Statistics Citadel
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Homework 8

Unless otherwise noted, $\varepsilon_t \sim iid\ N\left(0,\sigma^2\right)$

1. In the following problems $\varepsilon \sim iid\ N\left(0,\sigma^2\right)$. Simulate 200 data points from the following AR(1) models where $y_t = \beta_0 + \beta_1 y_{t-1} + \varepsilon_t$ and $\varepsilon_t \sim iid\ N\left(0,\sigma^2\right)$ (you might want to use arma.sim). For the simulations, fix the first value of the series at the unconditional mean of y ($\mu = \frac{\beta_0}{1-\beta_1}$). Then sequentially update the series using the previous value of the series and a random draw from a standard normal. You can generate random draws from a standard normal distribution using the Excel command =normsinv(rand()).

   i. $\beta_0 = 0,\ \beta_1 = 0,\ \sigma^2 = 1$
   ii. $\beta_0 = 0,\ \beta_1 = .9,\ \sigma^2 = 1$
   iii. $\beta_0 = 0,\ \beta_1 = .1,\ \sigma^2 = 1$
   iv. $\beta_0 = 0,\ \beta_1 = -.5,\ \sigma^2 = 1$
   v. $\beta_0 = 1,\ \beta_1 = .9,\ \sigma^2 = 1$

a. First, find the unconditional means for each series.
b. Second plot the simulated series. Do they look like you expect? What is the difference between series with large values of beta (near 1) and small values of beta?
c. Are all of these series mean reverting? If so, to which values?
d. Using the last observation in your sample (T=200), write down the one step ahead forecast of the series (i.e. find μ_{T+1}).
e. For your simulated data, find the one step ahead *conditional* distribution for the 201st observation. Find $f\left(y_{T+1} \mid y_T\right)$.
f. Write down a 95% predictive interval for this one step ahead forecast.
g. Plot the ACF and the PACF through lag 12

2. Simulate 200 observations the MA $y_t = \mu + \theta\varepsilon_{t-1} + \varepsilon_t$ where $\varepsilon_t \sim iid\ N\left(0,1\right)$. Plot the time series and present the ACF and the PACF though lag 10.
   a. μ=1, θ=.8
   b. μ=0, θ=-.8

a. First, find the unconditional means for each series.
b. Second plot the simulated series. Do they look like you expect? What is the difference between series with large values of beta (near 1) and small values of beta?
c. Are all of these series mean reverting? If so, to which values?
d. Using the last observation in your sample (T=200), write down the one step ahead forecast of the series (i.e. find μ_{T+1}).
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g. Plot the ACF and the PACF through lag 12

3. Use the Chicago housing price data (hpchicago.xls) for this homework.
    a. Fit an AR(1) model.
    b. Find a good ARMA model and describe how you arrived at the final model.
    c. Compare the adjusted R-squared of the AR(1) model and your new model. Which one is better?
    d. Plot the one-step ahead in-sample predictions from the two models and compare them.

4. Use the dataset fedfunds.xls. This data set contains (among other things) the overnight federal funds rate (the variable FFO) for the first day of the month from January 1989 through August 2008. Use this overnight rate for this problem.
    a. Plot the interest rates and calculate the sample mean return on the t-bills and report a standard error assuming the returns are iid. Is the iid assumption used to construct the standard errors correct? (We'll come back to this later in the class).
    b. Calculate the sample variance of the t-bill returns.
    c. Find two models that fit the data well. Start with simple models and check the residuals to see if they are uncorrelated. Report your models.
    d. For each model, report the ACF and PACF of the residuals.
    e. For both models, and for each observation in your sample, construct the one step ahead forecasts (i.e. $\mu_t = E\left(r_t \mid r_{t-1}, r_{t-2}, \ldots\right)$) using your fitted model in part c.

5. The second column of data in the spreadsheet fedfunds.xls is the 1 month ahead futures rate (Future). We will refer to FFO as the spot rate and Future as the futures rate. The month t futures rate is the interest rate that could have been locked in one month prior. Hence the data are aligned such that for a given month you have the spot interest rate (FFO) and the Futures rate that you could have locked into for that month – both rates are for the same month.
   We will consider a dynamic trading strategy in this problem that uses the one-step-ahead forecasts. Here is a simple strategy. If your forecast of a month's rate is higher than the future's rate for that month then you borrow at the futures rate and invest in the spot rate. Hence your payoff will be the overnight rate less the futures rate. If your forecast is lower than

the future's rate, then you invest at the futures rate and borrow at the spot rate. Hence your payoff will be the futures rate less the spot rate.

a. For both of your models in problem 2, construct the sequence of returns generated by implementing this strategy. Plot the returns.
b. Find the average return and the standard deviation of the strategy's returns for both models.
c. For both models, test the hypothesis that the average return is zero. Clearly state your conclusions. Recall that your test statistic is given by $t = \frac{\overline{r}}{se(\overline{r})}$ where $\overline{r}$ is the sample average and $se(\overline{r}) = \frac{s}{\sqrt{T}}$ where *s* is the sample standard deviation and T is the sample size. You reject if $|t| > 2$ (you could do a one sided test as well in which case you reject if $t > 1.65$).
d. Test the hypothesis that the average returns are the same for the two models. To do this, create a new series that is the difference between the returns for the two series. Call this new series D. Now, following the same procedure in part c., test the null that the mean of D is zero against the two-sided alternative that the mean is not zero. Your test statistic will be $t = \frac{\bar{D}}{se(\bar{D})}$. Clearly state your conclusions.

6. Go to Yahoo finance and download 20 years of daily returns data for the S&P500 (notice that the data are in reverse chronological order and you need to fix this). Look at the autocorrelations for the S&P500 continuously compounded returns. The efficient markets hypothesis says that stock returns should be unpredictable.
    a. Plot the series and look at the autocorrelations. What do the autocorrelations say about market efficiency?
    b. Fit an AR(1) model to the data. Does it appear to fit the data?
    c. Fit an MA(1) model to the data. Does it appear to fit the data?