in 2's complement form as follows:



Signbit 2's complement

* Represent -19. (in 2's complement form)

$$\begin{array}{r} 2 | 19 \\ 2 | 9 \quad 1 \\ 2 | 4 \quad 1 \\ 2 | 2 \quad 0 \\ \hline & 1 \quad 0 \end{array}$$

$$(19)_2 \rightarrow (10011)_2$$

$$2's \text{ of } -19 \rightarrow (01100)$$

$$\begin{array}{r} 1 \quad 1 \\ \cancel{+} 0 \quad 10011 \\ \hline 10100 \end{array}$$

$$\boxed{1 \quad 10100}$$

$$\begin{array}{r} 1 \quad 10110 \\ + 1 \quad 01110 \\ \hline 0 \quad 11110 \end{array}$$

$$\boxed{(110100)}_2$$

$$(19)_2 \rightarrow (10011)_2 \quad 10011 = 16$$

$$2's \text{ of } -19 \rightarrow (01100)$$

$$\begin{array}{r} 01100 \\ + 1 \\ \hline 01101 \end{array}$$

$$\boxed{1 \quad 01101}$$

$-19 = 101101$ in 2's complement form

represent * 34 in 2's complement form

$$\begin{array}{r} 34 \\ \hline 2 | 17 & 0 \\ 2 | 8 & 1 \\ 2 | 4 & 0 \\ 2 | 2 & 0 \\ \hline & 0 \end{array}$$

$$\begin{array}{r} 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ \hline 0 & 1 & 1 & 0 \end{array}$$

(Calculation of 2's complement)

exp. of 2's complement

$$(34)_2 \rightarrow (100010)$$

$$2's \text{ of } 34 = 6(011101)$$

$$\begin{array}{r} 011101 \\ + 000101 \\ \hline \underline{\underline{011110}} \end{array}$$

$$34 = 001110 \text{ (2's complement) form}$$

Binary Subtraction Using 1's Complement method

$$\begin{array}{r} 1111 - \\ 1010 \\ \hline 1111 + \\ 0101 \\ \hline \underline{\underline{01001}} \end{array}$$

- * Determined the 1's complement of negative (Second number)
- * Add to the positive number.
- * If there is a carry the result is positive and the magnitude is given by removing the carry and adding it to the LSB of the result
- * If there is no carry the result is negative. And magnitude of the result is obtained by taking the 1's complement of the result.

e.g. $1011 -$

$$\begin{array}{r} \underline{1111} \\ 1011 + \\ \hline 0000 \end{array}$$

~~1011 is added with 0000~~

~~Actual result = -ve 0100~~

~~1011 + 0000~~

~~1011~~

~~1101~~

~~0010~~

1)

$$\begin{array}{r}
 1000 \\
 1010 \\
 \hline
 1000 \\
 + \\
 0101 \\
 \hline
 1101
 \end{array}$$

result = -Ve 0010

2)

$$\begin{array}{r}
 1010 \\
 1110 \\
 \hline
 1010 \\
 + \\
 0001 \\
 \hline
 1011
 \end{array}$$

= -0100

Binary Subtraction Using 2's

Complement method

$$\begin{array}{r}
 1010 \\
 1111 \\
 \hline
 1010 \\
 + \\
 0001 \\
 \hline
 1011
 \end{array}$$

1's to 2's

$$\begin{array}{r}
 0000 \\
 + \\
 \hline
 0001
 \end{array}$$

2's of 1011

$$\begin{array}{r}
 0100 \\
 + \\
 \hline
 0101
 \end{array}$$

\therefore carry number is -ve $\underline{\underline{0101}}$

Answer is -ve $\underline{\underline{0101}}$

ii)

$$\begin{array}{r} 1111 - \\ 1010 \\ \hline 1111 + \\ 0110 \\ \hline 00101 \end{array}$$

-1010

-0101

-0101

0110

Answer is -ve

Answer is +ve = 0101

Binary Subtraction using 2's Complement

$$\begin{array}{r} 1111 - \\ 1010 \\ \hline 1111 + \\ 0110 \\ \hline 00101 \end{array}$$

~~$$\begin{array}{r} 1111' - \\ 1010 \\ \hline 0101 \end{array}$$~~

~~$$\begin{array}{r} 2^5 - \\ 0101 \\ \hline 0110 \end{array}$$~~

00101

If there is carry result is +ve

So final answer is +0101

$$\begin{array}{r} 1010 \\ \underline{1111} \\ 1010 \\ + \\ \underline{0011} \\ 1011 \end{array}$$

$$\begin{array}{r} 1111 \\ 0000 \\ + \\ \hline 0001 \end{array}$$

There is no carry

result is 1011

final result 1011

result is 0101

$$\begin{array}{r} 0100 \\ 0100 \\ + \\ \hline 0101 \end{array}$$

$$\begin{array}{r} 110101 \\ \underline{101110} \\ 110101 \\ + \\ \underline{010010} \\ 000111 \end{array}$$

$$\begin{array}{r} 101110 \\ 010001 \\ + \\ \hline 010010 \end{array}$$

$$+ 000111$$

Binary Codes

As digital system operate in a binary system the numerous, alphabets and other special characters has to be converted into binary this process of converting into binary format

is called Coding and Codes the generated are called binary codes.

binary codes are classified into two

Weighted codes	Unweighted codes
<p>the digital code with each bit shows the specific base assign to the bit positions is known as weighted codes.</p> <p>eg: BCD (8421), 24215211</p>	<p>the digital code with does not exhibit specific base assign to the bit positions is known as unweighted codes.</p> <p>eg: excess - 3 code Gray code</p>
<p><u>Applications</u></p> <p>Used to represents decimal digits in system like digital calculations, multimeters etc.</p>	<p><u>Applications</u></p> <p>To perform certain arithmetic operation in digital components encodes etc.</p>

Binary coded Decimal (BCD) or 8421 codes

Decimal	BCD (8421)
0	0 0 0 0
1	0 0 0 1
2	0 0 1 0
3	0 0 1 1
4	0 1 0 0
5	0 1 0 1
6	0 1 1 0
7	0 1 1 1
8	1 0 0 0
9	1 0 0 1

10 - 1010
 11 - 1011
 12 - 1100
 13 - 1101
 14 - 1110
 15 - 1111

} Indic

* Convert each of the following decimal numbers into BCD.

- i) 9 - 1001
- ii) 7.5 - 0111.0101
- iii) 7287 - 0111 0010 1000 0111
- iv) 26.8 - 0010 0110.1000

x) Find the decimal numbers represented by the following BCD or 8421 codes.

1) 1000 0001 - 81 0101 - 5

2) 0011 1000 0001 - 38.1

3) 1001 0110 0111 000 - 9678

BCD addition

i)
$$\begin{array}{r} 0001 \\ + 0100 \\ \hline 1000 \end{array}$$

$$\begin{array}{r} 1000 \\ + 0001 \\ \hline 1001 \end{array}$$

ii)
$$\begin{array}{r} 1001 \\ + 1001 \\ \hline 0101 \\ + 0111 \\ \hline 1000 \end{array}$$

Here BCD addition is invalid. Since there is a carry generated out of 4th bit. hence BCD addition cannot be performed.

iii)

$$\begin{array}{r}
 & 0100 \\
 + & 0110 \\
 \hline
 0101 & 0100 \\
 \hline
 1001 & 1010
 \end{array}$$

Hence BCD addition is invalid since the sum is greater than 9.

Gray Codes

Binary to Gray

The code which exhibits only a single bit change from 10th code number to the next is known as Gray code.

Binary to Gray

i)

$$\begin{array}{r}
 1 + 001101 \\
 \hline
 1\cancel{0}10101
 \end{array}$$

Binary	gray	decimal
0 0 0 0	0 0 0 0	0
0 0 0 1	0 0 0 1	1
0 0 1 0	0 0 1 1	2
0 0 1 1	0 0 1 0	3
0 1 0 0	0 1 1 0	4
0 1 0 1	0 1 1 1	5
0 1 1 0	0 1 0 1	6
0 1 1 1	0 1 0 0	7
1 0 0 0	1 1 0 0	8
1 0 0 1	1 1 0 1	9
1 0 1 0	1 1 1 1	10
1 0 1 1	1 1 1 0	11
1 1 0 0	1 0 1 0	12
1 1 0 1	1 0 1 1	13
1 1 1 0	1 0 0 1	14
1 1 1 1	1 0 0 0	15

gray to binary

1+0 01011

1 1101011

1001101

* 110110101

100100110

Gray	binary
0 0 00	0000
0 0 01	00010
0 0 10	00110
0 0 11	0010
0 1 00	0110
0 1 01	0111
0 1 10	0101
0 1 11	0100
1 0 00	1100
1 0 01	1101
1 0 10	1111
1 0 11	1110
1 1 00	1010
1 1 01	1011
1 1 10	1001
1 1 11	1000

Alphanumeric Code

The code which contains letters, numbers and other symbols are known as Alphanumeric Codes.

Eg: ASCII (American Standard for Information Interchange)

BCDIC Code (extend binary Code to Information Interchange Code)

ASCII Code

It is a widely used alphanumeric code. It is a 7 bit code in which decimal digits are represented by BCD on 8421 which is preceded by 011.

Eg: 0 → 01100000

7 → 01101110

A → 10000010

B → 10000010

a → 11000010

b → 11000010

Parity check code for error detection

Parity check code is more to widely used error detection code. Parity is noting but the number of 1's in binary.

Decimal	BCD	old parity bit	even parity bit
0	0000	1	0
1	0001	0	1
2	0010	0	1
3	0011	1	0
4	0100	0	1
5	0101	1	0
6	0110	1	0
7	0111	0	1
8	1000	10	0
9	1001	1	0

Error Correction Code - Hamming Codes.

Hamming Code is a 7 bit. Correcting Code in which one data bit can be transmitted this code formed is as follows.

$D_7 \ D_6 \ D_5 \ P_4 \ D_3 \ P_2 \ P_1$,
 where $D_7 \ D_6 \ D_5 \ P_4 \ D_3 \ P_2 \ P_1$ are data bits.
 And P_4, P_2, P_1 is parity bits.

- i) P₁ bit is even Parity bit over D₃ D₅ D₇
 - ii) P₂ bit is even Parity bit over D₃ D₆ D₇
 - iii) P₄ bit is even Parity bit over D₅ D₆ D₇

D₇ D₆ D₅ P₄ D₃ P₂ P₁
 1 1 1 1 1 0 0 0
 * Code checks bits 0110 into a 7 bit even Parity Hamming Code.

Ans:

MODULE - 2

Axiomatic definition of Boolean Algebra.

It is a set of rules, laws and theorems by which all the logical operations can be expressed.

Boolean Theorems / postulates.

- 1) $A + 0 = A$
- 2) $A + 1 = 1$
- 3) $A + A = A$
- 4) $A + \bar{A} = 1$
- 5) $A \cdot 0 = 0$
- 6) $A \cdot 1 = A$
- 7) $A \cdot A = A$
- 8) $A \cdot \bar{A} = 0$
- 9) $\bar{\bar{A}} = A$
- (10) $A + B = B + A$
- (11) $A \cdot B = B \cdot A$
- (12) $(A + B) + C = A + (B + C)$
- (13) $(A \cdot B) \cdot C = A \cdot (B \cdot C)$

14) A C
15) C
16)
17)

$$14) A(CB+C) = ACB + ACD \text{ (distributive law)}$$

$$15) (A+B)C = ACC + BCC$$

$$16) \overline{A+B} = \bar{A} \cdot \bar{B}$$

$$17) \overline{AB} = \bar{A} + \bar{B}$$

Demongani's theorem

The First theorem states that complement of a product of two variables is equal to the sum of the complements of the variables individually.

$$\overline{AB} = \bar{A} + \bar{B}$$

Second theorem states that complement of a sum of two variables is equal to products of the complements of variables individually.

$$\overline{A+B} = \bar{A} \cdot \bar{B}$$

first theorem proof

$$1, \overline{AB} = \bar{A} + \bar{B}$$

A	B	AB	\overline{AB}	\bar{A}	\bar{B}	$\bar{A} + \bar{B}$
0	0	0	1	1	1	1
0	1	0	1	1	0	1
1	0	0	1	0	1	1
1	1	1	0	0	0	0

Second theorem proof

$$2) \overline{A + B} = \bar{A} \cdot \bar{B}$$

A	B	$\overline{A + B}$	$\overline{A + B}$	$\bar{A} \cdot \bar{B}$	$\bar{A} \cdot \bar{B}$
0	0	0	1	1	1
0	1	1	0	0	0
1	0	1	0	0	0
1	1	1	0	0	0

By using boolean theorems Following logical expressions can be obtained.

- 1) $A + AB = A$
- 2) $A(A+B) = A$
- 3) $(A+B)(A+C) = A+BC$
- 4) $A + \bar{A}B = A+B$

Simplification of Boolean Functions Using Boolean theorems

$$A + AB = A$$

$$A(A+B) = A$$

$$(A+B)(A+C) = A+BC$$

$$A + A\bar{B} = A+B$$

- 1) Prove that $AB + BC + \bar{B}C = AB + C$

Ans: $AB + BC + \bar{B}C = AB + C(B + \bar{B})$

$$(AB + C)(B + \bar{B}) = AB + C \cdot 1 (\because B + \bar{B} = 1)$$

$$= \underline{\underline{AB + C}}$$

- 2) Simplify $A\bar{B} + AB + \bar{A}\bar{B}$

Ans: $\bar{A}\bar{B} + AB + \bar{A}\bar{B} = B(\bar{A} + A) + (\bar{A} \cdot B) + \bar{A}\bar{B}$

$$= B + \bar{B}\bar{A} \quad B + \bar{A}\bar{B}$$

$$= B + \bar{B}\bar{A} \quad (\text{Since } B + \bar{B} = 1)$$

$$= \underline{\underline{B + \bar{A}}}$$

3) Simplify $A + A\bar{B} + \bar{A}B$

Ans $A + A\bar{B} + \bar{A}B = A(1 + \bar{B}) + \bar{A}B$
 $= A + \bar{A}B$ ($\because 1 + \bar{B} = 1$)
 $= \underline{\underline{A + B}}$ ($\because A + \bar{A}B = A + B$)

4) Simplify $y = (A+B)(A+B)$

Ans $y = (A+B)(A+B)$
 $= B + \bar{A}A$
 $= B + 0 = \underline{\underline{B}}$ ($\because \bar{A}A = 0$)

5) Simplify $y = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}B\bar{C} + AB\bar{C}$

$$= \bar{C}(\bar{A}\bar{B} + \bar{A}\bar{B} + \bar{A}B + AB)$$

$$= \bar{C}(\bar{A}(\bar{B} + B) + A(\bar{B} + B))$$

$$= \bar{C}(\bar{A} \cdot 1 + A \cdot 1)$$

$$= \bar{C}(\bar{A} + A)$$

$$= \bar{C}$$

6) Simplify $y = \bar{A}\bar{B} + (\bar{A} + \bar{B})C$

$$= \bar{A}\bar{B} + \bar{A}\bar{B}C$$

$$= \bar{A}\bar{B}(1+C) = \bar{A}\bar{B}$$

Sum of Products and products of sums

Product term

A learning boolean expression where one or more uncomplimented or complimented variables are multiplied (ANDed)

Eg: $\bar{A}B$, $A\bar{B}c$, $A\bar{B}\bar{C}$, $\bar{A}D$, $\bar{B}E$

Sum terms

It is a term in boolean expression where one or more complimented or uncomplimented variables are added (ORED)

Eg: $A+B$, $\bar{A}+B$, $\bar{A}+\bar{B}+\bar{C}$, $\bar{B}+\bar{C}+\bar{E}$

Sum of product (SOP)

Boolean expression

It is a type of boolean expression where several product terms are added (ORED) together.

Eg: $\bar{A}B+\bar{B}C+B\bar{C}D$

$\bar{A}D+A\bar{B}\bar{C}+\bar{A}BC$

Product of sum (POS) Boolean expressions

It is a type of boolean expression where several some terms are multiplied (ANDed).

$$\text{eg: } (A+B)(B+C)(\bar{A}+B+\bar{C})$$

$$(\bar{A}+B)(\bar{A}+\bar{B}+\bar{C})(A+\bar{B})$$

Canonical or Standard SOP

- * It is a SOP in which all the product terms contains all the variables in the expression in either Complimented or uncomplimented form.
- * Product terms in canonical \Leftrightarrow SOP are called minterms.

$$Y = A'B + BC + \bar{B}\bar{C} \rightarrow \text{not canonical SOP}$$

$$Y = A'B + \bar{A}B \rightarrow \text{canonical} \rightarrow \text{minterm}$$

$$Y = AB\bar{C} + \bar{A}B\bar{C} + A\bar{B}C \rightarrow \text{canonical} \rightarrow$$

$$Y = A'B + \bar{A}B + AC \rightarrow \text{not canonical SOP}$$

$$Y = AB\bar{C}\bar{D} + A\bar{B}C\bar{D} + \bar{A}BCD$$

Canonical or Standard Pos

It is a pos in which all the sum terms contains all the variables in the expression in either Complemented or uncomplemented forms.

Sum terms in Canonical pos are called maxterms

e.g. $(A+B)$ $(\bar{A}+C)$ - not Canonical pos

$(A+\bar{B}+\bar{C})$ $(\bar{A}+\bar{B}+\bar{C})$ \rightarrow Canonical pos

$(A+\bar{B})$ $(\bar{A}+\bar{B})$ \rightarrow maxterms
 \rightarrow Canonical pos

$(A+\bar{B}+C)$ $(\bar{A}+\bar{B}+\bar{C})$ Non Canonical pos

minterms and maxterms
from truth table

	A	B	C	D	minterm	maxterm
0	0	0	0	0	$\bar{A}\bar{B}\bar{C}\bar{D}$	$A + B + C + D$
1	0	0	0	1	$\bar{A}\bar{B}\bar{C}D$	$A + B + C + \bar{D}$
2	0	0	1	0	$\bar{A}\bar{B}C\bar{D}$	$A + B + \bar{C} + D$
3	0	0	1	1	$\bar{A}\bar{B}CD$	$A + B + \bar{C} + \bar{D}$
4	0	1	0	0	$\bar{A}B\bar{C}\bar{D}$	$A + \bar{B} + \bar{C} + \bar{D}$
5	0	1	0	1	$\bar{A}B\bar{C}D$	$A + \bar{B} + \bar{C} + D$
6	0	1	1	0	$\bar{A}BC\bar{D}$	$A + \bar{B} + \bar{C} + \bar{D}$
7	0	1	1	1	$\bar{A}BCD$	$A + \bar{B} + \bar{C} + D$
8	1	0	0	0	$AB\bar{C}\bar{D}$	$A + \bar{B} + \bar{C} + \bar{D}$
9	1	0	0	1	$AB\bar{C}D$	$A + \bar{B} + C + D$
10	1	0	1	0	$AB\bar{C}D$	$A + \bar{B} + C + \bar{D}$
11	1	0	1	1	$ABCD$	$A + B + \bar{C} + D$
12	1	1	0	0	$AB\bar{C}\bar{D}$	$A + \bar{B} + \bar{C} + \bar{D}$
13	1	1	0	1	$AB\bar{C}D$	$A + \bar{B} + C + \bar{D}$
14	1	1	1	0	$ABC\bar{D}$	$A + \bar{B} + C + \bar{D}$
15	1	1	1	1	$ABC\bar{D}$	$A + \bar{B} + \bar{C} + \bar{D}$

SOP expression = $\bar{A}\bar{B}CD + \dots + A\bar{B}CD$

POS. expression = $(A+\bar{B}+C+\bar{D}) * (\bar{A}+B+C+\bar{D})$

* Obtain the Canonical SOP of the function
 $y = A+B$

Given $y = A+B$

$$= A \cdot 1 + B \cdot 1$$

$$= A(CB + \bar{B}) + B(A + \bar{A})$$

$$= AB + A\bar{B} + AB + \bar{A}B$$

$$= AB + A\bar{B} + \bar{A}B$$

~~AB + A \bar{B} + $\bar{A}B$~~

~~AB + A \bar{B} + $\bar{A}B$~~

8421

3 2 1

1 1
1 0
0 1

$$= \sum m(1, 2, 3)$$

Obtained the Canonical CF of the function

$$y = A + BC$$

$$= A \cdot 1 \cdot 1 + B C \cdot 1$$

$$= A \cdot (B + \bar{B}) + B C \cdot (C + \bar{C}) + B C (A + \bar{A})$$

$$= AB + A\bar{B} \cdot (C + \bar{C}) + B C + \bar{A} B C$$

$$= ABC + A\bar{B}C + A\bar{B}C + A\bar{B}\bar{C} + A\bar{B}C + \bar{A}BC$$

Obtain the canonical SOP of the function

$$Y = AB + \overline{B}CD$$
$$= AB \cdot 1 \cdot 1 + \overline{B}CD \cdot 1$$

$$= AB(C + \bar{C})(D + \bar{D}) + ACD + (B + \bar{B})$$

$$= ABCD + AB\bar{C}\bar{D}$$

$$= AB(C + \bar{C})(D + \bar{D}) + ACD + (B + \bar{B})$$

$$= ABC + AB\bar{C}(D + \bar{D}) + A\bar{B}CD + A\bar{B}\bar{C}D$$

$$= ABCD + AB\bar{C}\bar{D} + A\bar{B}\bar{C}D + A\bar{B}\bar{C}\bar{D} + A\bar{B}CD + A\bar{B}C\bar{D}$$

$$= ABCD + AB\bar{C}\bar{D} + A\bar{B}\bar{C}D + A\bar{B}\bar{C}\bar{D} + A\bar{B}CD + A\bar{B}C\bar{D}$$

$$= ABCD + AB\bar{C}\bar{D} + A\bar{B}\bar{C}D + A\bar{B}\bar{C}\bar{D} + A\bar{B}CD + A\bar{B}C\bar{D}$$

Obtain the Canonical POS expression

$$Y = (A+B)(B+C)(A+\bar{C})$$

$$= (A+\bar{B}+0)(B+C+0)(A+\bar{C}+0)$$

$$(A+\bar{B}+0)(A+\bar{B}+\bar{C})(B+C+0)$$

$$(B+C+0)(A+\bar{B}+C\bar{C})(B+C+C)$$

$$(A+\bar{C}+B)1$$

$$(A\bar{B} + \bar{B} + C) (A + \bar{B} + \bar{C}) (B\bar{C} + A)$$

$$(\bar{B} + \bar{C} + \bar{A}) (A\bar{C} + \bar{B}) (A + \bar{C} + \bar{B})$$

$$= (A + \bar{B} + C) (A + \bar{B} + \bar{C}) (A + B + C)$$

$$(\bar{A} + B + C) (A + B + C) (\bar{A} + \bar{B} + \bar{C})$$

Ans 0, 1, 2, 3, 4
Ans 0, 1, 2, 3, 4

Obtain the Canonical POS expression.

$$Y = (A + B) (\bar{A} + \bar{B}) (\bar{B} + \bar{C})$$

$$(\bar{A} + B + \bar{C}) (A + \bar{B} + \bar{C}) (\bar{B} + \bar{C} + A\bar{B})$$

$$(A + B + C) (\bar{A} + B + \bar{C}) (\bar{A} + \bar{B} + C)$$

$$(A + \bar{B} + C) (\bar{B} + \bar{C} + A) (\bar{B} + \bar{C} + \bar{A})$$

$$= (\overset{0}{A} \overset{0}{B} \overset{0}{C}) (\overset{0}{A} \overset{0}{B} \overset{1}{C}) (\overset{0}{A} \overset{\bar{1}}{B} \overset{0}{C})$$

$$(\overset{0}{A} \overset{1}{B} \overset{1}{C}) (\overset{0}{A} \overset{1}{B} \overset{1}{C}) (\overset{1}{A} \overset{1}{B} \overset{1}{C})$$

$$\Rightarrow (\cancel{0}; \cancel{0}; \cancel{0}) (\overset{0}{0})$$

$$= (\overset{1}{A} \overset{0}{B} \overset{0}{C}) (\overset{1}{A} \overset{0}{B} \overset{1}{C}) (\overset{1}{A} \overset{1}{B} \overset{0}{C})$$

$$(\overset{1}{A} \overset{1}{B} \overset{0}{C}) (\overset{1}{A} \overset{1}{B} \overset{1}{C})$$

$$\Pi_m = \{0, 1, 2, 3, 7\}$$

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000
001
010
011
111

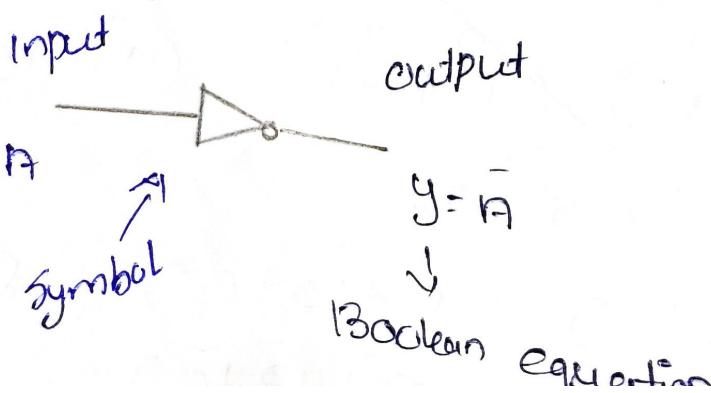
logic gates

- * logic gates is a electronic circuit which makes logic decisions
- * logic Gates are classified into basic gates and universal gates.

Basic Gates

OR, AND, NOT gates are called basic gates. On basic building blocks of a digital system

NOT Gate



Input	Output
A	$y = \bar{A}$
0	1
1	0

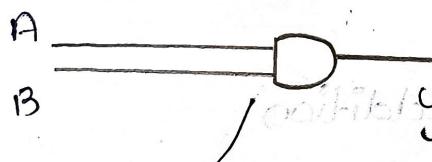
$$Y = \bar{A} \cdot 1 + A \cdot 0$$

$$= \bar{A} + 0$$

$$= \underline{\bar{A}}$$

- * Note Gate has only one input and one output.
- * It performs a basic logic function called inversion or complementation.
- * In this gate logic high appears at output only when input is logic low

And Gate



Input	Output
A B	0 0
	0 1
	1 0
	1 1

~~$$Y = A B \cdot 1 = \underline{A B}$$~~

- * And Gate has two or more inputs and one output
- * It performs logical multiplication.
- * Here output is logic high only when all the inputs are logic high

OR gate



and $y = A + B$
Symbol Boolean equation

Input		Output
A	B	y
0	0	0
0	1	1
1	0	1
1	1	1

$$y = \overline{A}\overline{B} + A\overline{B} + A\overline{B}$$

$$= B(A + \overline{A}) + A\overline{B}$$

$$= B + A\overline{B}$$

$$\text{and } \overline{B} + A = \overline{B}(\overline{A} + 1) + A = \overline{B} + A = A + B$$

* OR gate has two or more inputs and one output

* It performs logical addition

* Here output is logic high only when any of the input is high.

Universal Gates

NAND and NOR gates are called universal gates because all other gates can be realised using NAND gates only and NOR gates only.

NAND gate



Symbol

$$y = \overline{AB}$$

↑

Boolean equation

Input	Output			
A	B	y = \overline{AB}		
0	0	1		
0	1	1		
1	0	1		
1	1	0		

- * NAND gate is nothing but nested AND gate

- * It has two or more inputs and one output
- * Hence output is logic high whenever either one of the input is logic low.

NOE gate



Symbol

Boolean equation
 $y = \overline{A+B}$

Input	Output			
A	B	y = $\overline{A+B}$		
0	0	1		
0	1	0		
1	0	0		
1	1	0		

- * NOE gate is nothing but nested OR gate
- * It has two or more inputs and one output
- * Hence output is logic high whenever all the inputs are logic low.

Exor gate

A
B



$$y = A \oplus B$$

Symbol

$$= \bar{A}\bar{B} + A\bar{B}$$

↑
Input signal

Boolean equation

3)

Input		Output
A	B	$y = A \oplus B$
0	0	0
0	1	1
1	0	1
1	1	0

Exnor gate

A
B



$$y = A \oplus B$$

Symbol

$$= \bar{A}\bar{B} + A\bar{B}$$

Input signal
Boolean equation

Input		Output
A	B	$y = A \oplus B$
0	0	1
0	1	0
1	0	0
1	1	1

Realisation

of basic gates using

gates only

1)

Not gate using NAND Gate only



$A \oplus \bar{A}$



$$y = \bar{A}$$

2)

AND gate using NAND Gate only.



A

B

$$y = A \bar{B}$$

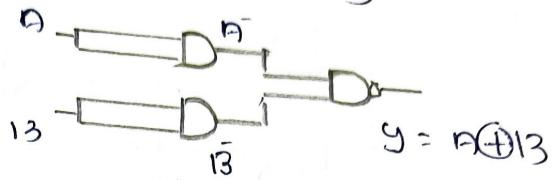
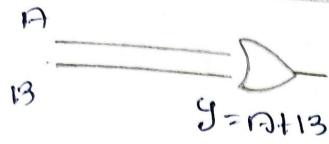


$\bar{A} \bar{B}$



$$y = \bar{A} B$$

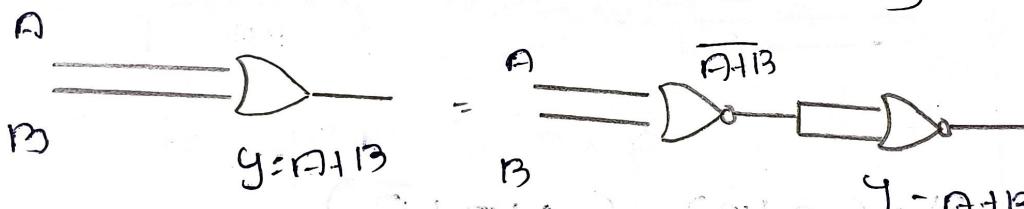
3) OR gate using NAND gate only



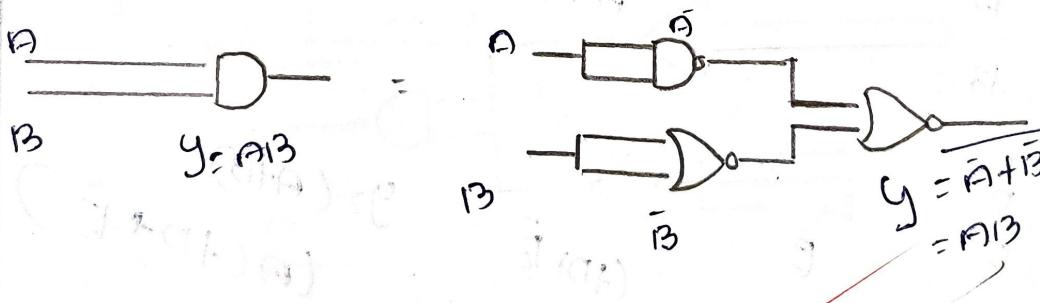
Realisation of basic gates using NOR gates only.



OR gate using NOR gates only

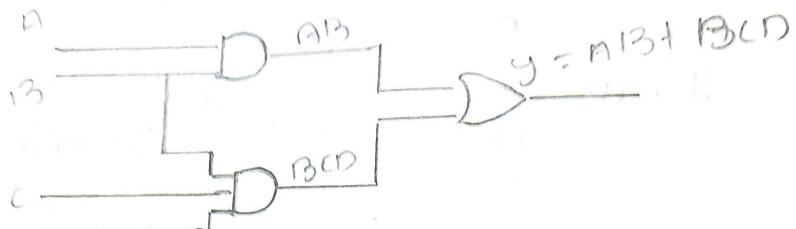


AND gate using NOR gate only.

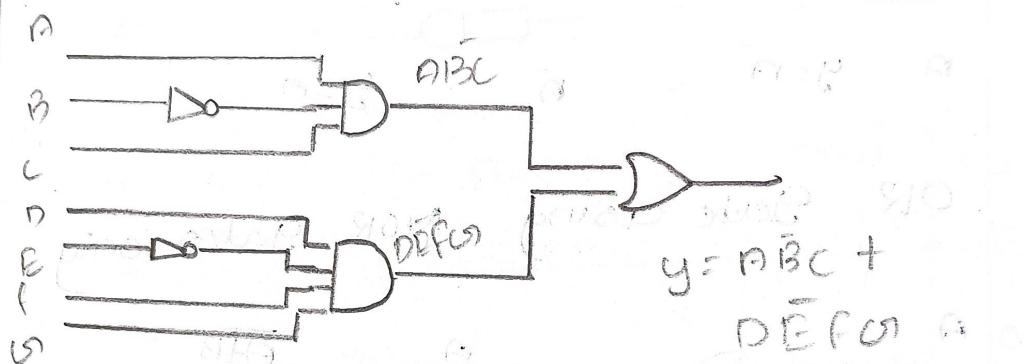


Implementation of logic circuits from boolean expressions.

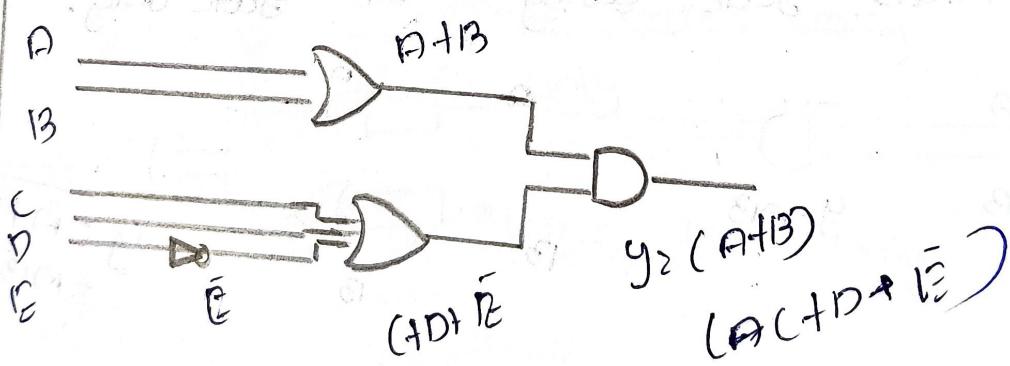
i) $y = AB + \bar{A}B$ (CN)



2) $Y = A\bar{B}C + D\bar{E}F$



3) $Y = (A+B)(C+D+\bar{E})$



2)

Umwandlung von additivem Logik
Schaltung in multiplikativen Form

Logikschaltung

3)

Karnaugh mapping (K map)

- * Karnaugh map is a graphical tool for finding the Sop or pos Simplification of a boolean function.
- * A k map work by arranging the terms of expression in such a way that variables can be cancelled by grouping minterms or maxterms

2 variable K map.

1)

		m_0	m_1
		0	1
A	0	1	0
	1	0	0

$$Y = \bar{A} \cdot \bar{B}$$

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

2)

		m_0	m_1
		0	1
A	0	1	0
	1	1	0

$$Y = \bar{B}$$

A	B	Y
0	0	1
0	1	0
1	0	1
1	1	0

3)

		m_0	m_1
		0	1
A	0	1	1
	1	1	0

$$Y = B + \bar{A}$$

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

4)

A	B	C
0	0	0
0	1	1
1	1	1

$$y = 1$$

A	B	C	y
0	0	0	1
0	0	1	1
0	1	0	1
1	0	0	1

3 variable K map (Sop)

A \ BC	00	01	11	10
0	m ₀	m ₁	m ₃	m ₂
1	m ₄	m ₅	m ₇	m ₆

- * Minimize the following function using K map

$$y = \sum m(1, 2, 3, 5, 7)$$

A \ BC	00	01	11	10
0	0	m ₁	m ₃	m ₂
1	m ₄	0	m ₇	m ₆

$$y = C + \bar{A}B$$

$$\bar{A} = 0$$

$$A = 1$$

- * Use a K map to minimize following expression

$$y = \bar{A}\bar{B}C$$

$$+ A\bar{B}\bar{C} + \bar{A}B\bar{C} + AB\bar{C}$$

$$\sum m(6, 1, 4, 5)$$

A \ BC	00	01	11	10
0	1	1	0	0
1	1	1	0	0

char
complement
value

$$y = \bar{B}$$

4 variable K map

A13 \ C0		00	01	11	10
00	m0	m1	m3	m2	
01	m4	m5	m7	m6	
11	m12	m13	m5	m14	
10	m8	m9	m11	m10	

Simplify the following expression using K map

$$Y = m_1 + m_3 + m_5 + m_7 + m_9 + m_{12} + m_{13}$$

A13 \ C0		00	01	11	10
00	m0	m1	m3	m2	
01	m4	m5	m7	m6	
11	m12	m13	m5	m14	
10	m8	m9	m11	m10	

$$Y = \overline{m_4} \overline{m_6} \overline{C_0} + \overline{A} + A_1 \overline{C}$$

Simplify the following expression using K map

$$Y = \Sigma m (3, 2, (0, 11))$$

AB\CD	00	01	11	10
00	m ₀	m ₁	m ₃	m ₂
01	m ₄	m ₅	m ₇	m ₆
11	m ₂	m ₃	m ₅	m ₄
10	m ₈	m ₉	m ₁₀	m ₁₁

fixed
m₂ m₃

o Boolean - 2

$$Y = \overline{E} \overline{A} \overline{B} \overline{C} \overline{D}$$

Simplify the boolean equation using
K mapping & general sets

$$Y = \sum m(8, 10, 11, 12, 13, 14, 15)$$

AB\CD	00	01	11	10
00	m ₀	m ₁	m ₃	m ₂
01	m ₄	m ₅	m ₇	m ₆
11	m ₂	m ₃	m ₅	m ₄
10	m ₈	m ₉	m ₁₀	m ₁₁

$$Y = \overline{A} + \overline{B} \overline{C} \overline{D}$$

$$Y = \sum m(8, 10, 11, 12, 13, 14, 15) + \sum d(9)$$

AB\CD	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	1	1	1	1
10	1	1	1	1

$$Y = A$$

$$Y = \overline{\pi} m(9, 10, 11, 12, 13, 14, 15)$$

$$SOP = \Sigma m(0, 1, 2, 3, 4, 5, 6, 7, 9)$$

$$\gamma = \overline{\pi} m(6, 2) = \Sigma m(1, 3)$$

Module - 3

Co 3 - Design Combinational Circuits

Companision Of Combinational And Sequential Circuits

Combinational	Sequential
* Here the output depends on present input only.	Here the Output depends on present Input as well as Previous output.
* The Circuit is time independent and it is very fast in operation.	Circuit is time independent and comparatively slowly operation.
* There is no feedback between past bw output and input	There is a Feedback past bw output and input.
* Combination main building of combination circuit is logic gates.	main building combination circuit is flip flops.

Combination Circuit
Circuit mainly used
for mainly arithmetic
matic and boolean
operation

Sequential Circuit
Circ used for data
Storing

It doesn't have
block clock

It does have block
clock

Operation of combi
nation is very
Simple

Operation Sequential
is very complex

Eg: Adder, Sub
tractors, en
Codes, decodes,
multiplexers,
demultiplexers

Eg: Flip flops, Shift
Registers, counters, etc.

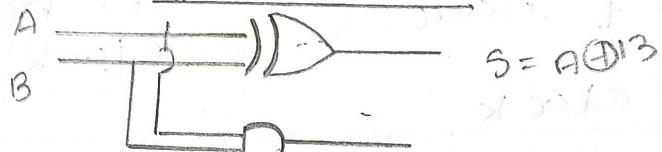
Half Adder

It is a logic circuit that adds
two binary bits.

It has two input A, b and two
output (all sum(s) and carry(c))

Truth Table			
Input	Output		
A	B	S	C
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

Half adder circuit



$$S = A \oplus B$$

$C = A B$ ~~and~~

A hand-drawn logic circuit diagram illustrating the expression $S = \bar{P}\bar{B} + \bar{P}\bar{A}\bar{B}$. The circuit consists of four logic gates: two NOT gates (inverted triangles), one AND gate (rectangle with a dot), and one OR gate (rectangle without a dot). The inputs are labeled P , A , and B . The first NOT gate takes input P and produces output \bar{P} . The second NOT gate takes input B and produces output \bar{B} . The AND gate takes inputs \bar{P} and \bar{B} and produces output $\bar{P}\bar{B}$. The OR gate takes inputs \bar{P} and $\bar{A}\bar{B}$ and produces output S .

full acceleration

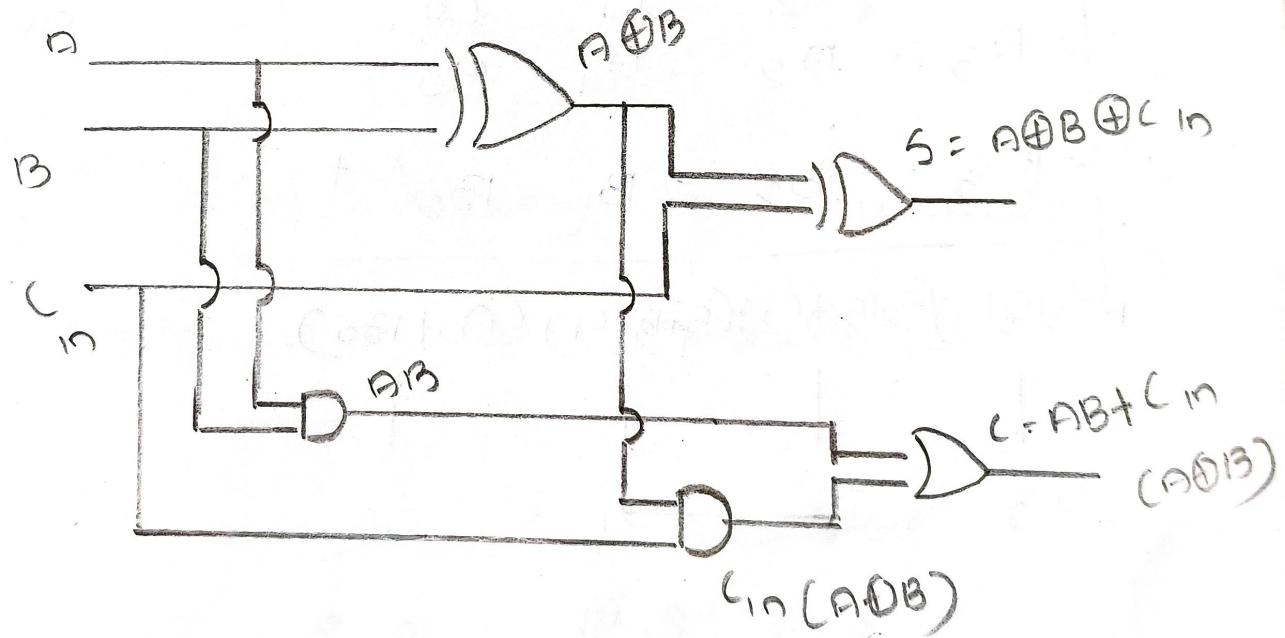
It is a logic circuit has three binary bits.

It has 3 inputs a, b, c and two outputs called sum and carry.

Input		Output		
A	B	C_{in}	S	C
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

$$S = A \oplus B \oplus C$$

$$C = AB + C_{in}(A \oplus B)$$



4 bit parallel adder

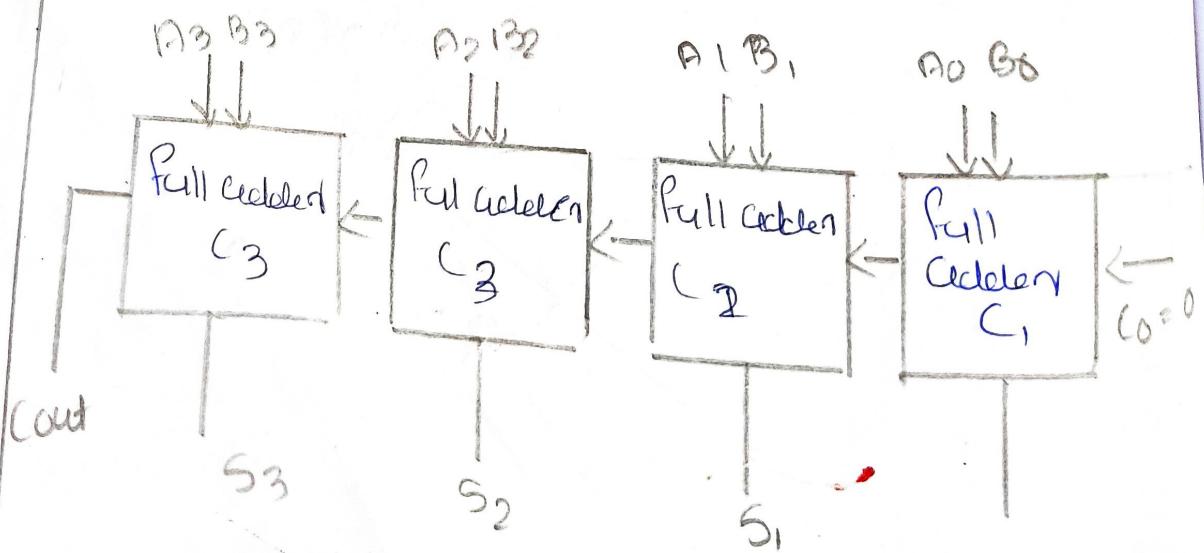
my se

- * To add two four bit numbers four bit parallel adder uses four full adder circuit.
- * Output carry from one full adder connected to input carry of the next full adder.
- * Let $A_3 A_2 A_1 A_0$ and $B_3 B_2 B_1 B_0$ be the two four bit numbers

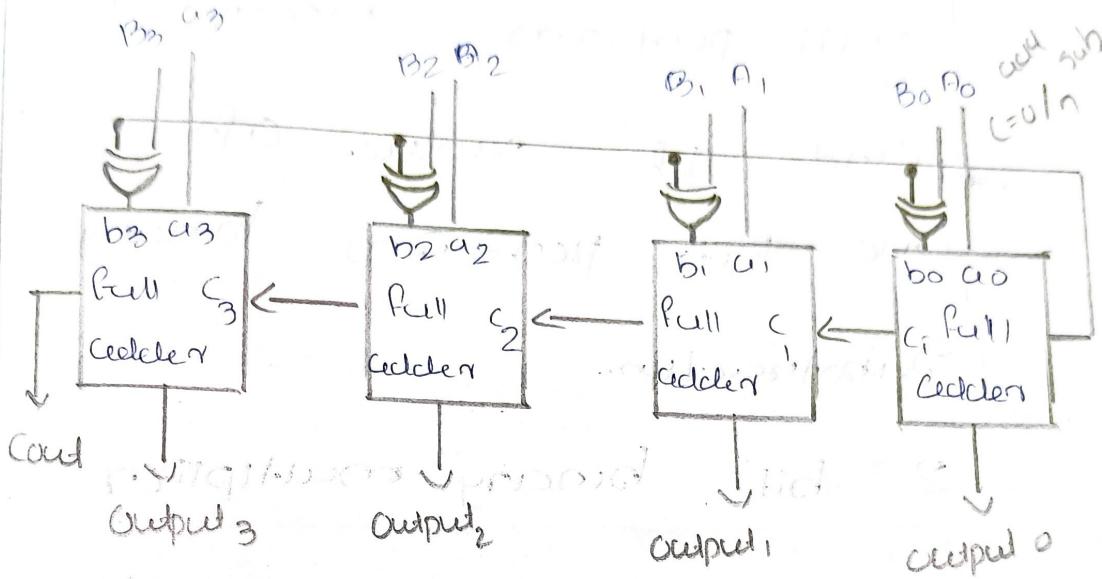
$$\begin{array}{r} B_3 \quad B_2 \quad B_1 \quad B_0 \\ + \end{array}$$

$$\text{out} (A_3 + B_3 + C_3) (A_2 + B_2 + C_2) (A_1 + B_1 + C_1) (A_0 + B_0)$$

$$\begin{array}{r} | & | & | & | \\ S_3 & S_2 & S_1 & S_0 \end{array}$$



4 bit binary adder / Subtractor



- * A bit binary is calculate both addition and subtraction.
- * The circuit consists of four full adder and a control line c which takes either zero or one.

* If receiver of C zero circuit
will perform binary addition
and If receiver of C is
one bit perform binary
Subtraction.

2 bit binary multiplication

$$A_1 A_0 \times$$

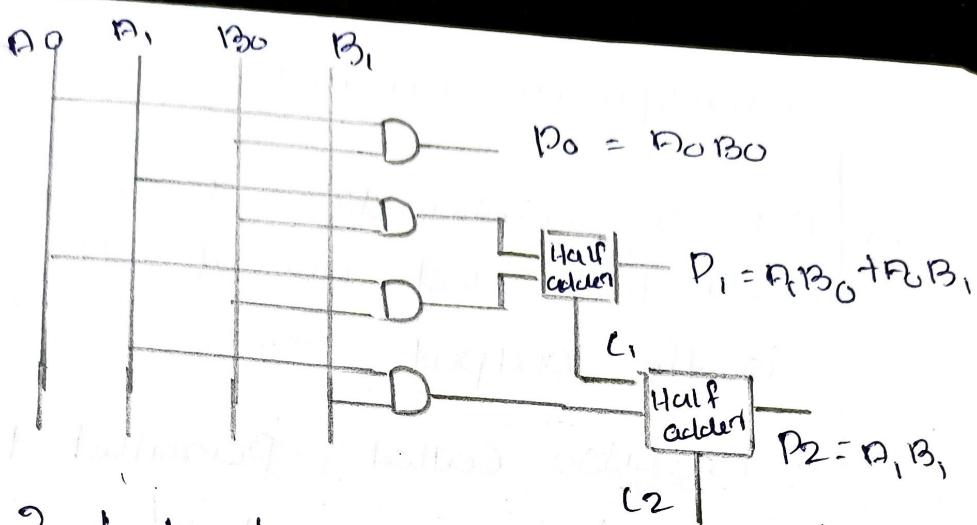
$$B_0 B_1 B_0$$

$$\begin{array}{r} \text{After subtraction of } A_1 \text{ from } A_1 B_0 \\ \hline A_1 B_0 \quad A_0 B_0 \\ \text{After subtraction of } B_0 \text{ from } A_0 B_0 \\ \hline A_1 B_1 \quad A_0 B_1 \end{array}$$

$$C_2 \quad A_1 B_1, \quad A_1 B_0 A_0 B_0$$

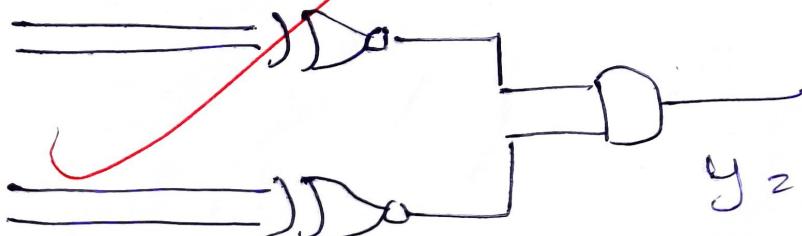
$$\begin{array}{c} \downarrow \\ P_3 \end{array} \quad \begin{array}{c} \downarrow \\ P_2 \end{array} \quad \begin{array}{c} \downarrow \\ P_1 \end{array} \quad \begin{array}{c} \downarrow \\ P_0 \end{array}$$

method of 3 stages will be followed



- * A 2 bit binary multiplier of a complex lesson lesson for two binary numbers.
 - * And gate is basic multiplier which is used for one bit multiplication
- Magnitude Complicated Digital Comparison

Digital comparators use to compare the magnitude of two numbers. The circuit determine where two numbers equal or not. NOR gate basic comparator its output is high when the two inputs are not equal and output is low when the input are equal.



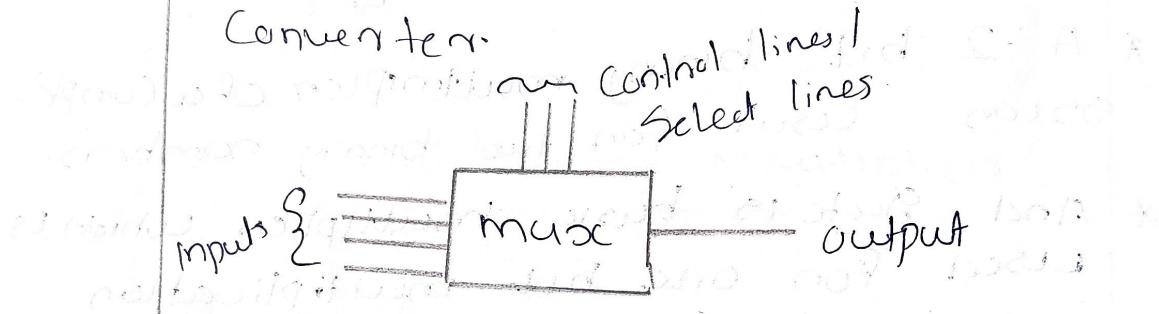
$$y = 1 \text{ for } \neq$$

$$y = 0 \text{ for } \neq$$

Multiplexer (Mux)

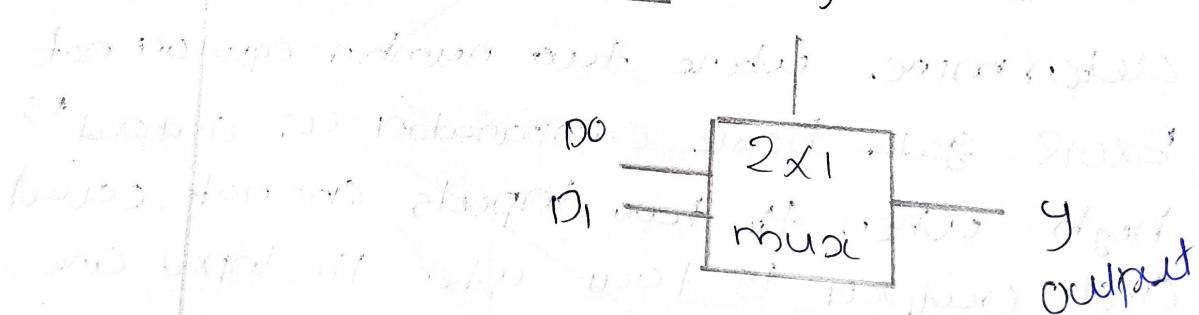
\bar{s} - HD
 s - AND
 t - OR

- * It is a combination logic circuit used to select one of its input as the output.
- * It is also called Parallel to Serial Converter.



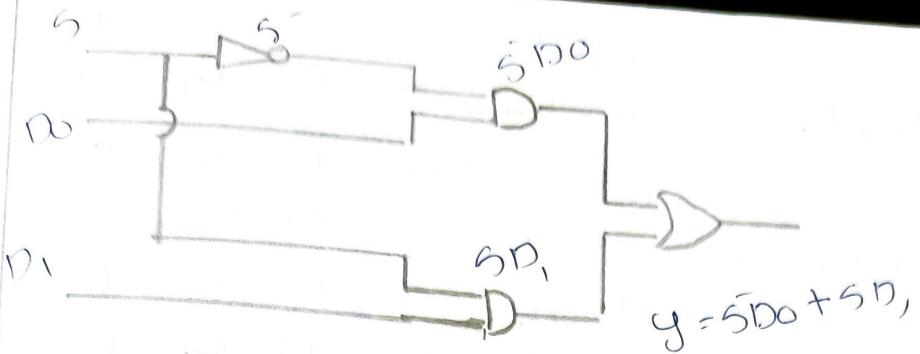
number of inputs = $2^{\text{number of control lines}}$

2x1 mux



Input	Output
0	D_0
1	D_1

Implementation: $Y = \bar{s} D_0 + s D_1$



two inputs and one output

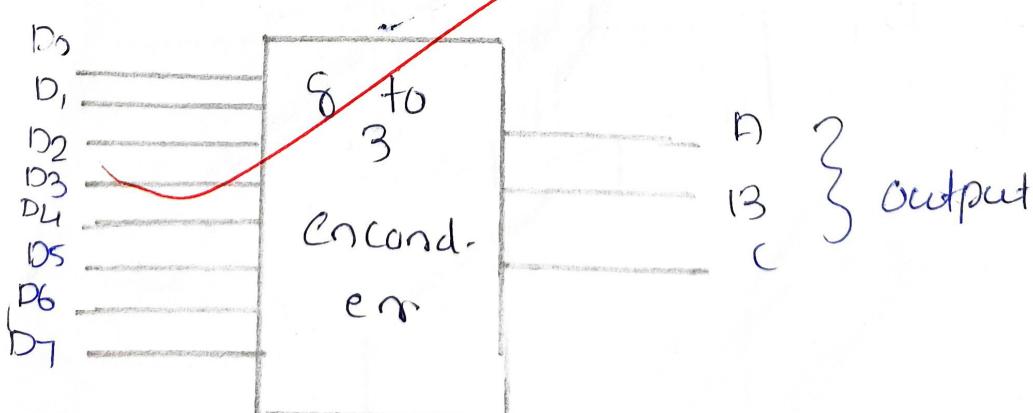
Encoder

It is a combination circuit which accepts one of its inputs which represent a digit and converts it to a coded outputs such as binary.

Here number of digits

No. of inputs = 2. no. of outputs.

8 to 3 encoder (Octal to binary encoder)



Inputs								Output	
D ₀	D ₁	D ₂	D ₃	D ₄	D ₅	D ₆	D ₇	A	B
1	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	1
0	0	0	1	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0	0
0	0	0	0	0	1	0	0	0	1
0	0	0	0	0	0	1	0	1	0
0	0	0	0	0	0	0	1	1	1
0	0	0	0	0	0	0	0	1	1

$$A = D_4, D_5, D_6, D_7$$

$$B = D_2, D_3, D_6, D_7$$

$$C = D_1, D_3, D_5, D_7$$

D₀

D₁

D₂

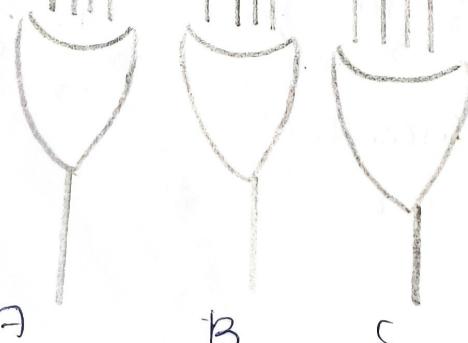
D₃

D₄

D₅

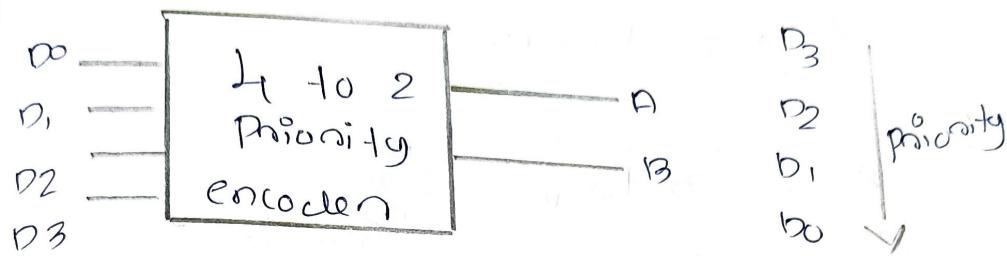
D₆

D₇



	Output			
	D ₃	D ₂	D ₁	D ₀
7	0	0	1	0
6	0	1	0	1
5	1	0	1	0
4	0	0	1	1
3	0	0	0	1
2	1	0	0	1
1	1	1	0	0
0	1	1	0	0

4 to 2 priority encoder



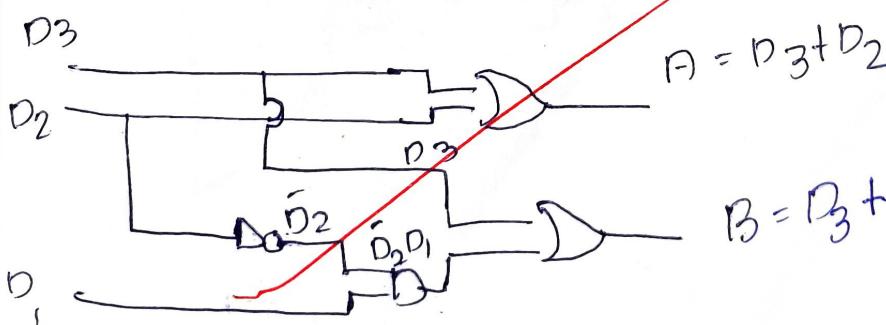
Input				Output	
D ₃	D ₂	D ₁	D ₀	A	B
0	0	0	0	X	X
0	0	0	1	0	0
0	0	1	X	0	1
0	1	X	X	1	0
1	X	X	X	1	1

$$A = \overline{D_3} D_2 + D_3$$

$$= \underline{\underline{D_3 + D_2}}$$

$$B = \overline{D_3} \overline{D_2} \overline{D_1} + D_3$$

$$= \underline{\underline{D_3 + \overline{D_2} D_1}}$$

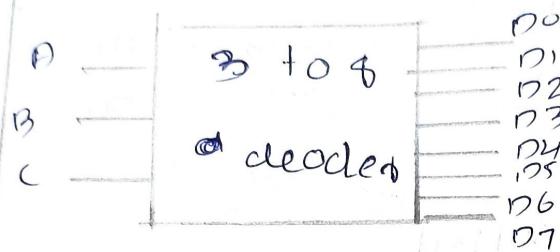


$$A = D_3 + D_2$$

$$B = D_3 + \overline{D_2} D_1$$

Decoded

3 to 8 decades



$$D_0 = \bar{A}\bar{B}C$$

$$D_1 = \bar{A}\bar{B}\bar{C}$$

$$D_2 = \bar{A}B\bar{C}$$

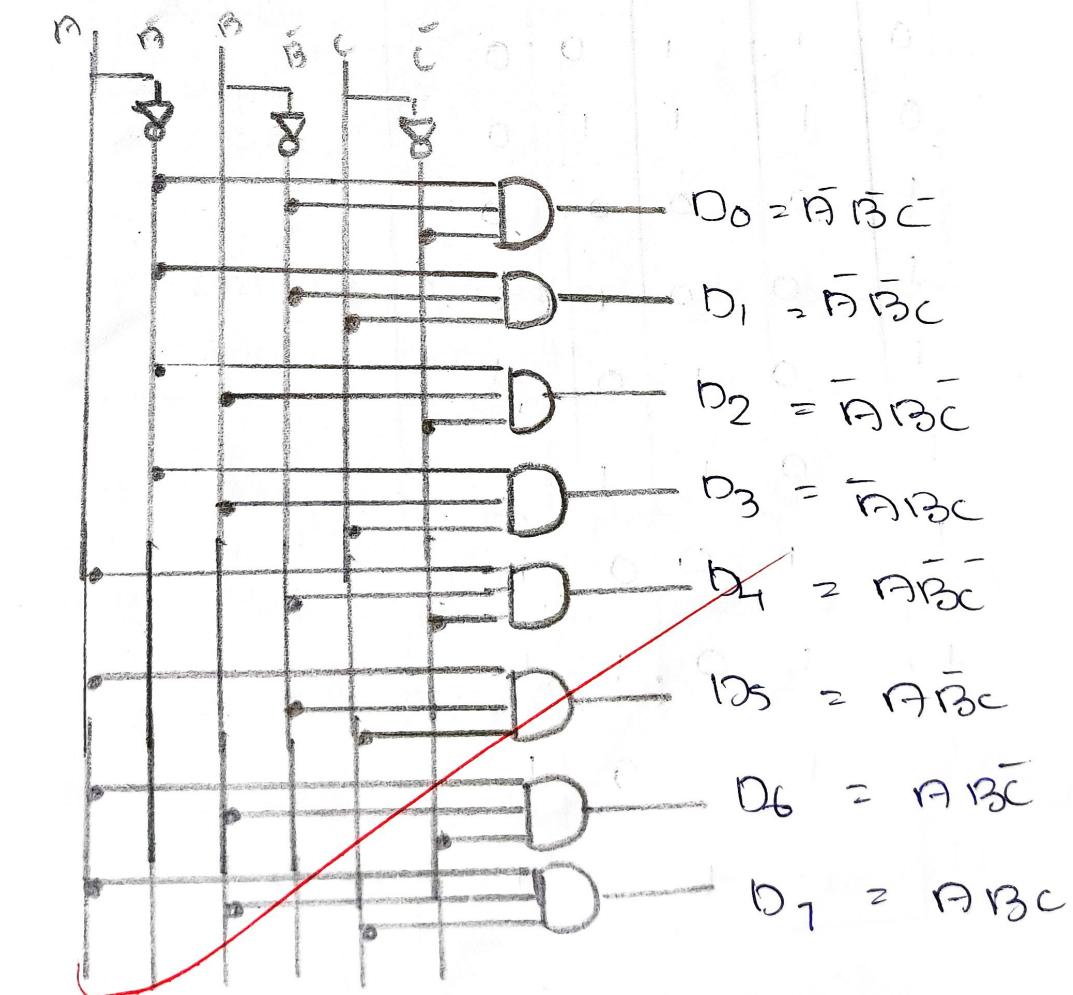
$$D_3 = \bar{A}\bar{B}C$$

$$D_4 = A\bar{B}\bar{C}$$

$$D_5 = A\bar{B}C$$

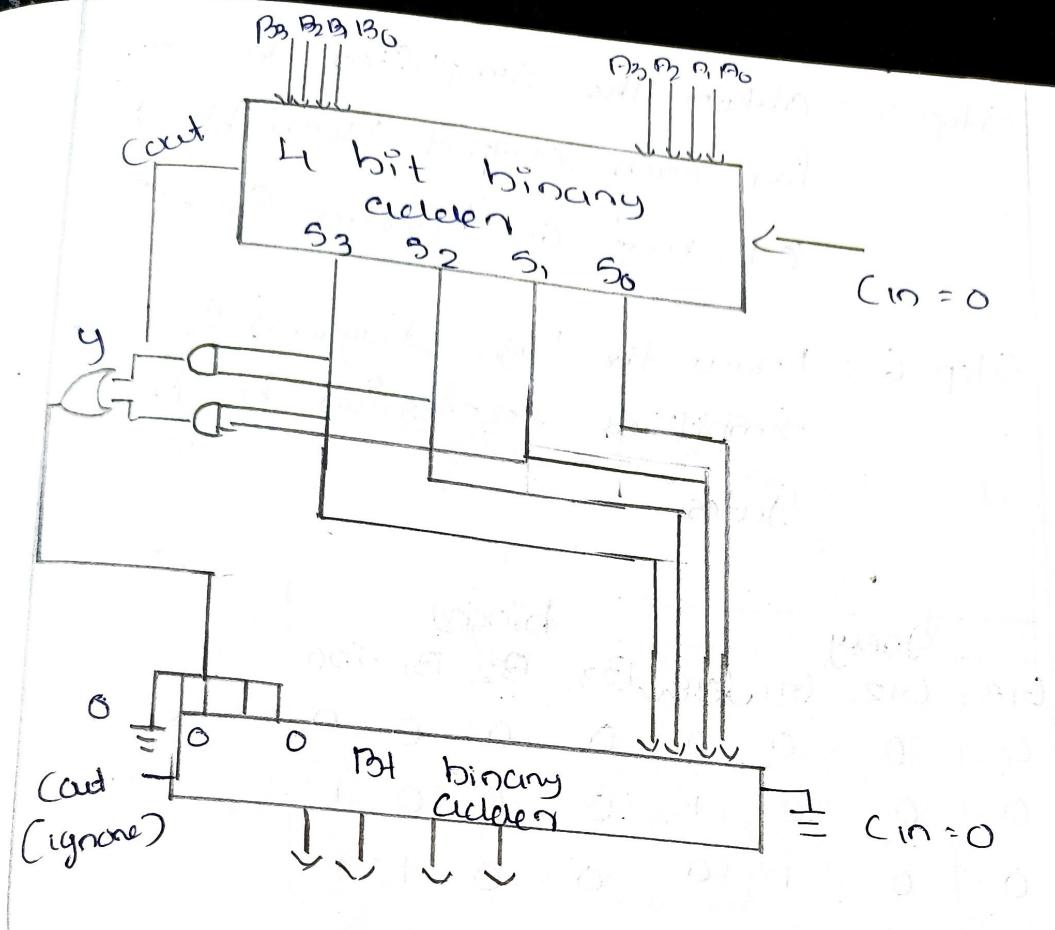
$$D_6 = AB\bar{C}$$

$$D_7 = ABC$$



BCD Adder

Input				Output
s_3	s_2	s_1	s_0	y
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1



Design and analysis procedure of combinational logic circuits

Step 1 : Analyze the given problem

Step 2 : Determine the number of input and output variables

Step 3 : Assigning letter symbols to input and output variables.

Step 4 : Derive the truth table indicating the relationship b/w input and output variables.

Step 5 : Obtain the simplified boolean functions for each output variable by using K-map or boolean Simplification.

Step 6 : Draw the logic diagram for the above Simplified expression by using logic Gates.

Gray				Binary			
G ₃	G ₂	G ₁	G ₀	B ₃	B ₂	B ₁	B ₀
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	1
0	0	1	0	0	0	1	1
0	0	1	1	0	0	1	0
0	1	0	0	0	1	0	0
0	1	0	1	0	1	1	1
0	1	1	0	0	1	0	1
0	1	1	1	0	1	0	0
1	0	0	0	1	1	1	1
1	0	0	1	0	1	0	1
1	0	1	0	1	1	1	0
1	0	1	1	1	1	0	0
1	1	0	0	1	0	1	0
1	1	0	1	1	0	1	1
1	1	1	0	1	0	0	1
1	1	1	1	1	0	0	0

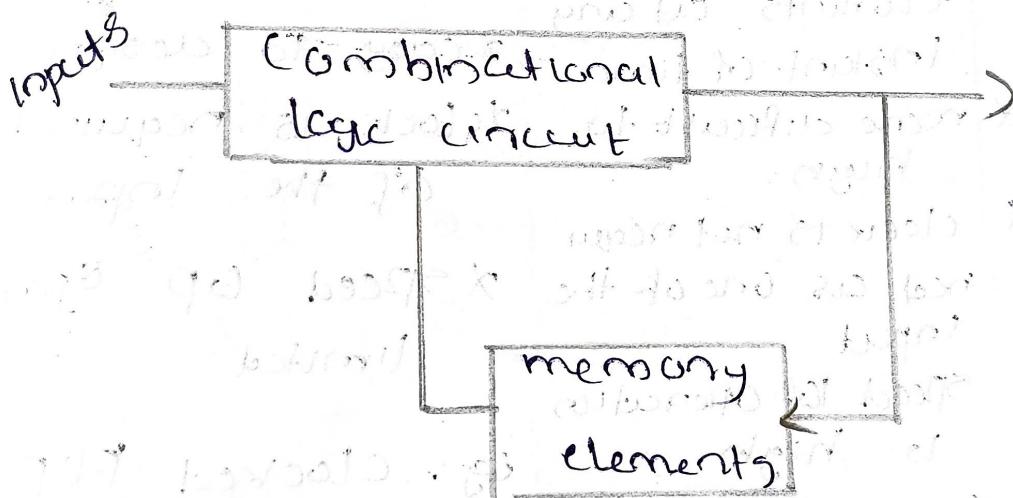
functions
using
equation.

above
logic

Module - 4

Combinational & Sequential

Combinational Circuit



Block diagram of

Sequential Circuit

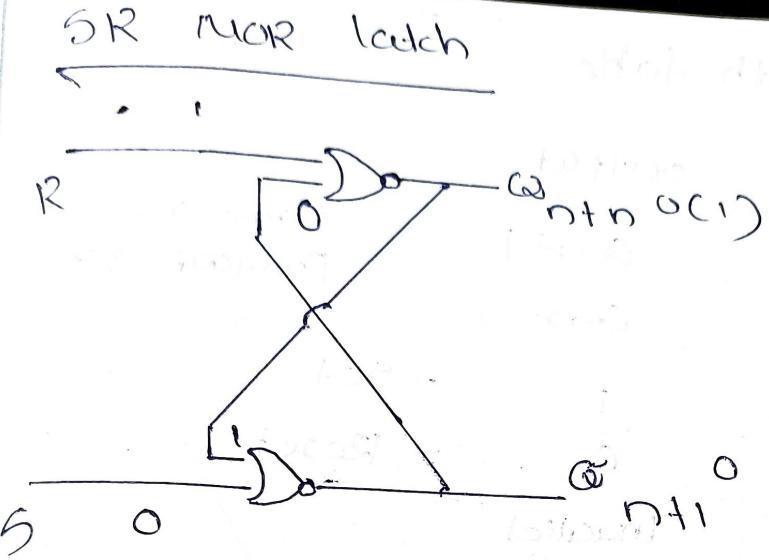
~~Sequential Circuit is digital Circuit whose output depends on present input as well as past output.~~

Comparison of Synchronous and Asynchronous Sequential Circuit

Asynchronous sequential circuit	Synchronous Sequential Circuit
* Input signal can affect memory elements at any instant of time	Input signals can affect memory at discrete instant of time
* more difficult to design	* easy to design
* clock is not required as one of the input	* declock is required as one of the input
* Speed of operation is high	* speed of operation limited
eg.: unclocked flipflops	eg.: Clocked flipflops

Latches (Unclocked flipflops)

- * It is the simplest kind of Sequential Circuit
- * It is capable of storing bit of information that is either logic 1 or logic 0.



$$R = 1 = S_0 \quad Q_{n+1} = 0$$

$$Q_{n+1} = 0$$

$$R = 1 \quad S = 0 \quad Q_n = 1$$

$$Q_{n+1} = 0$$

$$Q_{n+1} = 0$$

NOR

Input		Output
A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

Case - 4

$$R = 1 \quad S = 1$$

$$Q_{n+1} = \text{Indef}$$

Truth table

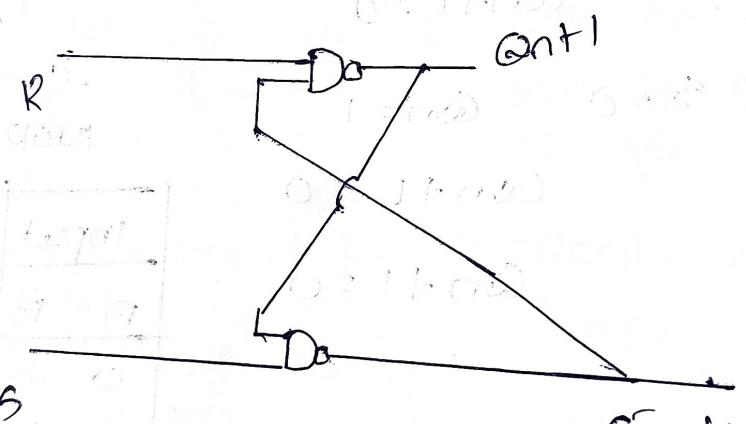
input	output
R S	Qnt1
0 0	Qn0
0 1	1
1 0	0
1 1	Invalide

→ Same as
Previous O/P

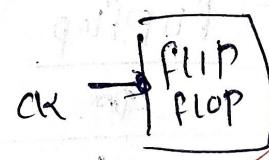
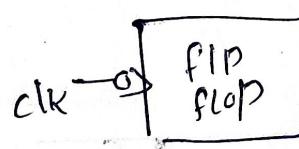
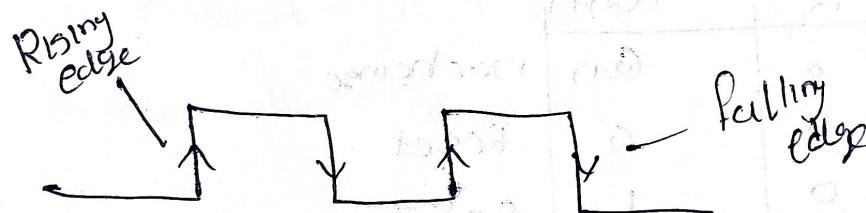
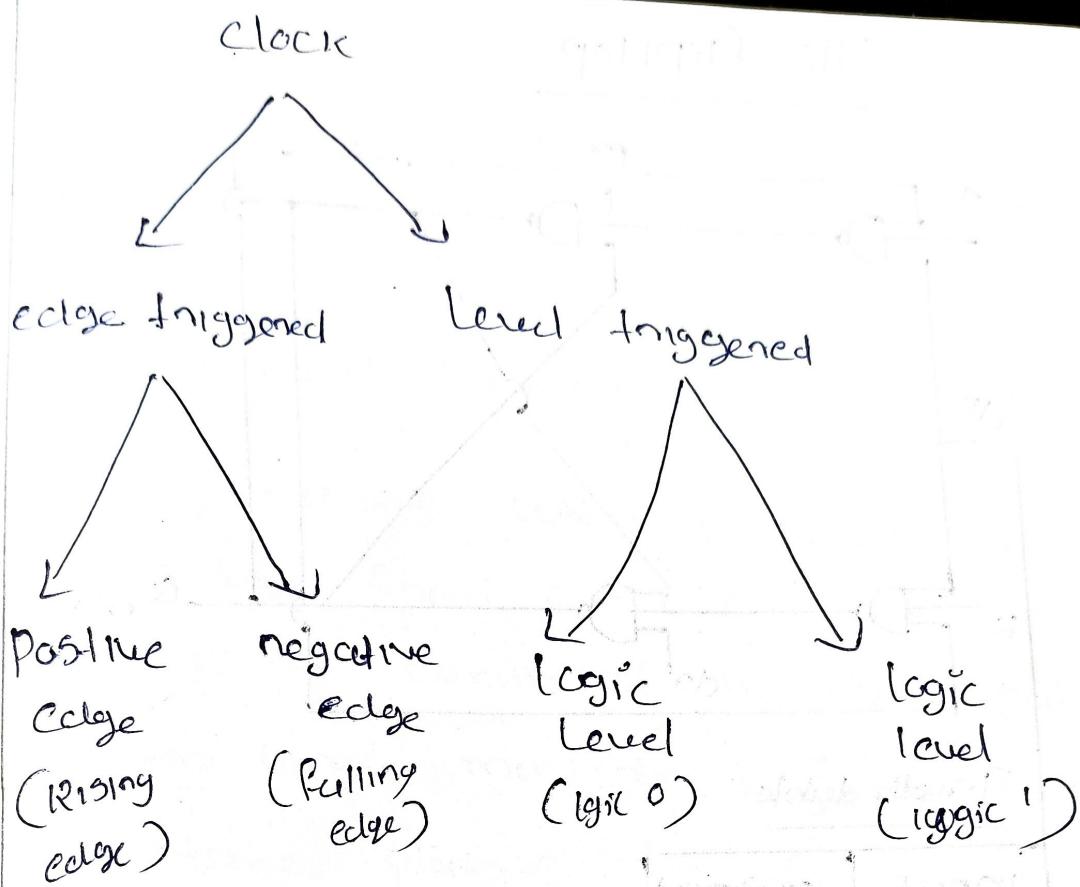
- set

- Reset

SIR AND Clatch



Truth table



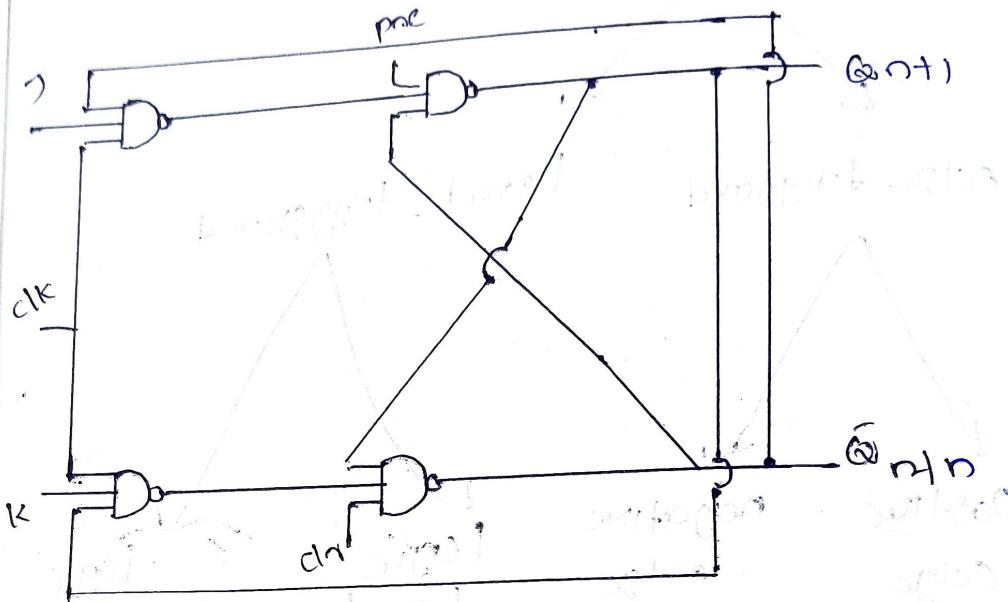
Symbol of
positive edge
triggered
flip flop

Symbol of
negative
edge triggered
flip flop

Symbol of
logic level
triggered /
Positive Level

Symbol
of logic
level trig-
gered / nega-
tive Level

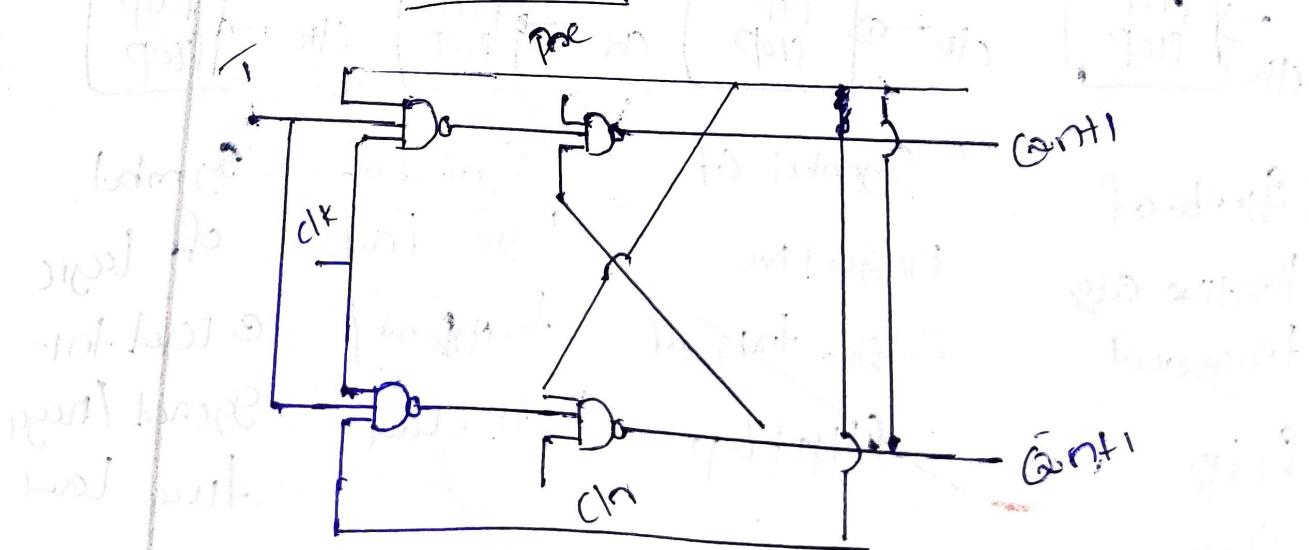
JK flipflop



Truth table

Input	Output	
J	1K	Qn+1
Q	0	Qn
0	1	No change
1	0	Reset
1	1	Set
		Qn
		toggle

T flipflop



Input	Output
T	Qent
0	Qen
1	\bar{Q}_{en}

No change

Toggle

Race Around Condition

- * In JK Flipflops when $J = K = 1$ the output will toggle from 0 to 1 or 1 to 0. But if the width of the clock pulse is too long multiple toggling will not take place.
- * As a result it will be difficult to determine the state of output.
- * To overcome this two methods are commonly used
 - 1) Use of negative edge triggering
 - 2) Use of master slave JK flipflops

master slave JK flip flop

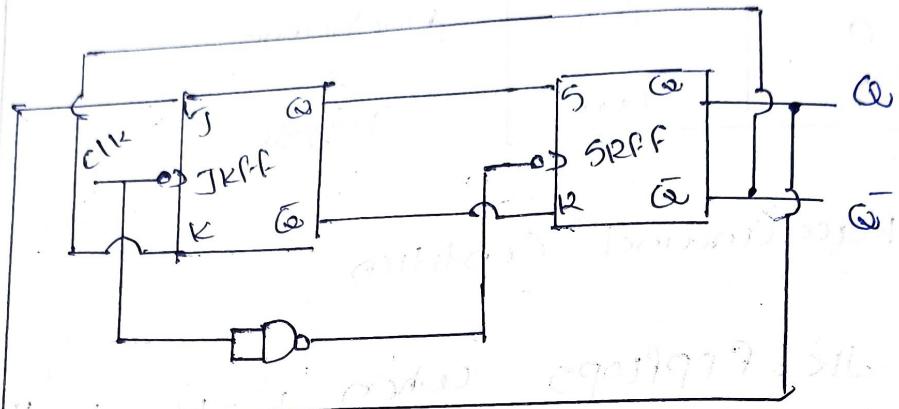


Diagram of master slave JK flip flop

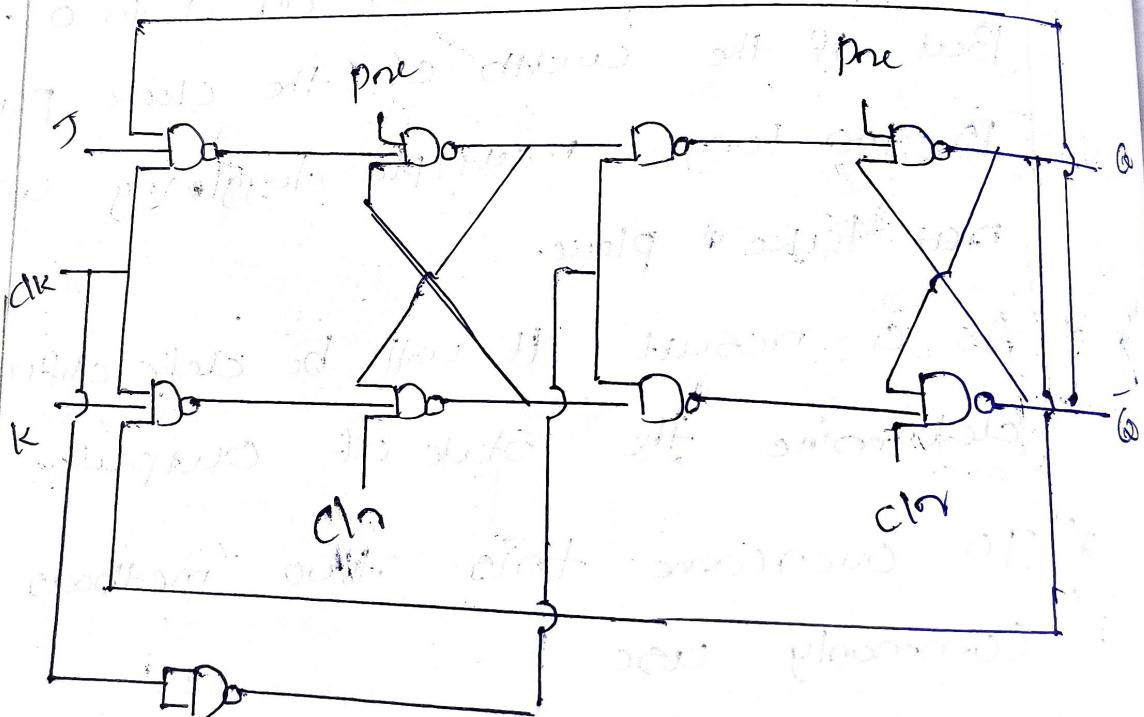
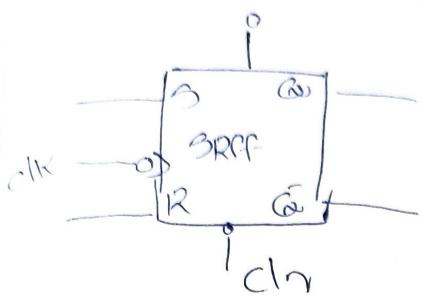


Diagram of master slave JK flip flop

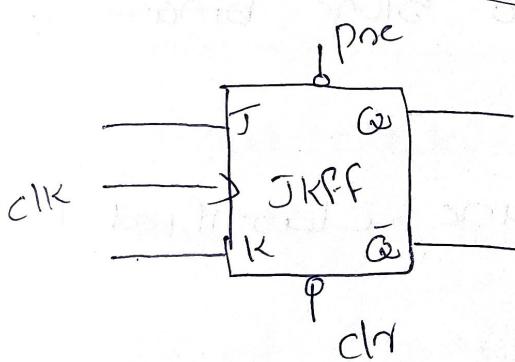
Logic Symbols of FlipFlops



negative edge

triggered SRFF

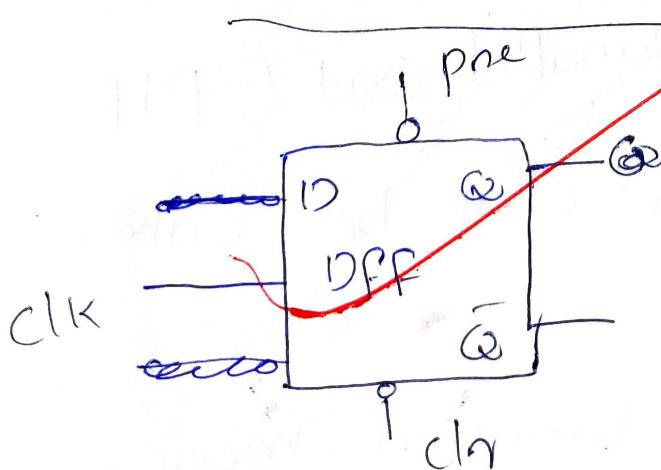
Positive edge triggered JKFF



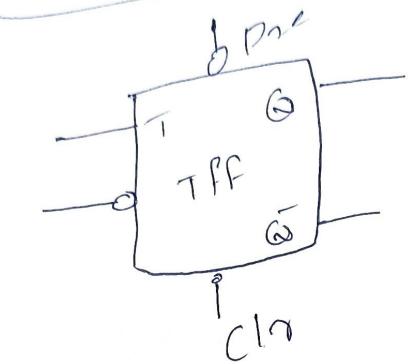
positive edge

triggered JKFF

Positive level triggered D flipflop



Negative level triggered T flip flop



Shift Registers

- * Shift register is a set of flip flops logically connected to store binary to n-bit binary data.
- * On the mode of shift registers are classified into four

1) Serial In Serial Out (SISO)

2) Serial In parallel out (SIPo)

3) Parallel In Serial Out (PISO)

4) Parallel In parallel out (PIPO)

* Data transfer can be either serial or parallel

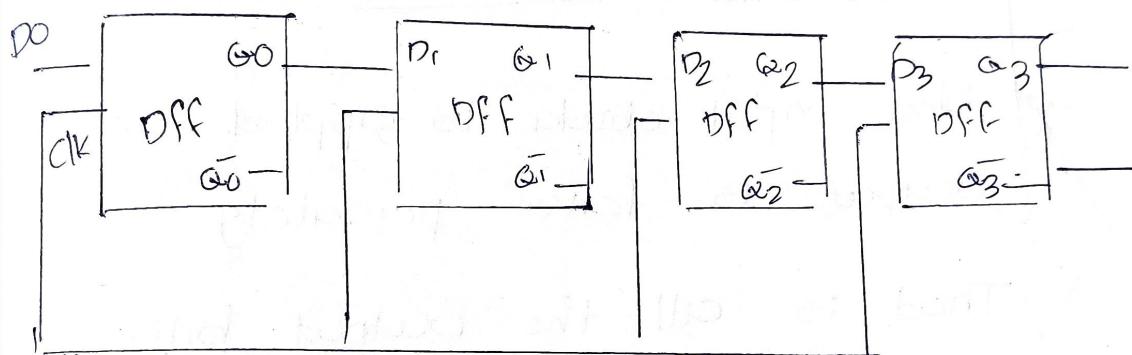
* Serial data transfer means we can

Input only a single data at a time.

Since parallel data transfer means

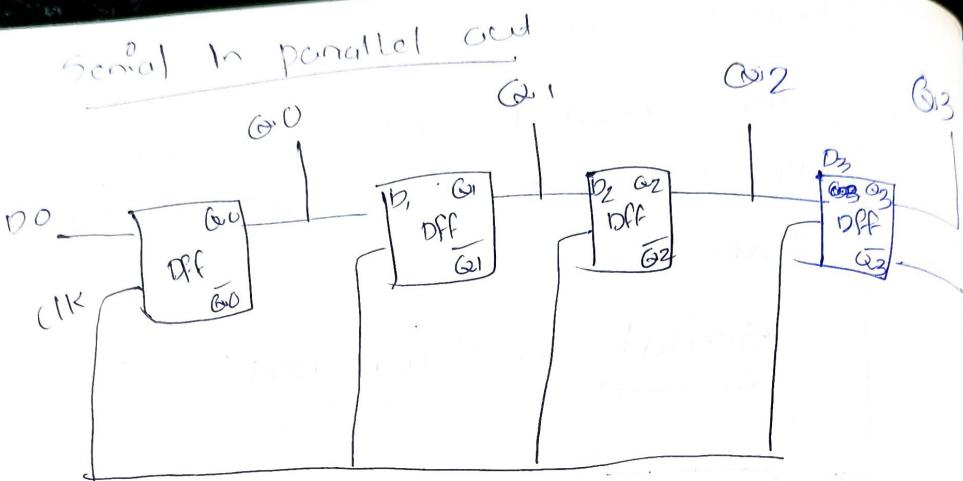
we can input a set of data same time.

Serial In Serial Out



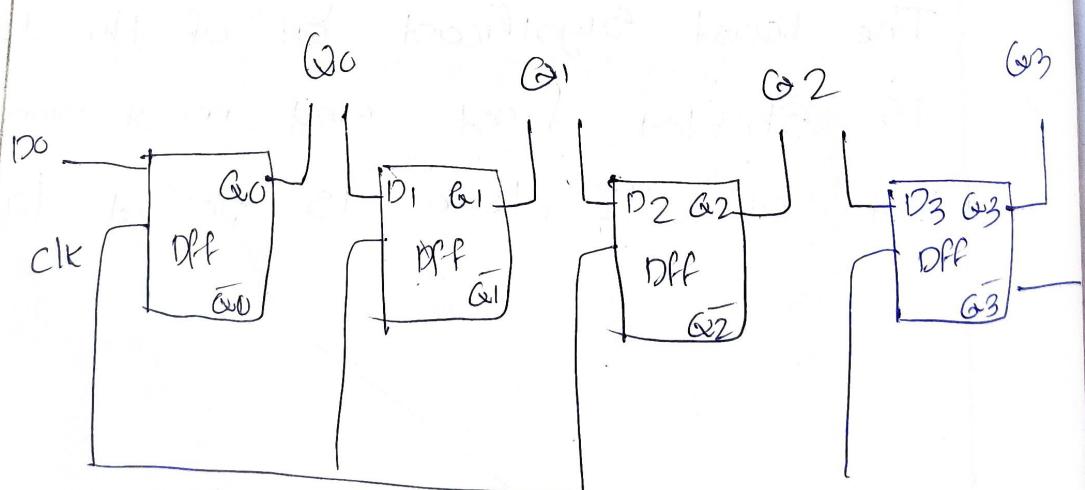
Here Input data is applied serially and output is also taken serially.

The least significant bit of the data is shifted first and most significant bit of the data is shifted last.



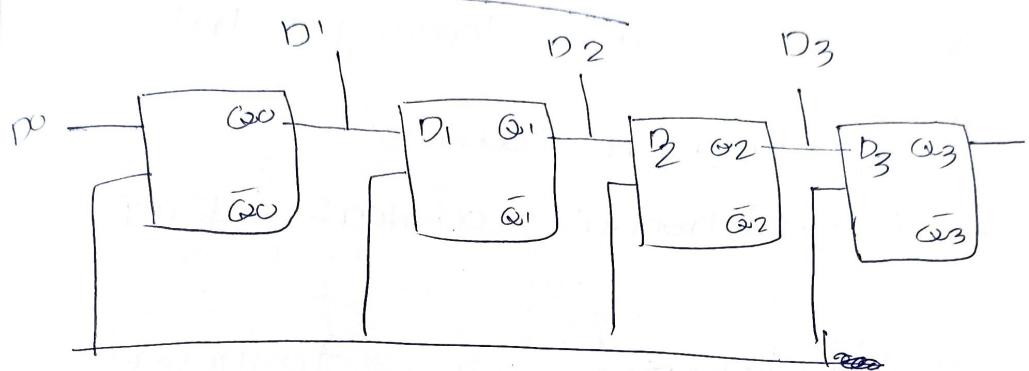
- * Here Input data is applied Serially and output is taken parallelly.
- * That is all the output bits are available simultaneously.

Parallel In serial out



- * Here input data is applied Parallelly and output is also taken parallelly.

Parallel In Parallel Out



Here Input data is applied parallelly
output is taken serially.

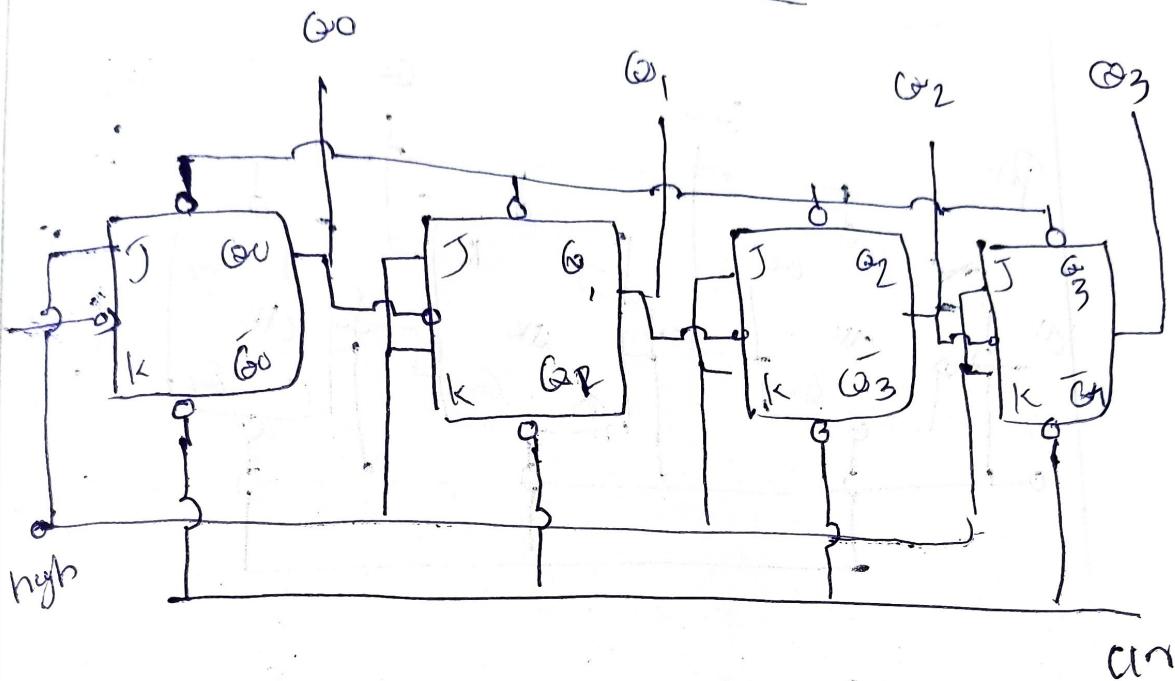
Counters

- * Counters is a sequential circuit where output states progresses in a repeated pattern advancing by one state for each per clock pulse
- * If a Counter has n flipflops then it may count clock pulses up to $2^n - 1$
- * Modulus of a counter means number of states of the counter.

- * Counters uses a chain of flip flops.
 - * Counters are classified into
 - 1) Synchronous counters
 - 2) Asynchronous counters (Ripple)
-

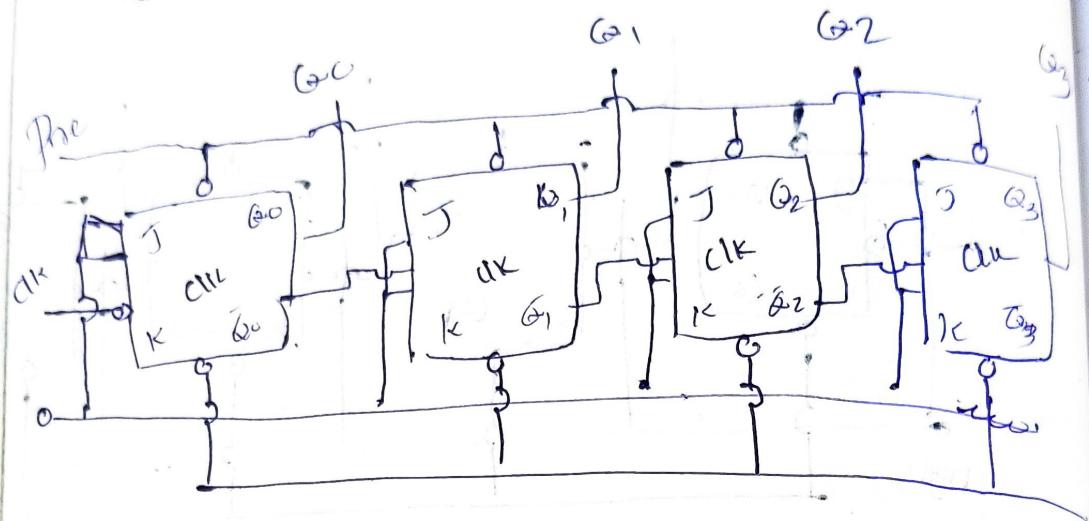
Synchronous Counters	Asynchronous Counters
* Common clock is connected to all the flip flops	* 1st first flip flop is clocked by external clock pulse and rests remaining flip flops are clocked from previous flip flops output.
* This design is complex	* Design is simple
* Operation is fast	* Operation is slow compared to synchronous
* Costly high	* Cost is less compared to synchronicity

4 bit Asynchronous Up Counter

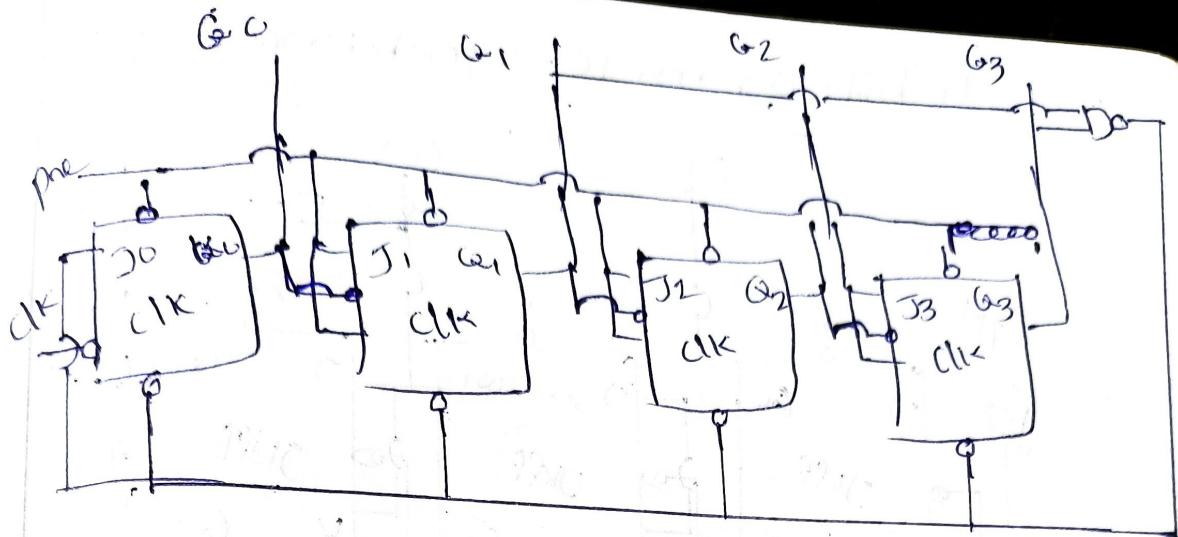


CLK	Q0	Q1	Q2	Q3
1	0	0	0	1
2	0	0	1	0
3	0	0	0	1
4	0	1	0	0
5	0	1	0	1
6	0	1	1	0
7	0	1	1	1
8	1	0	0	0
9	1	0	0	1
10	1	0	1	0
11	1	0	1	1
12	1	1	0	0
13	1	1	0	1
14	1	1	1	0
15	1	1	1	1
16	0	0	0	0

4 bit Asynchronous down Counter

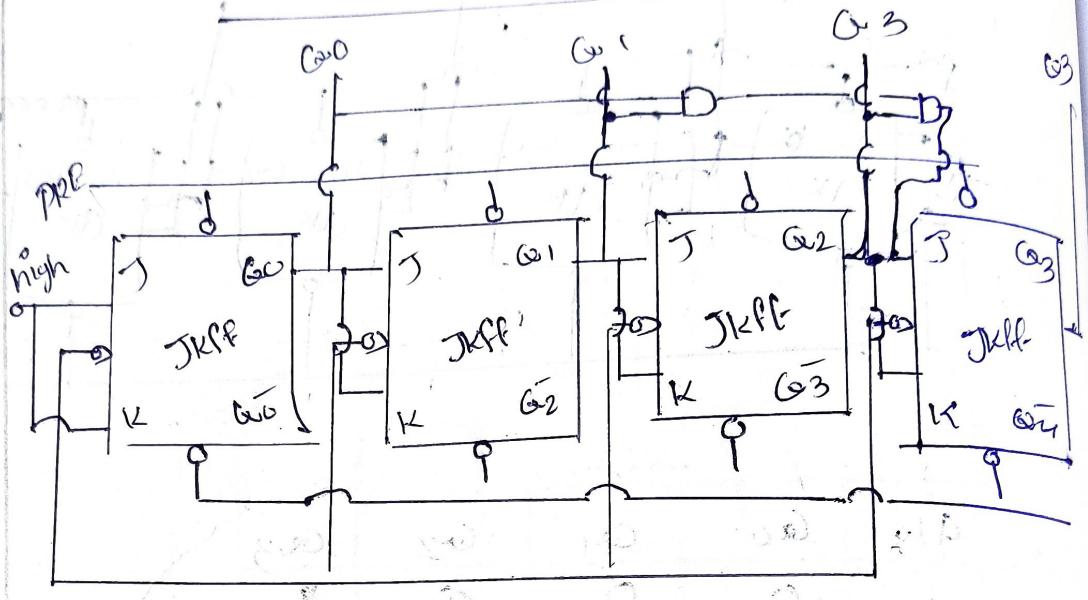


clk	Q0	Q1	Q2	Q3
1	1	1	1	1
2	1	1	1	0
3	1	1	0	1
4	1	1	0	0
5	1	0	1	1
6	1	0	1	0
7	1	0	0	1
8	1	0	0	0
9	0	1	1	1
10	0	1	1	0
11	0	1	0	1
12	0	1	0	0
13	0	0	1	1
14	0	0	1	0
15	0	0	0	1
16	0	0	0	0



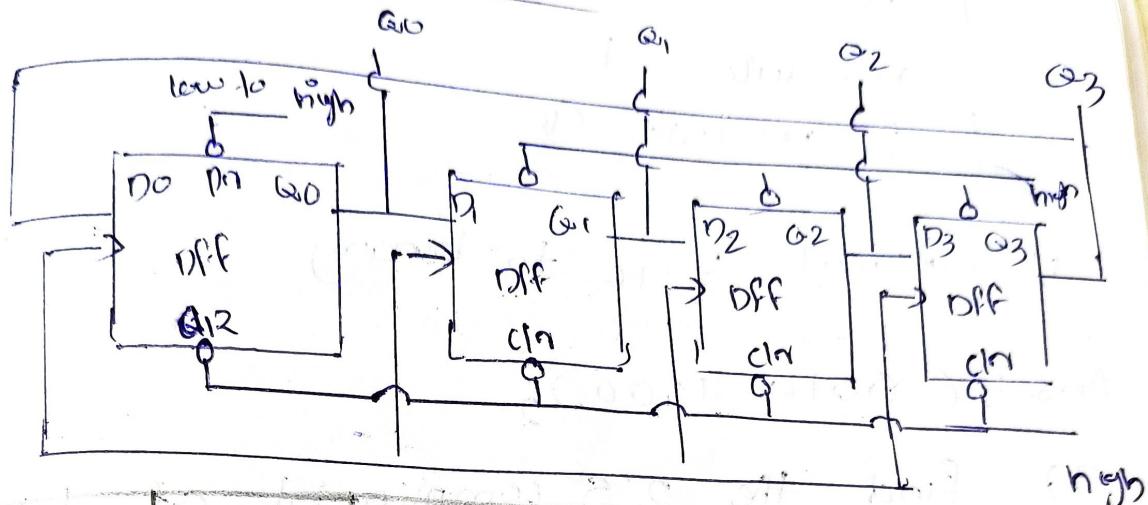
AK	Q0	Q1	Q2	Q3
1	0	0	0	0
2	0	0	1	0
3	0	0	1	0
4	0	1	0	0
5	0	1	0	0
6	0	1	1	0
7	1	1	1	0
8	1	0	0	0
9	1	0	0	0
10	0	0	0	0

4 bit or mo-16-Synchronous UpCounter



C1 k	Q3	Q2	Q1	Q0
0	0	0	0	0
1	0	0	0	1
2	0	0	1	0
3	0	0	1	1
4	0	1	0	0
5	0	1	1	0
6	0	1	1	1
7	0	1	0	0
8	1	0	0	1
9	1	0	0	0
10	1	0	1	0
11	1	0	1	1
12	1	1	0	0
13	1	1	0	1
14	1	1	1	0
15	1	1	1	1

Ring Counter



C_{in}	Q₀	Q₁	Q₂	Q₃
1	1	0	0	0
2	0	1	0	0
3	0	0	1	0
4	0	0	0	1
5	1	0	0	0

~~ring counter with 5 stages~~