FTAP Homework 10

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Problem 1

Read and Format Problem 1:

```
orderByDate <- function (x){
  x$DateTime <- as.Date(x$Date, "%Y-%m-%d")
  x <- x[order(x$DateTime, decreasing=F),]
  x <- x[complete.cases(x),]
  x
}

logRateReturn <- function(x){
  x$logReturn <- c(NA, diff(log(x$Adj.Close)))
  x
}

riskFreeReturn <- function(x){
  x$excessReturn <- x$logReturn - tbill$WeeklyReturn
  x
}

tbill <- read.xls("tbillWeek.xls", skip = 8)
tbill <- rename(x = tbill, c("observation_date" = "Date", "WTB3MS"="Return"))
tbill$RawReturn <- tbill$Return / 100
tbill$WeeklyReturn <- tbill$RawReturn / 52
spy <- read.csv("spy.csv")
spy <- orderByDate(spy)
spy <- logRateReturn(spy)[-1,]
spy <- riskFreeReturn(spy)

bestBuy <- read.csv("bby.csv")
bestBuy <- orderByDate(bestBuy)
bestBuy <- logRateReturn(bestBuy)[-1,]
bestBuy <- riskFreeReturn(bestBuy)

homeDepot <- read.csv("hd.csv")
homeDepot <- orderByDate(homeDepot)
homeDepot <- logRateReturn(homeDepot)[-1,]
homeDepot <- riskFreeReturn(homeDepot)
```

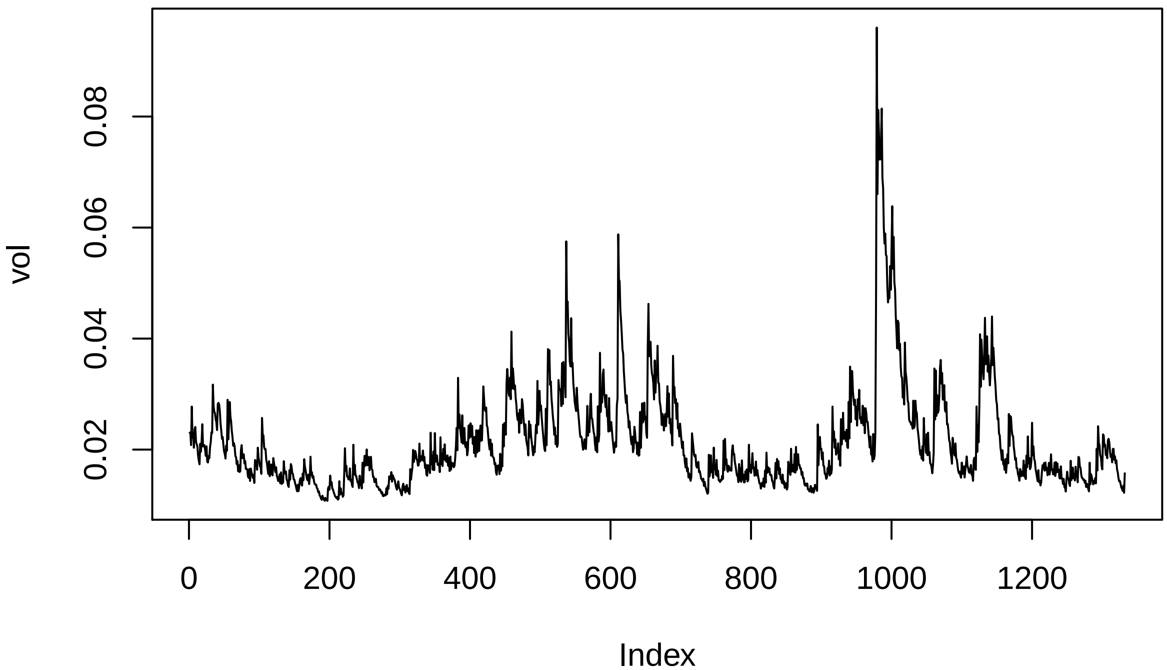
a

```
lm.bestBuy <-lm(bestBuy$excessReturn~spy$excessReturn)
v.bestBuy <- var(lm.bestBuy$residuals)
beta.bestBuy <- as.numeric(lm.bestBuy$coefficients["spy$excessReturn"])
```

```
lm.homeDepot <-lm(homeDepot$excessReturn~spy$excessReturn)
v.homeDepot <- var(lm.homeDepot$residuals)
beta.homeDepot <- as.numeric(lm.homeDepot$coefficients["spy$excessReturn"])
```

b

```
garch <- garchFit(~garch(2,2),data=spy$excessReturn, trace = F)
vol <- volatility(garch)
plot(vol, type="l")
```



c

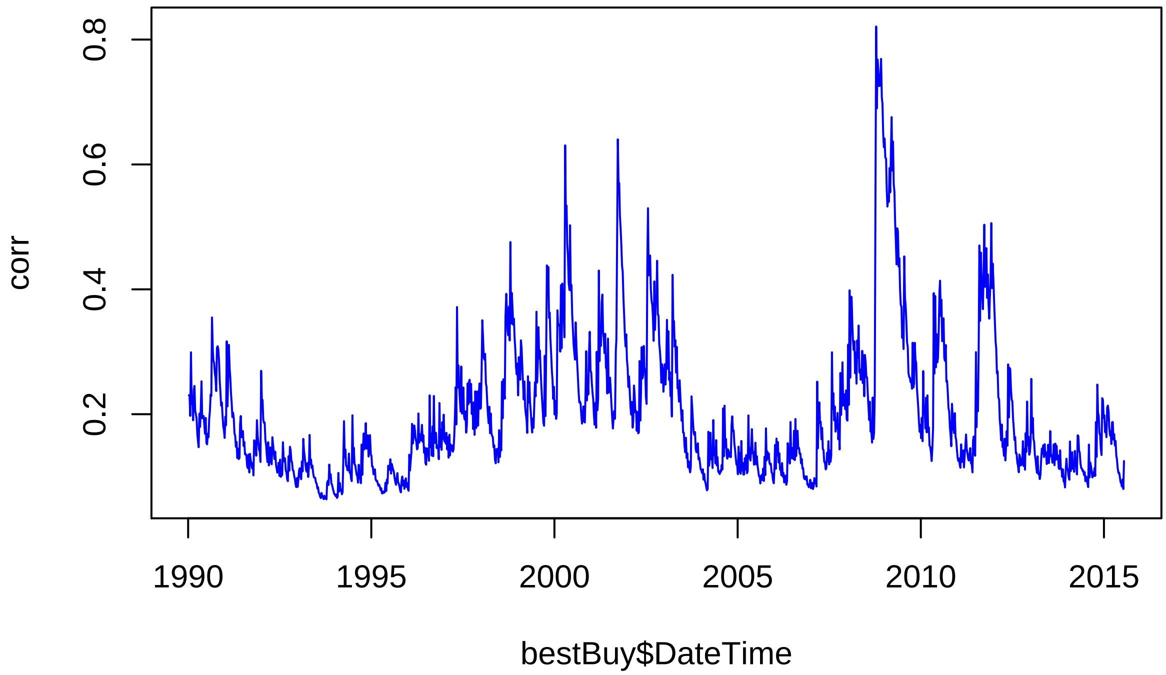
Conditional Correlation

```
cov <- beta.bestBuy*beta.homeDepot*vol^2
denom.bestBuy <- beta.bestBuy^2*vol^2+mean(residuals(lm.bestBuy)^2)
denom.homeDepot <- beta.homeDepot^2*vol^2+mean(residuals(lm.homeDepot)^2)

corr=cov/sqrt(denom.bestBuy*denom.homeDepot)
```

d

```r
plot(bestBuy$DateTime,corr,type="l", col="blue")
```



e

```r
forecast <- predict(garch,n.ahead=1)
oneDayVar <- .3^2*(beta.bestBuy^2*forecast$standardDeviation^2 +  var(lm.bestBuy$residuals))
  + .7^2*(beta.homeDepot^2*forecast$standardDeviation^2 + var(lm.homeDepot$residuals)) + 2 *
  cov[length(bestBuy$excessReturn)]*.3*.7
```

```
## [1] 0.0009479559
```

```r
oneDayVar
```

```
## [1] 0.0004818167
```

f

```r
valrisk_norm<-qnorm(.010,0,1)*sqrt((oneDayVar/length(bestBuy$excessReturn)))
valrisk_norm*1000000
```

```
## [1] -1399.147
```

Problem 2

Read and Format Problem 2:

```
riskFreeDaily <- function(x){
  x$excessReturn <- x$logReturn - tbillDaily$RawReturn
  x
}

spyDaily <- read.csv("spyDaily.csv")
spyDaily <- orderByDate(spyDaily)
spyDaily <- logRateReturn(spyDaily)[-1,]

tbillDaily <- read.xls("tbill.xls", skip = 8)
tbillDaily <- rename(x = tbillDaily, c("observation_date" = "Date", "DTB3"="Return"))
tbillDaily$DateTime <- as.Date(tbillDaily$Date, "%Y-%m-%d")
tbillDaily <- tbillDaily[tbillDaily$DateTime > as.Date("1999-12-31", "%Y-%m-%d"), ]
tbillDaily$RawReturn <- tbillDaily$Return / 100

att <- read.csv("tDaily.csv")
att <- orderByDate(att)
att <- logRateReturn(att)[-1,]

msft <- read.csv("msftDaily.csv")
msft <- orderByDate(msft)
msft <- logRateReturn(msft)[-1,]

sam <- read.csv("samDaily.csv")
sam <- orderByDate(sam)
sam <- logRateReturn(sam)[-1,]


ba <- read.csv("baDaily.csv")
ba <- orderByDate(ba)
ba <- logRateReturn(ba)[-1,]

gs <- read.csv("gsDaily.csv")
gs <- orderByDate(gs)
gs <- logRateReturn(gs)[-1,]
```

a

```
oneFactor <- function(x, y){
#   lm.x <- lm(x$logReturn~spyDaily$logReturn)
#   lm.x$coefficients <- rename(lm.x$coefficients, c("spyDaily$logReturn"=y))
#   lm.x
  lm(x$logReturn~spyDaily$logReturn)
}

lm.att <- oneFactor(att, "attBeta")
lm.msft <- oneFactor(msft, "msftBeta")
lm.sam <- oneFactor(sam, "samBeta")
lm.ba <- oneFactor(ba, "boeingBeta")
lm.gs <- oneFactor(gs, "gsBeta")
```

b

```r
all.betas <- c(lm.gs$coefficients[2], lm.ba$coefficients[2],
               lm.sam$coefficients[2], lm.msft$coefficients[2],
               lm.att$coefficients[2])

all.residVar <- sapply(list(gsResid = lm.gs$residuals, baResid = lm.ba$residuals,
               samResid = lm.sam$residuals, msftResid = lm.msft$residuals,
               attResid = lm.att$residuals), var)

kable(head(outer(all.betas, all.betas) * var(spyDaily$logReturn) + diag(5) * all.residVar))
```

	spyDaily$logReturn	spyDaily$logReturn	spyDaily$logReturn	spyDaily$logReturn	spyDaily$lo
spyDaily$logReturn	0.0006086	0.0002087	0.0001286	0.0002301	0.
spyDaily$logReturn	0.0002087	0.0003810	0.0000868	0.0001554	0.
spyDaily$logReturn	0.0001286	0.0000868	0.0004776	0.0000957	0.
spyDaily$logReturn	0.0002301	0.0001554	0.0000957	0.0004045	0.
spyDaily$logReturn	0.0001735	0.0001172	0.0000722	0.0001292	0.

Problem 3

a

(Assuming that the final marginal probability is irrelavant for cleanliness)

$$L = f(x_N|F_{N-1,\theta}) * ... * f(x_2|F_{1,\theta})$$
$$\mathcal{L} = log(f(x_N|F_{N-1,\theta})) + ... + log(f(x_2|F_{1,\theta}))$$
$$\mathcal{L} = \Sigma_2^N log(f(x_i|F_{i-1,\theta}))$$
$$\mathcal{L} = \Sigma_2^N log(\frac{1}{\omega + \alpha * x_{i-1}} e^{\frac{-x_i}{\omega+\alpha*x_{i-1}}})$$

$$\mathcal{L} = \Sigma_2^N - log(\omega + \alpha * x_{i-1}) + \frac{-x_i}{\omega + \alpha * x_{i-1}}$$

b

Assume x_i in the conditional probability function should be an x_{i-1} Let $U = \omega + \alpha * x_{i-1}$

$$\frac{d\mathcal{L}}{d\alpha} = \Sigma_2^N(-\frac{1}{U} * \frac{dU}{d\alpha} + x_{i-1}\frac{1}{U^2} * \frac{dU}{d\alpha})$$

$$\frac{dU}{d\alpha} = x_{i-1}$$

$$\frac{d\mathcal{L}}{d\alpha} = \Sigma_2^N(-\frac{x_{i-1}}{\omega + \alpha * x_{i-1}} + \frac{x_{i-1}^2}{(\omega + \alpha * x_{i-1})^2})$$

$$\frac{d\mathcal{L}}{d\omega} = \Sigma_2^N(-\frac{1}{U} * \frac{dU}{d\omega} + x_{i-1}\frac{1}{U^2} * \frac{dU}{d\omega})$$

$$\frac{dU}{d\omega} = 1$$

$$\frac{d\mathcal{L}}{d\omega} = \Sigma_2^N(-\frac{1}{\omega + \alpha * x_{i-1}} + \frac{x_{i-1}}{(\omega + \alpha * x_{i-1})^2})$$

c

1. $$\frac{d^2\mathcal{L}}{d^2\alpha} = \Sigma_2^N(\frac{x_{i-1}}{U^2} * \frac{dU}{d\alpha} - 2 * \frac{x_{i-1}^2}{U^3})\frac{dU}{d\alpha}$$

$$\frac{d^2\mathcal{L}}{d^2\alpha} = \Sigma_2^N(\frac{x_{i-1}^2}{(\omega + \alpha * x_{i-1})^2} - 2 * \frac{x_{i-1}^3}{(\omega + \alpha * x_{i-1})^3})$$

2. $$\frac{d^2\mathcal{L}}{d\alpha d\omega} = \Sigma_2^N(\frac{x_{i-1}}{U^2} * \frac{dU}{d\omega} - 2 * \frac{x_{i-1}^2}{U^3})\frac{dU}{d\omega}$$

$$\frac{d^2\mathcal{L}}{d^2\alpha} = \Sigma_2^N(\frac{x_{i-1}}{(\omega + \alpha * x_{i-1})^2} - 2 * \frac{x_{i-1}^2}{(\omega + \alpha * x_{i-1})^3})$$

3. $$\frac{d^2\mathcal{L}}{d^2\omega} = \Sigma_2^N(\frac{1}{U^2} * \frac{dU}{d\omega} - 2 * \frac{x_{i-1}}{(U)^3} * \frac{dU}{d\omega})$$

$$\frac{d^2\mathcal{L}}{d^2\omega} = \Sigma_2^N(\frac{1}{(\omega + \alpha * x_{i-1})^2} - 2 * \frac{x_{i-1}}{(\omega + \alpha * x_{i-1})^3})$$

4. $$\frac{d^2\mathcal{L}}{d\omega d\alpha} = \Sigma_2^N(\frac{1}{U^2} * \frac{dU}{d\alpha} - 2 * \frac{x_{i-1}}{(U)^3} * \frac{dU}{d\alpha})$$

$$\frac{d^2\mathcal{L}}{d\omega d\alpha} = \Sigma_2^N(\frac{x_{i_1}}{(\omega + \alpha * x_{i-1})^2} - 2 * \frac{x_{i-1}^2}{(\omega + \alpha * x_{i-1})^3})$$

$$\begin{pmatrix} \Sigma_2^N(\frac{x_{i-1}^2}{(\omega+\alpha*x_{i-1})^2} - 2*\frac{x_{i-1}^3}{(\omega+\alpha*x_{i-1})^3}) & \Sigma_2^N(\frac{x_{i-1}}{(\omega+\alpha*x_{i-1})^2} - 2*\frac{x_{i-1}^2}{(\omega+\alpha*x_{i-1})^3}) \\ \Sigma_2^N(\frac{x_{i_1}}{(\omega+\alpha*x_{i-1})^2} - 2*\frac{x_{i-1}^2}{(\omega+\alpha*x_{i-1})^3}) & \Sigma_2^N(\frac{1}{(\omega+\alpha*x_{i-1})^2} - 2*\frac{x_{i-1}}{(\omega+\alpha*x_{i-1})^3}) \end{pmatrix}$$