Class 2

Exercises

1. Suppose the LIBOR discount factor curve for the first year is as follows:

t	discount factor
0	1
0.25	0.99782972
0.5	0.994845185
1	0.987439389

(a) What is the fair rate for an FRA on 3M LIBOR where the floating rate is observed in 3 months?

(b) What is the present value of an FRA on 3M LIBOR where the floating rate is observed in 6 months if you receive the fixed rate of 1% on a notional of $10 million? (Interpolate by assuming that the instantaneous forward rate is constant between time points.)

(c) You encounter an FRA from 6M to 1Y where the fixed leg is quoted, as usual, as a simple rate of interest, but the floating rate is 3M LIBOR compounded over the term of the FRA. That is, in 6M the 3M LIBOR rate r_1 is observed, then in 9M the 3M LIBOR rate r_2 is observed, and the floating amount paid on the terminating date is:

$$N\left[(1+r_1\Delta t)(1+r_2\Delta t)-1\right]$$

Determine the fair fixed rate for this contract. Support your answer by showing how such a contract may be hedged or replicated.

2. When the term structure of zero rates is…

t	r (continuous)
0.25	0.012
0.5	0.0155
1	0.018
2	0.02

…you are asked to report certain characteristics of an FRN with the following terms and conditions:

principal: 100
reset and coupon frequency: quarterly
time to maturity: 1.2 years
floating rate margin: 2%
most recent reset rate: 0.0145

For the purposes of this question, disregard the possibility that the bond may default. Interpolate in the term structure of interest rates by assuming that the continuously compounded zero rates are linear, and that the rate is constant between 0 and 0.25 years.

(a) What are the projected cash flows of the bond?
(b) What is the present value of the bond?
(c) What is the bond's duration? Its convexity? (Use the more general form for these analytics based on dollar duration.)

3. For the following term structure of discount factors….

t	df
0.5	0.975309912
1	0.946485148
1.5	0.917364861
2	0.884263663

(a) Determine the par swap rate for a 2-year swap whose fixed leg pays annually.
(b) Determine the par swap rate for a 2-year swap whose fixed leg pays semiannually.
(c) Determine the forward par swap rate for a 1-year swap commencing in 1 year whose fixed leg pays semiannually.

4. The term structure of zero rates is as follows:

t	rate (continuous)
1	0.03
2	0.035

(a) A bond paying annual coupons of 5% per year matures 1 year from today and currently trades at 101.25. What is the z-spread for this bond?
(b) A bond from a different issuer paying annual coupons of 5% per year matures 2 years from today and currently trades at 102. What is the constant z-spread for this bond?
(c) Suppose a second bond from the same issuer as in part (b) is found. This bond pays a 2% annual coupon and matures in 1 year; its price is 98.75. Use this bond and the one from part (b) to find the 2-year term structure of z-spreads for this issuer.

5. The z-spreads for a particular issuer at years 1 and 2 are:

t	z-spread
1	0.005
2	0.0068

For the purposes of these questions, assume no recovery in the event of default.

(a) What is the unconditional probability of default in the second year?
(b) What is the probability of default in the second year conditional on survival for the first year?
(c) What is the forward hazard rate for the second year?

6. A particular issuer has a constant hazard rate of 0.01. For the purposes of this question, assume that the risk-free interest rate is 0.05 to all maturities.

(a) A semiannual bond with maturity 10 years paying a coupon rate of 4% trades at a price of 89. What is the present value of the recovery on the bond?
(b) What is the recovery rate implied by this value?
(c) Suppose that the bond in question is backed by collateral posted by the issuer, and due to recent changes, that collateral has appreciated to such a degree that you now consider the recovery rate on the bond to be 100%. What is the price of the bond under this assumption?
(d) Compare your price above in (c) to the price this bond would have if it were free of credit risk. The two prices differ. Explain why one is greater. What change to the bond's coupon, if any, would cause the other to be greater?

Applications

1. *A Simple Example of Bootstrapping Swaps Curves*

As with Treasury bonds, swaps can be used to determine a term structure of discount factors for pricing other instruments. Swaps may, for many applications, be a better choice for discounting, since Treasury bonds have unique characteristics that other instruments (e.g., corporate debt) do not share.

For this application, we follow a simplified version of the procedure for deriving two key curves in the EUR market: The EONIA (OIS) discounting curve, and the 6M EURIBOR curve. All swaps will be assumed to pay both fixed and floating cash flows semiannually.

(a) The first step is to use fixed-for EONIA swaps to determine the discounting curve. We observe the following market rates:

Fixed-for-EONIA

maturity (years)	par swap rate (percent)
1	-0.366%
2	-0.275%
3	-0.160%
4	-0.035%
5	0.015%
7	0.275%
10	0.660%

(note rates are for illustration purposes, not actual market quotes)

For these swaps, the EONIA discounting curve you solve for will be used both to determine the forward rates appropriate to the floating leg and to discount the cash flows.

Assume that the continuously compounded spot rate is constant from now to the 1-year maturity; interpolate linearly in the continuously compounded spot rates for cash flows between the maturities given above. The output of this step should be a set of discount factors at the maturities corresponding to the market quotes above, with the final known point at 10 years.

(b) The second step is to use fixed-for 6M EURIBOR swaps to determine a 6M EURIBOR curve. We observe the following market rates:

Fixed-for-6M EURIBOR	
maturity (years)	par swap rate (percent)
1	-0.250%
2	-0.160%
3	-0.040%
4	0.100%
5	0.250%
7	0.520%
10	0.890%

(note rates are for illustration purposes, not actual market quotes)

For these swaps, the 6M EURIBOR curve you solve for will be used to determine the forward rates appropriate to the floating leg, while the EONIA curve you generated in part (a) above will be used to discount all cash flows. The output of this step should be a set of discount factors at the maturities corresponding to the market quotes above, just as in the EONIA step.

(c) Using your calculator, show the result of a shock upward of 1bp in all EONIA rates while keeping the EURIBOR swap rates unchanged. This will give you a different 6M EURIBOR curve. Was the result what you expected? What do you think would happen if we left our 6M EURIBOR discounting curve unchanged and recalculated the EURIBOR par swap rates after a 1bp shock up to the EONIA rates?