

Investments HW 3

Zachary Fogelson

August 9, 2015

Problem 1

a

$$\begin{aligned} \text{Var}(A + B + C + D) &= w_a^2 \text{Var}(A) + w_b^2 \text{Var}(B) + w_c^2 \text{Var}(C) + w_d^2 \text{Var}(D) + \\ &2 * (w_a w_b \text{cov}(A, B) + w_a w_c \text{cov}(A, C) + w_a w_d \text{cov}(A, D) + w_b w_c \text{cov}(B, C) + w_b w_d \text{cov}(B, D) + w_c w_d \text{cov}(C, D)) \end{aligned}$$

$$\text{Var}(Mrkt) = .0715$$

$$\text{Cov}(A, A + B + C + D) = \text{Var}(A) + \text{Cov}(A, B) + \text{Cov}(A, C) + \text{Cov}(A, D)$$

Similarly for B,C,D. Interestingly, this corresponds to the sum of the elements in any given row of the Variance Covariance matrix!

$$\text{Cov}(A, P) = 0.2308$$

$$\text{Cov}(B, P) = 0.2660$$

$$\text{Cov}(C, P) = 0.3012$$

$$\text{Cov}(D, P) = 0.3364$$

b

$$\beta = \frac{\text{Cov}(P, A)}{\sigma_P^2}$$

$$\beta_A = 3.226$$

$$\beta_B = 3.718$$

$$\beta_C = 4.210$$

$$\beta_D = 4.702$$

c

$$r_A - r_f = \beta_A * [E(r_m) - r_f]$$

$$E(r_A - r_f) = 0.226$$

$$E(r_B - r_f) = 0.260$$

$$E(r_C - r_f) = 0.295$$

$$E(r_D - r_f) = 0.329$$

d

MVE Weights of the Portfolio:

A: 24%

B: 25%

C: 25%

D: 26%

The MVE weights are exactly the percentage of each firm over the value of the market.

e MVE Weights of the Portfolio:

A: 25%

B: 25%

C: 25%

D: 25%

The weights have changed.

f

MVE Weights of the Portfolio:

A: 29%

B: 23%

C: 24%

D: 24%

$\text{Var}(P) = .0028$

Problem 2

- Format Data

```
openFormat <- function(x, skips){
  y <- read.csv(x, skip=skips, nrows=1068)
  y <- rename(y, c("X"="Date"))
  y <- y[y$Date > 193200,]
  tmp <- y$Date
  y <- y[,-1]/100
  y$Date <- tmp
  y
}

mrkt <- openFormat("F-F_Research_Data_Factors.CSV", 3)
ports <- openFormat("25_Portfolios_5x5.CSV", 19)
dates <- ports$Date
ports <- ports - mrkt$RF
ports$Date <- dates
```

a

```
regressors <- names(ports)[-length(ports)]
coefs <- data.frame(Index=c(1,2), row.names = c("alpha", "beta"))
for(i in regressors){
  cs <- coefficients(lm(ports[,i] ~ mrkt$Mkt.RF))
  coefs <- cbind(coefs, rbind(round(as.numeric(cs["(Intercept)"]),5), round(as.numeric(cs["mrkt$Mkt.RF"]))))
}
```

```

}
coefs <- coefs[,-1]
names(coefs) <- regressors
# pandoc.table(coefs,split.tables=90 )

```

| | SMALL.LoBM | ME1.BM2 | ME1.BM3 | ME1.BM4 | SMALL.HiBM |
|--------------|------------|----------|---------|---------|------------|
| alpha | -0.00396 | -0.00117 | 0.00199 | 0.00443 | 0.00524 |
| beta | 1.651 | 1.444 | 1.395 | 1.274 | 1.402 |

| | ME2.BM1 | ME2.BM2 | ME2.BM3 | ME2.BM4 | ME2.BM5 |
|--------------|----------|---------|---------|---------|---------|
| alpha | -0.00161 | 0.0013 | 0.00259 | 0.00347 | 0.00375 |
| beta | 1.306 | 1.262 | 1.23 | 1.243 | 1.394 |

| | ME3.BM1 | ME3.BM2 | ME3.BM3 | ME3.BM4 | ME3.BM5 |
|--------------|----------|---------|---------|---------|---------|
| alpha | -0.00082 | 0.00183 | 0.00219 | 0.00293 | 0.00309 |
| beta | 1.281 | 1.132 | 1.149 | 1.18 | 1.375 |

| | ME4.BM1 | ME4.BM2 | ME4.BM3 | ME4.BM4 | ME4.BM5 |
|--------------|---------|---------|---------|---------|---------|
| alpha | 0.00011 | 0.00046 | 0.0018 | 0.00245 | 0.0014 |
| beta | 1.085 | 1.095 | 1.144 | 1.159 | 1.413 |

| | BIG.LoBM | ME5.BM2 | ME5.BM3 | ME5.BM4 | BIG.HiBM |
|--------------|----------|---------|---------|---------|----------|
| alpha | -0.00029 | 2e-05 | 0.00084 | 0.00028 | 0.00147 |
| beta | 0.9492 | 0.943 | 0.9748 | 1.089 | 1.245 |

b

```

coefs <- rbind(coefs, sapply(ports, mean))
rownames(coefs) <- c("alpha", "beta", "actual")
coefs <- data.frame(t(coefs))
kable(coefs)

```

| | alpha | beta | actual |
|------------|----------|---------|-----------|
| SMALL.LoBM | -0.00396 | 1.65058 | 0.0080490 |
| ME1.BM2 | -0.00117 | 1.44401 | 0.0093410 |
| ME1.BM3 | 0.00199 | 1.39526 | 0.0121487 |
| ME1.BM4 | 0.00443 | 1.27417 | 0.0137078 |
| SMALL.HiBM | 0.00524 | 1.40240 | 0.0154456 |
| ME2.BM1 | -0.00161 | 1.30590 | 0.0078980 |
| ME2.BM2 | 0.00130 | 1.26222 | 0.0104836 |
| ME2.BM3 | 0.00259 | 1.22953 | 0.0115339 |
| ME2.BM4 | 0.00347 | 1.24267 | 0.0125122 |

| | alpha | beta | actual |
|----------|----------|---------|-----------|
| ME2.BM5 | 0.00375 | 1.39384 | 0.0138949 |
| ME3.BM1 | -0.00082 | 1.28106 | 0.0085043 |
| ME3.BM2 | 0.00183 | 1.13201 | 0.0100712 |
| ME3.BM3 | 0.00219 | 1.14924 | 0.0105557 |
| ME3.BM4 | 0.00293 | 1.18019 | 0.0115219 |
| ME3.BM5 | 0.00309 | 1.37529 | 0.0130966 |
| ME4.BM1 | 0.00011 | 1.08544 | 0.0080107 |
| ME4.BM2 | 0.00046 | 1.09543 | 0.0084292 |
| ME4.BM3 | 0.00180 | 1.14362 | 0.0101255 |
| ME4.BM4 | 0.00245 | 1.15873 | 0.0108878 |
| ME4.BM5 | 0.00140 | 1.41345 | 0.0116846 |
| BIG.LoBM | -0.00029 | 0.94915 | 0.0066222 |
| ME5.BM2 | 0.00002 | 0.94301 | 0.0068857 |
| ME5.BM3 | 0.00084 | 0.97485 | 0.0079334 |
| ME5.BM4 | 0.00028 | 1.08938 | 0.0082060 |
| BIG.HiBM | 0.00147 | 1.24520 | 0.0105276 |

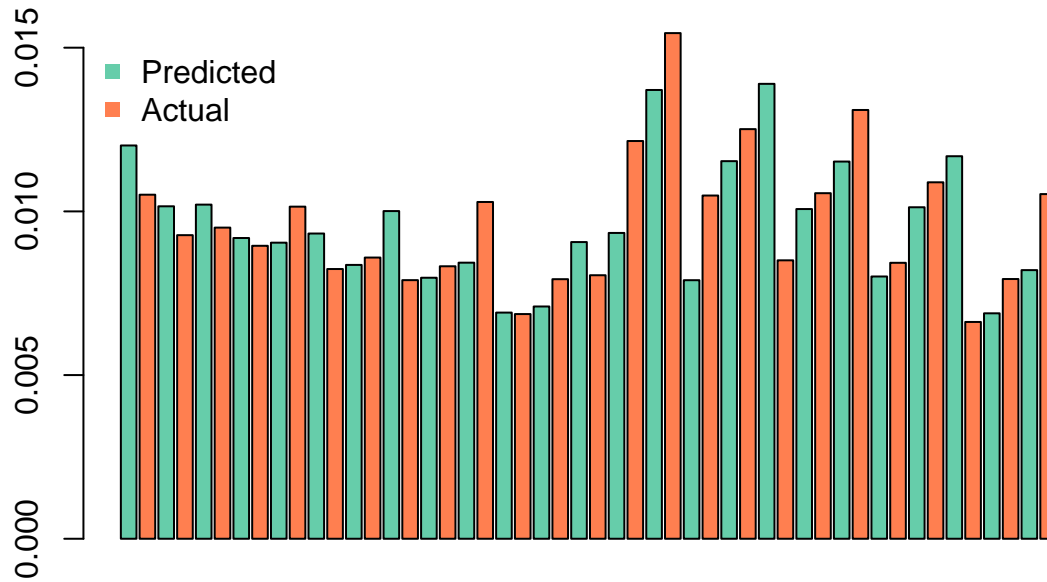
c

```
lm.capm <- lm(actual ~ beta, coefs)
summary(lm.capm)
```

```
##
## Call:
## lm(formula = actual ~ beta, data = coefs)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.0050694 -0.0013277  0.0001523  0.0012318  0.0039872
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.002078   0.003135   0.663   0.5140
## beta         0.006689   0.002521   2.654   0.0142 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.002092 on 23 degrees of freedom
## Multiple R-squared:  0.2344, Adjusted R-squared:  0.2011
## F-statistic: 7.042 on 1 and 23 DF,  p-value: 0.01419
```

d

```
coefs$predictions <- mean(mrkt$Mkt.RF) * coefs$beta
barplot(c(coefs$predictions, coefs$actual), beside = T, col=c("aquamarine3","coral"))
legend("topleft", c("Predicted","Actual"), pch=15, col=c("aquamarine3","coral"), bty="n")
```



Problem 3

a

```
coefs <- data.frame(Index=c(1,2,3,4), row.names = c("alpha", "betaMrkt", "betaSMB", "betaHML"))
for(i in regressors){
  cs <- coefficients(lm(ports[,i] ~ mrkt$Mkt.RF + mrkt$SMB + mrkt$HML))
  coefs <- cbind(coefs, rbind(round(as.numeric(cs["(Intercept)"]),5), round(as.numeric(cs["mrkt$Mkt.RF"]))))
}
coefs <- coefs[,-1]
names(coefs) <- regressors
```

b

```
coefs <- rbind(coefs, sapply(ports, mean))
rownames(coefs) <- c(rownames(coefs)[-length(rownames(coefs))], "actual")
# pandoc.table(coefs, split.tables=90 )
```

| | SMALL.LoBM | ME1.BM2 | ME1.BM3 | ME1.BM4 | SMALL.HiBM |
|-----------------|------------|----------|----------|---------|------------|
| alpha | -0.00723 | -0.00373 | -0.00112 | 0.00105 | 0.00063 |
| betaMrkt | 1.251 | 1.07 | 1.051 | 0.918 | 0.9758 |
| betaSMB | 1.513 | 1.54 | 1.227 | 1.227 | 1.327 |
| betaHML | 0.4489 | 0.2297 | 0.5043 | 0.5848 | 0.917 |
| actual | 0.008049 | 0.009341 | 0.01215 | 0.01371 | 0.01545 |

| | ME2.BM1 | ME2.BM2 | ME2.BM3 | ME2.BM4 | ME2.BM5 |
|-----------------|----------|----------|---------|---------|----------|
| alpha | -0.00215 | -0.00033 | 0.00038 | 0.00062 | -0.00029 |
| betaMrkt | 1.088 | 1.027 | 0.9935 | 0.9797 | 1.066 |
| betaSMB | 1.136 | 0.9661 | 0.8235 | 0.8167 | 0.8945 |
| betaHML | -0.2344 | 0.1497 | 0.3718 | 0.568 | 0.8982 |
| actual | 0.007898 | 0.01048 | 0.01153 | 0.01251 | 0.01389 |

| | ME2.BM1 | ME2.BM2 | ME2.BM3 | ME2.BM4 | ME2.BM5 |
|-----------------|----------|----------|----------|----------|----------|
| | | | | | |
| | ME3.BM1 | ME3.BM2 | ME3.BM3 | ME3.BM4 | ME3.BM5 |
| alpha | -0.00098 | 0.00109 | 0.00061 | 0.00055 | -0.00053 |
| betaMrkt | 1.134 | 1.013 | 1.007 | 1 | 1.116 |
| betaSMB | 0.8219 | 0.5067 | 0.4327 | 0.4491 | 0.5997 |
| betaHML | -0.2384 | 0.04692 | 0.3229 | 0.5565 | 0.8743 |
| actual | 0.008504 | 0.01007 | 0.01056 | 0.01152 | 0.0131 |
| | | | | | |
| | ME4.BM1 | ME4.BM2 | ME4.BM3 | ME4.BM4 | ME4.BM5 |
| alpha | 0.00091 | -0.00012 | 4e-04 | 0.00031 | -0.00217 |
| betaMrkt | 1.064 | 1.034 | 1.048 | 1.027 | 1.207 |
| betaSMB | 0.3312 | 0.2162 | 0.2006 | 0.2261 | 0.305 |
| betaHML | -0.3546 | 0.09609 | 0.3503 | 0.5639 | 0.9602 |
| actual | 0.008011 | 0.008429 | 0.01013 | 0.01089 | 0.01168 |
| | | | | | |
| | BIG.LoBM | ME5.BM2 | ME5.BM3 | ME5.BM4 | BIG.HiBM |
| alpha | 0.00073 | 0.00017 | 7e-05 | -0.00166 | -0.00164 |
| betaMrkt | 1.019 | 0.9854 | 0.981 | 1.034 | 1.134 |
| betaSMB | -0.15 | -0.2122 | -0.2313 | -0.1686 | -0.1353 |
| betaHML | -0.2519 | 0.0313 | 0.311 | 0.6385 | 0.9772 |
| actual | 0.006622 | 0.006886 | 0.007933 | 0.008206 | 0.01053 |

```

coefs <- data.frame(t(coefs))
kable(coefs)

```

| | alpha | betaMrkt | betaSMB | betaHML | actual |
|------------|----------|----------|---------|----------|-----------|
| SMALL.LoBM | -0.00723 | 1.25073 | 1.51317 | 0.44893 | 0.0080490 |
| ME1.BM2 | -0.00373 | 1.06993 | 1.54024 | 0.22968 | 0.0093410 |
| ME1.BM3 | -0.00112 | 1.05073 | 1.22688 | 0.50435 | 0.0121487 |
| ME1.BM4 | 0.00105 | 0.91796 | 1.22694 | 0.58482 | 0.0137078 |
| SMALL.HiBM | 0.00063 | 0.97585 | 1.32734 | 0.91703 | 0.0154456 |
| ME2.BM1 | -0.00215 | 1.08840 | 1.13642 | -0.23440 | 0.0078980 |
| ME2.BM2 | -0.00033 | 1.02676 | 0.96611 | 0.14971 | 0.0104836 |
| ME2.BM3 | 0.00038 | 0.99346 | 0.82347 | 0.37182 | 0.0115339 |
| ME2.BM4 | 0.00062 | 0.97966 | 0.81675 | 0.56803 | 0.0125122 |
| ME2.BM5 | -0.00029 | 1.06580 | 0.89446 | 0.89824 | 0.0138949 |
| ME3.BM1 | -0.00098 | 1.13374 | 0.82191 | -0.23845 | 0.0085043 |
| ME3.BM2 | 0.00109 | 1.01309 | 0.50674 | 0.04692 | 0.0100712 |
| ME3.BM3 | 0.00061 | 1.00672 | 0.43267 | 0.32289 | 0.0105557 |
| ME3.BM4 | 0.00055 | 1.00021 | 0.44911 | 0.55647 | 0.0115219 |
| ME3.BM5 | -0.00053 | 1.11593 | 0.59972 | 0.87431 | 0.0130966 |
| ME4.BM1 | 0.00091 | 1.06353 | 0.33118 | -0.35457 | 0.0080107 |
| ME4.BM2 | -0.00012 | 1.03368 | 0.21618 | 0.09609 | 0.0084292 |
| ME4.BM3 | 0.00040 | 1.04850 | 0.20055 | 0.35027 | 0.0101255 |
| ME4.BM4 | 0.00031 | 1.02701 | 0.22608 | 0.56395 | 0.0108878 |

| | alpha | betaMrkt | betaSMB | betaHML | actual |
|----------|----------|----------|----------|----------|-----------|
| ME4.BM5 | -0.00217 | 1.20685 | 0.30503 | 0.96021 | 0.0116846 |
| BIG.LoBM | 0.00073 | 1.01884 | -0.15002 | -0.25193 | 0.0066222 |
| ME5.BM2 | 0.00017 | 0.98542 | -0.21217 | 0.03130 | 0.0068857 |
| ME5.BM3 | 0.00007 | 0.98097 | -0.23125 | 0.31101 | 0.0079334 |
| ME5.BM4 | -0.00166 | 1.03420 | -0.16865 | 0.63847 | 0.0082060 |
| BIG.HiBM | -0.00164 | 1.13357 | -0.13526 | 0.97718 | 0.0105276 |

c

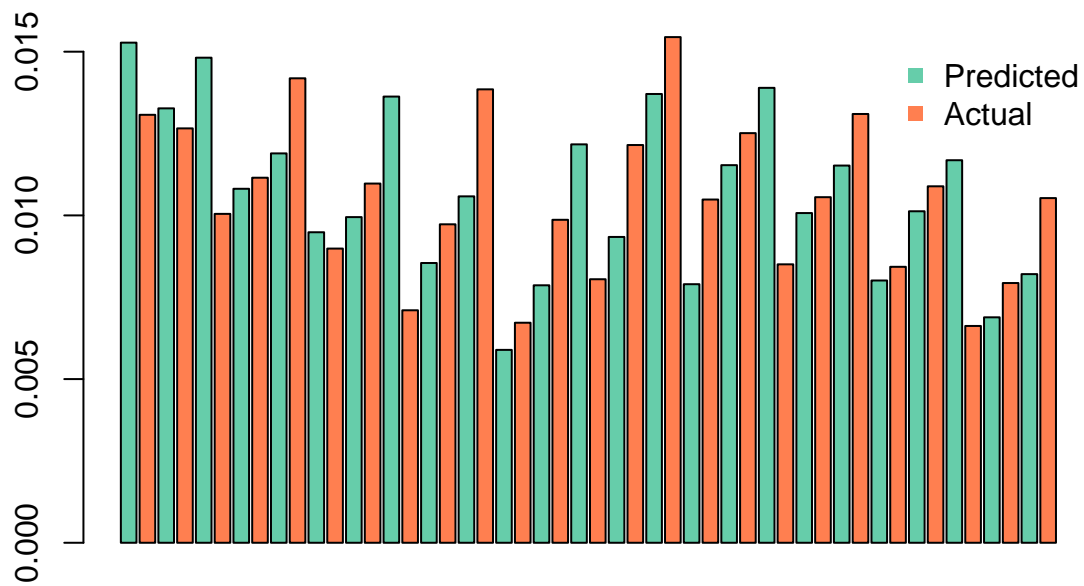
```
lm.ff3f <- lm(actual ~ betaMrkt + betaSMB + betaHML, coefs)
summary(lm.ff3f)
```

```
##
## Call:
## lm(formula = actual ~ betaMrkt + betaSMB + betaHML, data = coefs)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.0020975 -0.0001985  0.0001491  0.0006034  0.0014887
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.0190361  0.0031442   6.054 5.23e-06 ***
## betaMrkt     -0.0108778  0.0030279  -3.593 0.001714 **
## betaSMB       0.0018208  0.0004059   4.485 0.000204 ***
## betaHML       0.0043667  0.0005628   7.758 1.34e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.00109 on 21 degrees of freedom
## Multiple R-squared:  0.8102, Adjusted R-squared:  0.7831
## F-statistic: 29.88 on 3 and 21 DF,  p-value: 9.103e-08
```

d

```
coefs$predictions <- mean(mrkt$Mkt.RF) * coefs$betaMrkt +
  mean(mrkt$SMB) * coefs$betaSMB +
  mean(mrkt$HML) * coefs$betaHML

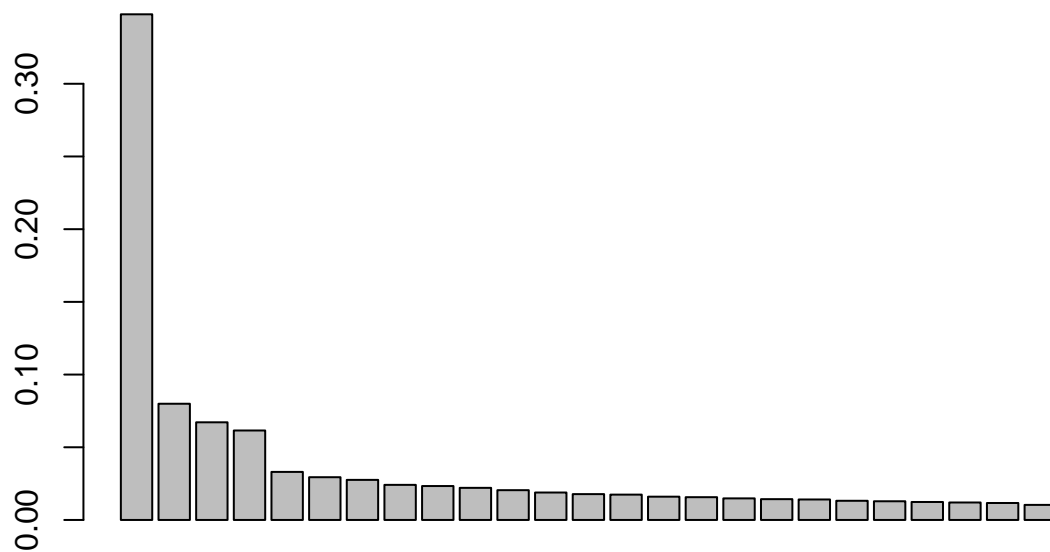
barplot(c(coefs$predictions, coefs$actual), beside = T, col=c("aquamarine3","coral"))
legend("topright", c("Predicted","Actual"), pch=15, col=c("aquamarine3","coral"), bty="n")
```



Problem 4

a

```
covMat <- cov(subset(ports, select = -(Date)))
eigens <- eigen(covMat)
barplot(sqrt(eigens$values))
```



```
loadings <- eigens$vectors[c(1:5),]
pc1 <- matrix(loadings[1,], nrow = 5, ncol = 5)
pc2 <- matrix(loadings[2,], nrow = 5, ncol = 5)
pc3 <- matrix(loadings[3,], nrow = 5, ncol = 5)
pc4 <- matrix(loadings[4,], nrow = 5, ncol = 5)
kable(pc1)
```


| | | | | |
|------------|------------|------------|------------|------------|
| -0.2902098 | 0.2012026 | 0.0259154 | -0.0405035 | 0.0058789 |
| -0.6087921 | -0.0772449 | -0.0780085 | 0.0052111 | 0.0203759 |
| -0.4981649 | -0.0702421 | -0.0427339 | -0.0051010 | 0.0246795 |
| 0.4712854 | 0.0345290 | 0.0695276 | 0.0051879 | -0.0053488 |
| -0.0533622 | -0.0088743 | -0.0234012 | -0.0509555 | 0.0010785 |

`kable(pc2)`

| | | | | |
|------------|------------|------------|------------|------------|
| -0.2486496 | 0.0430704 | -0.2147033 | -0.0199065 | 0.0404258 |
| -0.2721463 | -0.0174940 | -0.0976253 | 0.0292416 | 0.0248087 |
| 0.3545691 | -0.0774821 | 0.0783678 | 0.0715920 | -0.0152345 |
| -0.2524731 | 0.1829801 | -0.0551747 | -0.0744769 | 0.0288245 |
| -0.7391359 | 0.0564767 | -0.0580795 | -0.0081577 | -0.0000387 |

`kable(pc3)`

| | | | | |
|------------|------------|------------|------------|------------|
| -0.2385009 | -0.0285400 | 0.0487068 | -0.1692358 | -0.0550929 |
| -0.1457029 | 0.6923218 | -0.0035629 | -0.0359713 | 0.0163645 |
| 0.0228324 | -0.3072733 | 0.3476169 | 0.0607575 | -0.0168612 |
| -0.1635031 | 0.1531875 | -0.1650339 | -0.0715323 | -0.0103402 |
| 0.2937583 | -0.1102879 | -0.0125181 | 0.0699665 | -0.0148553 |

`kable(pc4)`

| | | | | |
|------------|------------|------------|------------|------------|
| -0.2224945 | 0.0215300 | 0.2229385 | 0.1656413 | -0.2794113 |
| -0.1454130 | 0.1185867 | 0.0827552 | -0.0843345 | 0.0513415 |
| 0.0808813 | -0.0126698 | -0.2626865 | -0.0738195 | 0.0865126 |
| -0.2768872 | -0.3047850 | 0.0058318 | 0.4801660 | -0.2038050 |
| 0.0152032 | 0.0602267 | -0.1348032 | -0.4097434 | 0.1508923 |