Investments HW 3

Zachary Fogelson

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Problem 1

a

$$Var(A+B+C+D) = w_a^2 Var(A) + w_b^2 Var(B) + w_c^2 Var(C) + w_d^2 Var(D) +$$
$$2*(w_a w_b cov(A,B) + w_a w_c cov(A,C) + w_a w_d cov(A,D) + w_b w_c cov(B,C) + w_b w_d cov(B,D) + w_c w_d cov(C,D))$$

$$Var(Mrkt) = .0715$$

$$Cov(A, A+B+C+D) = Var(A) + Cov(A,B) + C(A,C) + Cov(A,D)$$

Similarly for B,C,D. Interestingly, this corresponds to the sum of the elements in any given row of the Variance Covariance matrix!

$$Cov(A,P) = 0.2308$$
$$Cov(B,P) = 0.2660$$
$$Cov(C,P) = 0.3012$$
$$Cov(D,P) = 0.3364$$

b

$$\beta = \frac{Cov(P,A)}{\sigma_P^2}$$

$$\beta_A = 3.226$$
$$\beta_B = 3.718$$
$$\beta_C = 4.210$$
$$\beta_D = 4.702$$

c

$$r_A - r_f = \beta_A * [E(r_m) - r_f]$$

$$E(r_A - r_f) = 0.226$$
$$E(r_B - r_f) = 0.260$$
$$E(r_C - r_f) = 0.295$$
$$E(r_D - r_f) = 0.329$$

d

MVE Weights of the Portfolio:

A: 24%

B: 25%

C: 25%

D: 26%

The MVE weights are exactly the percentage of each firm over the value of the market.

e MVE Weights of the Portfolio:

A: 25%

B: 25%

C: 25%

D: 25%

The weights have changed.

f

MVE Weights of the Portfolio:

A: 29%

B: 23%

C: 24%

D: 24%

Var(P) = .0028

Problem 2

- Format Data

```
openFormat <- function(x, skips){
  y <- read.csv(x, skip=skips, nrows=1068)
  y <- rename(y, c("X"="Date"))
  y <- y[y$Date > 193200,]
  tmp <- y$Date
  y <- y[,-1]/100
  y$Date <- tmp
  y
}

mrkt <- openFormat("F-F_Research_Data_Factors.CSV", 3)
ports <- openFormat("25_Portfolios_5x5.CSV", 19)
dates <- ports$Date
ports <- ports - mrkt$RF
ports$Date <- dates
```

a

```
regressors <- names(ports)[-length(ports)]
coefs <- data.frame(Index=c(1,2), row.names = c("alpha", "beta"))
for(i in regressors){
  cs <- coefficients(lm(ports[,i] ~ mrkt$Mkt.RF))
  coefs <- cbind(coefs, rbind(round(as.numeric(cs["(Intercept)"]),5), round(as.numeric(cs["mrkt$Mkt.RF"]
```

```r
}
coefs <- coefs[,-1]
names(coefs) <- regressors
# pandoc.table(coefs,split.tables=90 )
```

	SMALL.LoBM	ME1.BM2	ME1.BM3	ME1.BM4	SMALL.HiBM
alpha	-0.00396	-0.00117	0.00199	0.00443	0.00524
beta	1.651	1.444	1.395	1.274	1.402

	ME2.BM1	ME2.BM2	ME2.BM3	ME2.BM4	ME2.BM5
alpha	-0.00161	0.0013	0.00259	0.00347	0.00375
beta	1.306	1.262	1.23	1.243	1.394

	ME3.BM1	ME3.BM2	ME3.BM3	ME3.BM4	ME3.BM5
alpha	-0.00082	0.00183	0.00219	0.00293	0.00309
beta	1.281	1.132	1.149	1.18	1.375

	ME4.BM1	ME4.BM2	ME4.BM3	ME4.BM4	ME4.BM5
alpha	0.00011	0.00046	0.0018	0.00245	0.0014
beta	1.085	1.095	1.144	1.159	1.413

	BIG.LoBM	ME5.BM2	ME5.BM3	ME5.BM4	BIG.HiBM
alpha	-0.00029	2e-05	0.00084	0.00028	0.00147
beta	0.9492	0.943	0.9748	1.089	1.245

b

```r
coefs <- rbind(coefs, sapply(ports, mean))
rownames(coefs)  <- c("alpha", "beta", "actual")
coefs <- data.frame(t(coefs))
kable(coefs)
```

	alpha	beta	actual
SMALL.LoBM	-0.00396	1.65058	0.0080490
ME1.BM2	-0.00117	1.44401	0.0093410
ME1.BM3	0.00199	1.39526	0.0121487
ME1.BM4	0.00443	1.27417	0.0137078
SMALL.HiBM	0.00524	1.40240	0.0154456
ME2.BM1	-0.00161	1.30590	0.0078980
ME2.BM2	0.00130	1.26222	0.0104836
ME2.BM3	0.00259	1.22953	0.0115339
ME2.BM4	0.00347	1.24267	0.0125122

	alpha	beta	actual
ME2.BM5	0.00375	1.39384	0.0138949
ME3.BM1	-0.00082	1.28106	0.0085043
ME3.BM2	0.00183	1.13201	0.0100712
ME3.BM3	0.00219	1.14924	0.0105557
ME3.BM4	0.00293	1.18019	0.0115219
ME3.BM5	0.00309	1.37529	0.0130966
ME4.BM1	0.00011	1.08544	0.0080107
ME4.BM2	0.00046	1.09543	0.0084292
ME4.BM3	0.00180	1.14362	0.0101255
ME4.BM4	0.00245	1.15873	0.0108878
ME4.BM5	0.00140	1.41345	0.0116846
BIG.LoBM	-0.00029	0.94915	0.0066222
ME5.BM2	0.00002	0.94301	0.0068857
ME5.BM3	0.00084	0.97485	0.0079334
ME5.BM4	0.00028	1.08938	0.0082060
BIG.HiBM	0.00147	1.24520	0.0105276

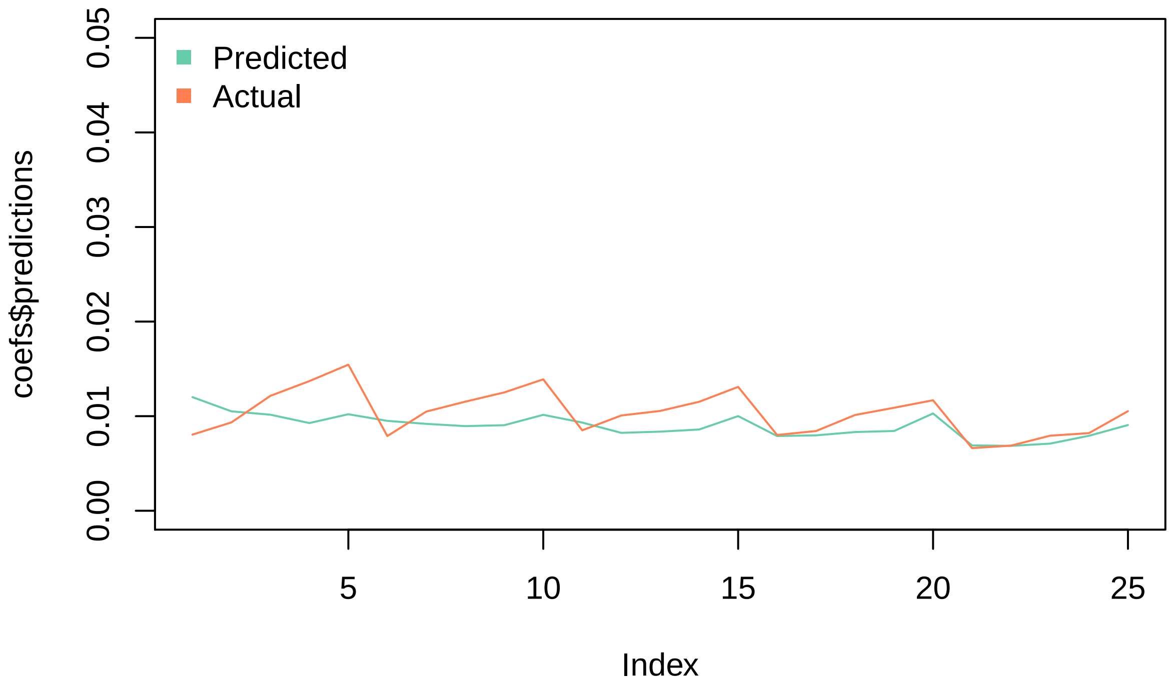
c

```r
lm.capm <- lm(actual ~ beta, coefs)
summary(lm.capm)
```

```
##
## Call:
## lm(formula = actual ~ beta, data = coefs)
##
## Residuals:
##        Min         1Q     Median         3Q        Max
## -0.0050694 -0.0013277  0.0001523  0.0012318  0.0039872
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 0.002078   0.003135   0.663   0.5140
## beta        0.006689   0.002521   2.654   0.0142 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.002092 on 23 degrees of freedom
## Multiple R-squared:  0.2344, Adjusted R-squared:  0.2011
## F-statistic: 7.042 on 1 and 23 DF,  p-value: 0.01419
```

d

```r
coefs$predictions <- mean(mrkt$Mkt.RF) * coefs$beta
plot(coefs$predictions, type="l", col="aquamarine3", ylim = c(0,.05))
lines(coefs$actual, col="coral")
legend("topleft", c("Predicted","Actual"), pch=15, col=c("aquamarine3","coral"), bty="n")
```



Problem 3

a

```
coefs <- data.frame(Index=c(1,2,3,4), row.names = c("alpha", "betaMrkt", "betaSMB", "betaHML"))
for(i in regressors){
  cs <- coefficients(lm(ports[,i] ~ mrkt$Mkt.RF + mrkt$SMB + mrkt$HML))
  coefs <- cbind(coefs, rbind(round(as.numeric(cs["(Intercept)"]),5), round(as.numeric(cs["mrkt$Mkt.RF"
}
coefs <- coefs[,-1]
names(coefs) <- regressors
```

b

```
coefs <- rbind(coefs, sapply(ports, mean))
rownames(coefs)  <- c(rownames(coefs)[-length(rownames(coefs))], "actual")
# pandoc.table(coefs,split.tables=90 )
```

	SMALL.LoBM	ME1.BM2	ME1.BM3	ME1.BM4	SMALL.HiBM
alpha	-0.00723	-0.00373	-0.00112	0.00105	0.00063
betaMrkt	1.251	1.07	1.051	0.918	0.9758
betaSMB	1.513	1.54	1.227	1.227	1.327
betaHML	0.4489	0.2297	0.5043	0.5848	0.917
actual	0.008049	0.009341	0.01215	0.01371	0.01545

	ME2.BM1	ME2.BM2	ME2.BM3	ME2.BM4	ME2.BM5
alpha	-0.00215	-0.00033	0.00038	0.00062	-0.00029

	ME2.BM1	ME2.BM2	ME2.BM3	ME2.BM4	ME2.BM5
betaMrkt	1.088	1.027	0.9935	0.9797	1.066
betaSMB	1.136	0.9661	0.8235	0.8167	0.8945
betaHML	-0.2344	0.1497	0.3718	0.568	0.8982
actual	0.007898	0.01048	0.01153	0.01251	0.01389

	ME3.BM1	ME3.BM2	ME3.BM3	ME3.BM4	ME3.BM5
alpha	-0.00098	0.00109	0.00061	0.00055	-0.00053
betaMrkt	1.134	1.013	1.007	1	1.116
betaSMB	0.8219	0.5067	0.4327	0.4491	0.5997
betaHML	-0.2384	0.04692	0.3229	0.5565	0.8743
actual	0.008504	0.01007	0.01056	0.01152	0.0131

	ME4.BM1	ME4.BM2	ME4.BM3	ME4.BM4	ME4.BM5
alpha	0.00091	-0.00012	4e-04	0.00031	-0.00217
betaMrkt	1.064	1.034	1.048	1.027	1.207
betaSMB	0.3312	0.2162	0.2006	0.2261	0.305
betaHML	-0.3546	0.09609	0.3503	0.5639	0.9602
actual	0.008011	0.008429	0.01013	0.01089	0.01168

	BIG.LoBM	ME5.BM2	ME5.BM3	ME5.BM4	BIG.HiBM
alpha	0.00073	0.00017	7e-05	-0.00166	-0.00164
betaMrkt	1.019	0.9854	0.981	1.034	1.134
betaSMB	-0.15	-0.2122	-0.2313	-0.1686	-0.1353
betaHML	-0.2519	0.0313	0.311	0.6385	0.9772
actual	0.006622	0.006886	0.007933	0.008206	0.01053

```
coefs <- data.frame(t(coefs))
kable(coefs)
```

	alpha	betaMrkt	betaSMB	betaHML	actual
SMALL.LoBM	-0.00723	1.25073	1.51317	0.44893	0.0080490
ME1.BM2	-0.00373	1.06993	1.54024	0.22968	0.0093410
ME1.BM3	-0.00112	1.05073	1.22688	0.50435	0.0121487
ME1.BM4	0.00105	0.91796	1.22694	0.58482	0.0137078
SMALL.HiBM	0.00063	0.97585	1.32734	0.91703	0.0154456
ME2.BM1	-0.00215	1.08840	1.13642	-0.23440	0.0078980
ME2.BM2	-0.00033	1.02676	0.96611	0.14971	0.0104836
ME2.BM3	0.00038	0.99346	0.82347	0.37182	0.0115339
ME2.BM4	0.00062	0.97966	0.81675	0.56803	0.0125122
ME2.BM5	-0.00029	1.06580	0.89446	0.89824	0.0138949
ME3.BM1	-0.00098	1.13374	0.82191	-0.23845	0.0085043
ME3.BM2	0.00109	1.01309	0.50674	0.04692	0.0100712
ME3.BM3	0.00061	1.00672	0.43267	0.32289	0.0105557
ME3.BM4	0.00055	1.00021	0.44911	0.55647	0.0115219
ME3.BM5	-0.00053	1.11593	0.59972	0.87431	0.0130966

	alpha	betaMrkt	betaSMB	betaHML	actual
ME4.BM1	0.00091	1.06353	0.33118	-0.35457	0.0080107
ME4.BM2	-0.00012	1.03368	0.21618	0.09609	0.0084292
ME4.BM3	0.00040	1.04850	0.20055	0.35027	0.0101255
ME4.BM4	0.00031	1.02701	0.22608	0.56395	0.0108878
ME4.BM5	-0.00217	1.20685	0.30503	0.96021	0.0116846
BIG.LoBM	0.00073	1.01884	-0.15002	-0.25193	0.0066222
ME5.BM2	0.00017	0.98542	-0.21217	0.03130	0.0068857
ME5.BM3	0.00007	0.98097	-0.23125	0.31101	0.0079334
ME5.BM4	-0.00166	1.03420	-0.16865	0.63847	0.0082060
BIG.HiBM	-0.00164	1.13357	-0.13526	0.97718	0.0105276

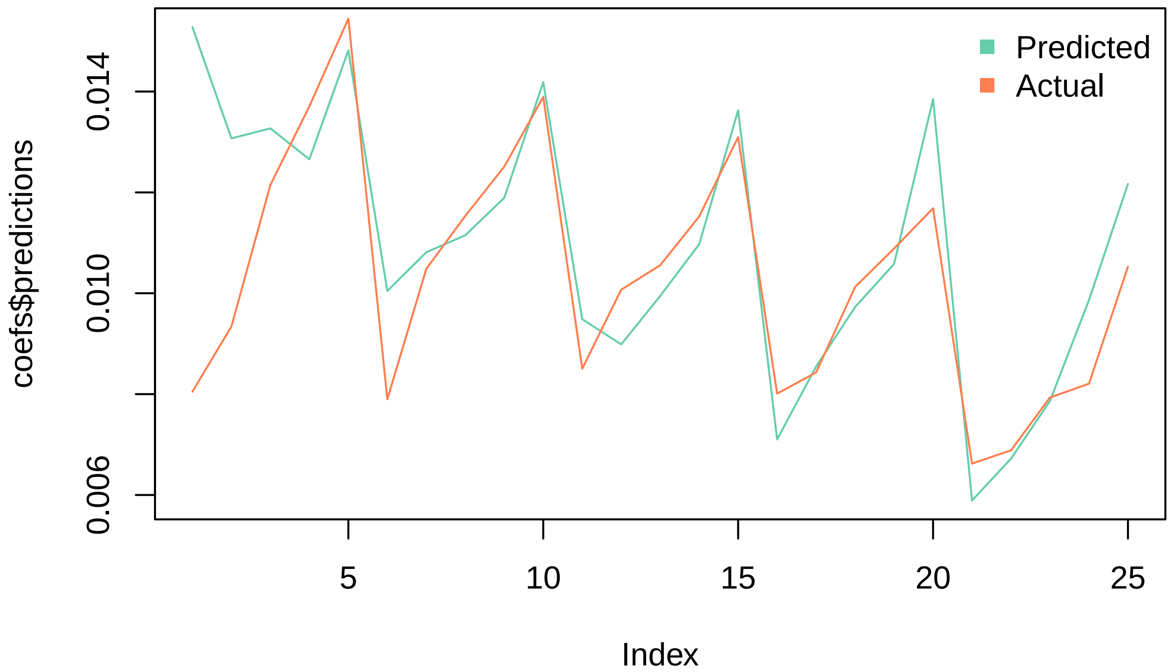
c

```r
lm.ff3f <- lm(actual ~ betaMrkt + betaSMB + betaHML, coefs)
summary(lm.ff3f)
```

```
##
## Call:
## lm(formula = actual ~ betaMrkt + betaSMB + betaHML, data = coefs)
##
## Residuals:
##        Min         1Q     Median         3Q        Max
## -0.0020975 -0.0001985  0.0001491  0.0006034  0.0014887
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.0190361  0.0031442   6.054 5.23e-06 ***
## betaMrkt    -0.0108778  0.0030279  -3.593 0.001714 **
## betaSMB      0.0018208  0.0004059   4.485 0.000204 ***
## betaHML      0.0043667  0.0005628   7.758 1.34e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.00109 on 21 degrees of freedom
## Multiple R-squared:  0.8102, Adjusted R-squared:  0.7831
## F-statistic: 29.88 on 3 and 21 DF,  p-value: 9.103e-08
```

d

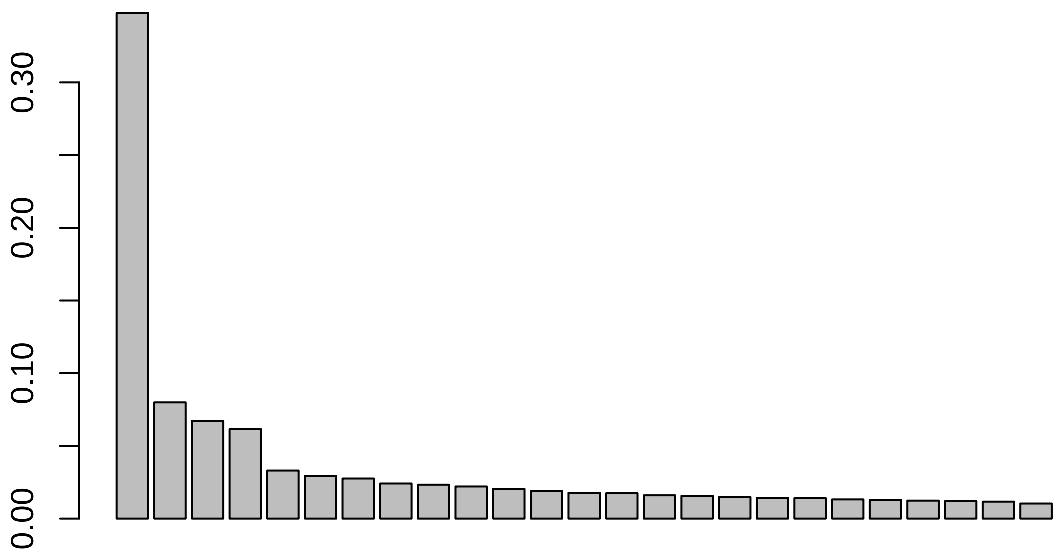
```r
coefs$predictions <- mean(mrkt$Mkt.RF) * coefs$betaMrkt +
  mean(mrkt$SMB) * coefs$betaSMB +
  mean(mrkt$HML) * coefs$betaHML

plot(coefs$predictions, type="l", col="aquamarine3")
lines(coefs$actual, col="coral")
legend("topright", c("Predicted","Actual"), pch=15, col=c("aquamarine3","coral"), bty="n")
```



Problem 4

a

```
covMat <- cov(subset(ports, select = -(Date)))
eigens <- eigen(covMat)
barplot(sqrt(eigens$values))
```




```
loadings <- eigens$vectors[c(1:5),]
pc1 <- matrix(loadings[,1], nrow = 5, ncol = 5)
pc2 <- matrix(loadings[,2], nrow = 5, ncol = 5)
pc3 <- matrix(loadings[,3], nrow = 5, ncol = 5)
```

```r
pc4 <- matrix(loadings[,4], nrow = 5, ncol = 5)
kable(pc1)
```

-0.2902098	-0.2902098	-0.2902098	-0.2902098	-0.2902098
-0.2486496	-0.2486496	-0.2486496	-0.2486496	-0.2486496
-0.2385009	-0.2385009	-0.2385009	-0.2385009	-0.2385009
-0.2224945	-0.2224945	-0.2224945	-0.2224945	-0.2224945
-0.2499112	-0.2499112	-0.2499112	-0.2499112	-0.2499112

```r
kable(pc2)
```

-0.6087921	-0.6087921	-0.6087921	-0.6087921	-0.6087921
-0.2721463	-0.2721463	-0.2721463	-0.2721463	-0.2721463
-0.1457029	-0.1457029	-0.1457029	-0.1457029	-0.1457029
-0.1454130	-0.1454130	-0.1454130	-0.1454130	-0.1454130
-0.1663922	-0.1663922	-0.1663922	-0.1663922	-0.1663922

```r
kable(pc3)
```

-0.4981649	-0.4981649	-0.4981649	-0.4981649	-0.4981649
0.3545691	0.3545691	0.3545691	0.3545691	0.3545691
0.0228324	0.0228324	0.0228324	0.0228324	0.0228324
0.0808813	0.0808813	0.0808813	0.0808813	0.0808813
-0.0856666	-0.0856666	-0.0856666	-0.0856666	-0.0856666

```r
kable(pc4)
```

0.4712854	0.4712854	0.4712854	0.4712854	0.4712854
-0.2524731	-0.2524731	-0.2524731	-0.2524731	-0.2524731
-0.1635031	-0.1635031	-0.1635031	-0.1635031	-0.1635031
-0.2768872	-0.2768872	-0.2768872	-0.2768872	-0.2768872
-0.3571149	-0.3571149	-0.3571149	-0.3571149	-0.3571149