6. Forwards and Futures

Forward Contracts

A forward contract is an agreement between two parties to exchange an asset for some fixed price at a specified time in the future. The payoff at expiry T of a forward contract is $(S_T - K)$ to the party receiving the underlying asset and paying the strike price K, where S_T is the terminal spot price of the asset. Typically, at inception it is arranged that the fair value of the forward contract is zero.

A static hedge for forward contracts

If the underlying asset of a forward contract can be traded at spot, then the parties to the agreement can hedge away exposure to the final asset price S_T, which is random, by means of a simple static hedge. The party receiving the asset and paying K, for example, is long the asset; therefore, hedging this exposure requires taking a short position in the asset. (As usual, we assume that assets can be continuously traded, and can be just as easily bought or sold at the same price.)

Shorting the asset now yields cash in the amount S_0, the asset's spot price. The cash proceeds are invested in a zero-coupon bond maturing at the expiry of the forward contract; that bond's price today is P_T per unit notional. At T, the bond matures, yielding S_0 / P_T in cash. At the same time, the forward contract expires, so the party receives the asset in exchange for K. The asset is then used to close the short position. We assume for the moment that the underlying asset pays no cash flows during the term of the forward contract.

Thus, the net cash flow at T for the portfolio consisting of the forward contract and the static hedge is $S_0 / P_T - K$, a quantity which is nonrandom at time zero. The value of this portfolio at time zero is thus:

$$V_\Pi = P_T\left(\frac{S_0}{P_T} - K\right) = S_0 - KP_T = S_0 - Ke^{-r_T T}$$

…where r_T is the risk-free zero rate to expiry of the forward contract.

Repeating an argument we have seen before, we observe that the portfolio at time zero consists of the forward contract and a static hedge that can be entered into at zero cost, so we may say that:

$$V_\Pi = V + V_{hedge} = V$$

$$V = S_0 - KP_T = S_0 - Ke^{-r_T T}$$

This relation can be used to price any forward contract, no matter its strike. At inception, however, it is most often the case that K is chosen to make this zero, in which case the fair strike of the contract is called the forward price, which satisfies:

$$0 = S_0 - FP_T$$

$$F = \frac{S_0}{P_T} = S_0 e^{r_T T}$$

This may seem an odd result. The assumption of no arbitrage appears to lead to the conclusion that all assets not paying cash flows must increase in value at the risk-free rate. Recall, however, that this result is not a forecast of future asset values; it is simply the condition that must hold in order for the forward price of the asset to be arbitrage-free, under the assumptions we have repeatedly used.

A forward contract on a risk-free zero-coupon bond

To illustrate the case of a forward contract on an asset that pays no cash flows before expiry, we return to an example we have already encountered twice before: a zero-coupon bond. For this case, we adjust our notation somewhat, taking T_b to be the maturity of the underlying bond, and $T < T_b$ to be the expiry of the forward.

To determine the forward price of the asset, we observe that the price today of a zero-coupon bond with face amount N maturing at T_b is NP_{Tb}. Thus:

$$F = \frac{NP_{T_b}}{P_T} = NP_{T,T_b}$$

…where $P_{T,Tb}$ is the quantity we earlier called the forward discount factor from T to T_b.

Similarly, we can price a contract struck at any other price:

$$V_{fwd} = NP_{T_b} - KP_T$$

This seems quite natural, since the effect of a long forward position in a zero-coupon bond with strike K is that the party pays K at T (the expiry of the forward contract) to purchase an instrument that pays N at T_b (the maturity of the bond).

Forward price of an asset paying fixed cash flows

When the underlying asset produces cash flows before the expiry of the forward contract, our valuation expression requires an adjustment. To see how to handle this case, we return to the original hedging argument we used with assets that pay no cash flows.

The static hedge we used to value the contract requires the party to go short the asset, in which case the party becomes responsible for paying the asset's cash flows until expiry of the forward. Suppose the asset's cash flows c_i before expiry are known, and will occur at times t_i between now and the expiry T of the forward contract.

As before, we suppose a party is long a forward contract on the asset and wishes to hedge away the dependence upon S_T, the terminal spot price. At time zero the hedge involves going short the asset and putting the proceeds in a risk-free asset. Since we suppose that the cash flows that the short position will require are nonrandom, it is possible to set aside the money to pay them at time zero: For each cash flow, we purchase a risk-free zero-coupon bond that at time t_i will pay c_i to cover the short obligation. So the amount of cash not committed to paying the intervening cash flows is:

$$V_{cash,0} = S_0 - \sum_{i=1}^{n} P_{t_i} c_i = S_0 - \sum_{i=1}^{n} c_i e^{-r_{t_i} t_i}$$

This is the amount of cash we invest in a risk-free asset maturing at T. At the intervening t_i, we are assured of having sufficient cash in the zero-coupon bonds we have set aside for paying the asset's cash flows. At time T, we receive the proceeds of our zero-coupon bond maturing at this time. We receive the asset via the forward contract and use it to close the short; finally, we owe K from the forward. So the net cash flow of the hedged forward in this case is, in terms of the risk-free zero-coupon bonds at time zero…

$$V_{\Pi,T} = \frac{S_0}{P_T} - \sum_{i=1}^{n} c_i \frac{P_{t_i}}{P_T} - K = \frac{S_0}{P_T} - \sum_{i=1}^{n} \frac{c_i}{P_{t_i,t}} - K$$

…or, alternatively, in terms of risk-free zero rates:

$$V_{\Pi,T} = S_0 e^{r_T T} - \sum_{i=1}^{n} c_i e^{r_T T - r_{t_i} t_i} - K$$

Once again, since this cash flow is nonrandom, we may discount to obtain its present value and be assured that this is also the value of the forward contract itself:

$$V = S_0 - \sum_{i=1}^{n} c_i P_{t_i} - K P_T = S_0 - \sum_{i=1}^{n} c_i e^{-r_{t_i} t_i} - K e^{-r_T T}$$

This is essentially the same expression as the one we obtained in the case where the asset pays no cash flows before expiry. The only difference is that the spot price of the asset is adjusted down by the present value of the intervening cash flows. This seems quite reasonable, since after all the party long the asset via the forward is not entitled to any of these cash flows.

We thus introduce the idea of the adjusted spot price of an asset paying cash flows before expiry of the forward contract…

$$S_{0,a} = S_0 - \sum_{i=1}^{n} c_i P_{t_i} = S_0 - \sum_{i=1}^{n} c_i e^{-r_{t_i} t_i}$$

…and can adapt the expressions we initially obtained for the value of the forward contract and the forward price of the asset:

$$V = S_{0,a} - K P_T = S_{0,a} - K e^{-r_T T}$$

$$F = \frac{S_{0,a}}{P_T} = S_{0,a} e^{r_T T}$$

This is a nice formulation, since it extends our initial result without changing its form to any significant degree. It also makes it clear that all we need to do in the case of assets that throw off cash flows in other ways is to determine what their present value is. If this can be done, then we can obtain the adjusted spot price and determine forward prices as well.

A forward contract on a risk-free coupon-bearing bond

If the underlying asset of a forward contract is a bond with notional N paying a fixed coupon rate c with frequency m to maturity T_b, then we may use the above approach to determine the bond's forward price and to value the contract. Once again, we call the forward contract's expiry time $T < T_b$.

Since potentially one or more of the bond's scheduled cash flows occurs before expiry of the forward, we may need to adjust the bond's spot price as described above before calculating the forward price.

The risk-free bond can be priced as the sum of its discounted cash flows, and thus has as its value today:

$$S_0 = \sum_{i=1}^{n} N \frac{c}{m} P_{t_i} + NP_T$$

Suppose some of these t_i occur before the forward contract's expiry, and some after. If we denote by j the number of coupons occurring on or before expiry T, then we may further elaborate this expression:

$$S_0 = \sum_{i=1}^{j} N \frac{c}{m} P_{t_i} + \sum_{i=j+1}^{n} N \frac{c}{m} P_{t_i} + NP_{T_b}$$

$$S_0 - \sum_{i=1}^{j} N \frac{c}{m} P_{t_i} = \sum_{i=j+1}^{n} N \frac{c}{m} P_{t_i} + NP_{T_b}$$

The expression on the left is the adjusted spot price—today's preset value less the present value of the cash flows between now and expiry.

$$S_{0,a} = S_0 - \sum_{i=1}^{j} N \frac{c}{m} P_{t_i}$$

The present value of a forward contract on this asset is thus:

$$V = S_{0,a} - KP_T = S_0 - \sum_{i=1}^{j} N \frac{c}{m} P_{t_i} - KP_T$$

…and the forward price is:

$$F = \frac{1}{P_T} \left(S_0 - \sum_{i=1}^{j} N \frac{c}{m} P_{t_i} \right)$$

Interestingly, while we recognize the left-hand side of the expression…

$$S_0 - \sum_{i=1}^{j} N \frac{c}{m} P_{t_i} = \sum_{i=j+1}^{n} N \frac{c}{m} P_{t_i} + NP_{T_b}$$

…as the adjusted spot price, of course the equation itself indicates that the right-hand side takes exactly the same value. So we can clearly calculate the forward price of the asset by the alternative method of dividing the right-hand side by the discount factor to expiry, obtaining the result in terms of cash flows occurring *after* expiry of the forward:

$$F = \frac{1}{P_T} \left(\sum_{i=j+1}^{n} N \frac{c}{m} P_{t_i} + NP_{T_b} \right) = \sum_{i=j+1}^{n} N \frac{c}{m} P_{T,t_i} + NP_{T,T_b}$$

This is an appealing result indeed, since it reassures us that the forward price can also be thought of as the value of the cash flows after expiry, using the *forward* discount factors. That is, the instrument is also a forward contract on a portfolio of zero-coupon bonds. Seen this way, today's spot price and the value of intervening cash flows is not strictly relevant: We can just as easily obtain the result by looking at cash flows after expiry of the forward contract and discounting them back to expiry using the forward discount factors.

Forward price of an asset paying proportional dividends
It's clear that the previous approach of reducing the spot price by the present value of the known cash flows between today and expiry is useful not only for bonds, but also potentially for equities. Certainly this approach will work if a dollar projection of future dividends is available. Many assets available in the equity markets do pay quite regular dividends—both in terms of size and timing—and can be reasonably handled in this way.

It may make more sense, in some cases, to model an equity dividend not in terms of a set dollar amount, but as a percentage of the equity's value. Certainly from a capital structure standpoint, this seems a reasonable model of how dividends might be paid. We wish to extend the approach above to cover this case.

We suppose as usual a forward contract on an asset with spot price S_0, with contract expiry T and strike K. Further, we suppose that the asset pays a one-time cash flow of a known proportion k of its value at dividend time $t < T$. That is, the cash flow at this time will be kS_{t-}, where the time t- occurs immediately before the dividend-payment time.

A seeming problem with handling this sort of dividend is that the price S_{t-} of the asset at the time of dividend payment is not known at pricing time. If we short the asset at time zero, receiving S_0 in cash, we cannot simply set aside some of the proceeds in a zero-coupon bond to supply the needed amount of cash at dividend-payment time.

Fortunately, there is a handy hedging asset available for this purpose: the equity itself. We imagine entering the short position, receiving S_0 in cash, purchasing k shares of the equity and then setting the remaining cash—$(1 - k)S_0$—aside in a risk-free account as before.

At dividend time t, the equity price jumps down from S_{t-} to $S_t = (1 - k)S_{t-}$, with holders of the equity in that instant also receiving a cash flow of kS_{t-}. At t-, to provide the needed payment for our short position in the equity, we sell the long position in the shares that we set up initially. This provides precisely the cash needed to cover the commitment on the short. That is, in the case of a known fixed dividend, the appropriate asset to use for the dividend was a zero-coupon bond; here, it is the equity.

Mechanically, you obviously would not need to take both a long and short position in the same asset at inception. The same result can be achieved by initially taking a short position in $1 - k$ shares of the asset referenced by the contract. At S_{t-}, you pay $S_{t-}(1 - k)$ to close the short immediately before the dividend is paid. Whatever cash amount this requires, an instant later—after the equity price has jumped—you short 1 full share of the asset, receiving $(1 - k)S_{t-}$, the precise amount you paid before the dividend.

However you arrive at the conclusion, the important thing to understand is that in this case the adjusted spot price is $(1 - k)S_0$, making the value of a forward contract on this asset…

$$V = S_0(1-k) - KP_T = S_0(1-k) - Ke^{-rT}$$

…and its forward price:

$$F = \frac{S_0}{P_T}(1-k) = S_0 e^{rT}(1-k)$$

There is another interesting thing to note about this result, which is that—although we specified a dividend payment time $t < T$, we actually do not need it in order to arrive at this result. The only important thing is that the dividend is paid before expiry of the contract. Indeed, this agrees with another observation we might make about the formula above for the forward contract. In the case of fixed cash dividends, we saw that the forward could be written in terms of an adjusted spot…

$$F = S_{0,a} e^{rT}$$

…where, in the case of fixed cash flows, the adjusted spot was today's spot adjusted down by the present value of the dividends paid between now and expiry:

$$S_{0,a} = S_0 - \sum_{i-1}^{n} c_i e^{-rt_i} = S_0 - V_{dividends}$$

Applying this same pattern to the case of proportional dividends, it is clear that…

$$S_{0,a} = S_0(1-k) = S_0 - kS_0 = S_0 - V_{dividends}$$

$$V_{dividends} = kS_0$$

…irrespective of when the dividend is paid. This makes sense in connection with the idea that the dividend itself is in essence a forward contract, since it pays some fixed multiple of the price at a future time t. The present value of such a contract should always be the same multiple of the spot price.

It is not difficult to extend this to the case of multiple proportional dividends. If before expiry the asset pays proportional dividends with proportions $k_1, k_2, \ldots, k_n$ at times before expiry T, then the adjusted spot is clearly…

$$S_{0,a} = S_0 \prod_{i=1}^{n} (1-k_i)$$

…and the forward…

$$F = S_0 e^{rT} \prod_{i=1}^{n} (1-k_i)$$

Forward price of an asset paying continuous dividends

If instead of paying proportional dividends at discrete times, it is often useful to think of an equity—particularly an equity index—paying a dividend yield continuously, rather than paying a proportion of the equity's value at fixed future times. As we have already seen, one fortunate feature of proportional dividends is that the exact timing of them is less important than the proportion itself. A nice consequence of this is that the present value of a proportional dividend is evidently independent of interest rates, which greatly simplifies an asset modeled using this approach.

If we imagine an asset paying a dividend rate q_m with frequency m, then the expression for the forward price under proportional dividends seen in the previous section becomes:

$$F = S_0 e^{rT}\left(1 - \frac{q}{m}\right)^{mT}$$

It should come as no surprise that we are interested in passing to the continuous-time limit to obtain the result:

$$F = S_0 e^{rT} e^{-qT} = S_0 e^{(r-q)T}$$

In terms we have seen before, this means that the adjusted spot price and present value of the dividends must be:

$$S_{0,a} = S_0 - V_{dividends} = S_0 e^{-qT}$$

$$V_{dividends} = S_0\left(1 - e^{-qT}\right)$$

Note that, as one would expect, this agrees with the present value of the dividends obtained by assuming that the forward price is the actual price path of the equity:

$$V_{dividends} = \int_0^T qS_t e^{-rt}\,dt = \int_0^T qS_0 e^{(r-q)t} e^{-rt}\,dt = S_0 \int_0^T qe^{-qt}\,dt$$

$$V_{dividends} = -S_0 e^{-qt}\Big|_0^T = S_0\left(1 - e^{-qT}\right)$$

A forward contract on a foreign currency

If the asset underlying a forward contract is a foreign currency, then we must consider the fact that a currency generates a risk-free return in pricing a forward contract on it. Suppose the price in domestic currency of a single unit of the foreign currency is X_0 today; the contract is to receive a unit of foreign currency in exchange for an amount X_K at time T. The payoff at expiry is thus:

$$V_T = X_T - X_K$$

To hedge this exposure, we must short the currency. Imagine for the moment we do so by selling a zero-coupon bond in the foreign currency. We wish to deliver 1 unit of the foreign currency at T, to close the short so we receive $P_{T,f}$ in foreign currency today, which is the discount factor in foreign currency to maturity T. We convert this amount to domestic currency at the spot exchange rate X_0 and purchase a zero-coupon bond in domestic currency with the proceeds.

So, at time T we owe 1 unit of foreign currency from the hedge; its value at that time is X_T. Similarly, we receive the payoff from our zero-coupon bond in domestic currency:

$$V_{hedge,T} = X_0 \frac{P_{T,f}}{P_T} - X_T$$

So, the total payoff of the hedged portfolio, in domestic currency, is…

$$V_{\Pi,T} = V_T + V_{hedge,T} = X_0 \frac{P_{T,f}}{P_T} - X_K = X_0 e^{(r-q)T} - X_K$$

…where q is the applicable risk-free interest rate in the foreign currency. As we have seen before, since the portfolio can be hedged to guarantee this payoff without incurring any additional cost, the instrument itself must have this value, discounted to the present…

$$V = X_0 P_{T,f} - X_K P_T = X_0 e^{-qT} - X_K e^{-rT}$$

…making the forward price of one unit of foreign currency…

$$X_F = X_0 \frac{P_{T,f}}{P_T} = X_0 e^{(r-q)T}$$

…more commonly called the forward exchange rate. The interest rate in the foreign currency behaves like a dividend rate, since between now and T this is the yield obtained on the underlying asset.

Generally, an FX forward contract is not specified in terms of exchange rates, but instead in terms of a forward exchange of A units in one currency for B units in another. Clearly this ratio can be used to determine the strike rate X_K implied by these terms.

The futures contract

Forward contracts are still quite common in the FX market, and they are still used at times in other asset classes as well. However, they do have drawbacks. A forward contract can entail significant credit risk if the spot price of the asset moves very far from the agreed strike price, since the contract is settled only at expiry. Too, since the forward contract is a private agreement between two parties—an over-the-counter (OTC) instrument—it may be difficult or expensive to liquidate such a contract at need. The exchange-traded futures contract is the method of choice for trading many assets forward, since it circumvents these two main difficulties while retaining many essential characteristics of a forward contract.

Futures contracts are traded in a standard set of maturities, with a standard contract size and a standard minimum fluctuation ("tick size"). While most futures contracts on financial assets are cash-settled, in commodities markets, where futures are the primary method of setting prices, contracts stipulate the allowed time and location of delivery, as well as the permissible types and qualities of assets that may be delivered. These features make futures standard, enhancing their liquidity.

Credit risk in futures is far less than in forward contracts. All futures trades are margined and cleared, meaning that collateral is posted to back some portion of the commitment in the contract, and clearing parties guarantee the performance of parties to the contract. To lessen the credit risk of trading still further, each trade is marked to market at the end of each day.

Mechanically, a party wishing to go long a particular asset in the futures market always, in essence, buys at par. The futures price at the time the trade is entered behaves like a forward price: In concept, this is the price the party is willing to pay to receive the asset at the time, and in the manner, specified in the futures contract, and is thus essentially equivalent to the forward price.

Since the trade is entered at par, there is no cost of entry *per se*. However, to collateralize the trade, the party is required to post an initial margin; at the end of each day, the gain or loss on the position is marked against this margin account. If the trade makes money so that the balance exceeds the maintenance margin, the party may take the difference in cash; similarly, if the account balance dips below the maintenance margin, the party must supply the difference in cash.

In the commodities markets, only a small percentage of contracts actually lead to physical delivery. Most participants—whether speculators or hedgers—typically roll their positions forward at expiry by closing positions in the contract about to expire and taking an offsetting position in the next month's contract.

Is the futures price a forward price?
Consider a modification of the forward contract with an additional settlement time. As before, we enter the contract with no net cash outlay. At time t with $0 < t < T$, the contract throws off a cash flow, and its strike resets so that, after the cash flow is paid, the contract's value is once again zero. The cash flow amount is the difference between the contract's initial strike and its reset strike. At expiry, the contract pays the difference between the spot price and the reset strike.

Clearly the reset strike K_t will have to be the forward price as seen at that time, since after the cash flow is paid out, the modified contract simply becomes a forward contract. Assuming that the asset pays no cash flows itself, this makes the reset strike…
$$K_t = S_t e^{r_{obs,t,T}(T-t)}$$
…where both the spot price and the interest rate are observed at the reset time.

The value at time t to a holder of the contract before the cash flow is therefore:
$$V_t = S_t e^{r_{obs,t,T}(T-t)} - K_0$$
…since after the reset the value of the contract is by construction zero.

The question is what fair strike K_0 ought to be chosen so that we can enter this contract at zero cost. Adopting the hedging reasoning we've used before, we imagine being long the asset through this contract. To hedge our exposure, we wish to take a position in the asset at time zero that will remove our exposure to S_t.

Unfortunately, we cannot. The factor…
$$e^{r_{obs,t,T}(T-t)}$$
…functions essentially as the number of shares we will be long when time t arrives. But this value is random; we do not know it at time zero. And of course we cannot hedge

statically to completely remove this exposure, since we do not know the value the equity will have at that time and thus we cannot know our exact exposure to the rate.

We can certainly conclude, however, that if we assume interest rates are nonrandom, then the forward rate can be substituted for the observed rate. For the moment, we proceed under the assumption that the forward rate is realized, replacing the observed rate to arrive at the value of the payoff…

$$V_t^* = S_t e^{r_{t,T}(T-t)} - K_0$$

…at time t. To hedge this contract, at time zero we would go short…

$$e^{r_{t,T}(T-t)}$$

…units of the underlying, receiving…

$$S_0 e^{r_{t,T}(T-t)}$$

…in cash. We invest this at the risk-free rate to maturity t. At that time, we have…

$$S_0 e^{r_{t,T}(T-t)} e^{r_t t} = S_0 e^{rT}$$

…in cash from this initial investment. At that time, the first reset occurs, and we receive the cash flow V_t*. If we use the proceeds of the reset to close the short, it costs us the amount K_0 to do so. All that remains is a single-period forward contract, which we already know how to hedge.

In order for the expected value of the contract at t to be zero, the cost K_0 should equal the cash we have available from the hedge. That is, if we assume the forward rate is realized, then the initial fair strike…

$$K_0 = S_0 e^{rT} = F_0$$

…is also the forward price.

This argument can be extended to the case of multiple resets to become a futures contract. All of these arguments, however, require the assumption that forward rates are realized. Once interest rates are random, we cannot be sure that the hedge will eliminate all of the risk of the futures contract.

Fortunately, we can still use this result in expectation. What we need to be able to conclude in the case above is that:

$$E^*\left[K_t\right] = E^*\left[S_t e^{r_{obs,t,T}(T-t)}\right] = E^*\left[S_t\right] E^*\left[e^{r_{obs,t,T}(T-t)}\right]$$

…where E^* denotes what we will for now call, without further elaboration, the expected values of these quantities. In order for this final step to be valid, we require that the spot price at t and the interest rate observed at that time be stochastically independent. If they are, then we may safely conclude that the forward price and futures price are equal in expectation.

One case where this assumption is not safe, it bears mentioning, is futures contracts referencing interest rates themselves. The tendency of interest rates at different maturities to move in correlated fashion means that contracts such as Eurodollar futures cannot be exactly priced off interest-rate curves without accounting for this correlation.

Considerations specific to commodities

For non-financial assets—or for physical assets such as precious metals—the relation between spot and forward is less direct than it is for the other assets we will study. Commodities require storage, and in most cases cannot be stored for an arbitrary length of time. For some commodities—gold, for example—storage costs may be largely offset by the ability of the owner of the storage space to lend the asset, but in other cases these costs may be significant.

If we define the storage cost s as the net cost of holding the asset—i.e., true storage costs less any income that may be generated from the stored asset—then this behaves in effect like a negative dividend. For example, if you are short an asset via a futures contract, then the way to hedge this exposure would be to borrow to acquire the asset; but the cost to finance the borrowing—r—is increased by the net storage cost s of holding the asset at spot. So the forward price in this circumstance would be expected to be:

$$F = S_0 e^{(r+s)T}$$

The seemingly logical consequence of this insight is that futures prices should be higher than the spot price, and the term structure of futures prices would be expected to be upward-sloping. This situation, called contango, does indeed arise very often.

However, it is not impossible to see the opposite situation—backwardation—in real commodities markets. To the degree that the spot asset is not fungible with the asset in the future—i.e., if it is apt to spoil, of if like electricity it cannot be effectively stored—the different maturities behave like separate assets. In particular, for many assets that are inputs to manufacturing processes, the spot asset may be preferred to the asset in the future, since it may be delivered at need. Thus, we may find that:

$$F = S_0 e^{(r+s-\delta)T}$$

…where the quantity δ may in some cases be greater than the sum of the risk-free rate and net storage costs. This δ is called the convenience yield.

Finally, it is worth noting that for some assets such as heating oil, there is a natural seasonal fluctuation in demand. Of course, producers of commodity are also aware of this seasonality of the demand; nevertheless, it is common in the term structure of futures prices for the commodity to observe this seasonality effect. For most consumption commodities, the different expiries of futures contracts represent related but distinct financial instruments.

Futures as a hedge

As was mentioned above, the vast majority of futures contracts do not lead to physical delivery. It does not therefore follow, however, that the majority of futures activity is speculative. Many market participants may not wish to take delivery in the manner specified by the contract, instead preferring to purchase at spot, but nevertheless can use futures contracts on the asset to protect against adverse price movements.

For example, a consumer of a traded asset may intend to purchase an asset at spot at a time coinciding with the expiry of the front-month futures contract. To lock in a price, the consumer goes long the asset via the futures contract today. At expiry, the party intends to close the contract and purchase the asset at spot. If the price has appreciated in the meantime, the proceeds of the long futures contract are available to cover the additional cost; similarly, if the price has dropped in the meantime, the party has sustained a loss that is compensated for by the lower purchase price of the asset.

There is an additional risk to this method of hedging, which is called basis risk. Theoretically, the futures price and spot price should converge to the same value at expiry, but the realities of supply and demand and other frictions in the market dictate that this is only approximately true. The futures hedge may not precisely compensate for losses sustained through appreciation of the spot price over the term of the contract. This difference—the basis—can be quite significant if the asset is needed in quantity.

Futures contracts may also be used as a hedge by those whose business operations involve commodities that are similar, but not identical, to the traded assets. Oil refiners, for example, tend only to be able to process certain varieties of crude, and the benchmark crude varieties represent only a fraction of the crudes that are commercially important in real production.

Nevertheless, futures contracts on benchmark crudes can be used to hedge price risk in other varieties, since it is generally the case that the prices of crude varieties move in a correlated fashion. Suppose a producer of a crude oil variety wishes to hedge the risk in the price Y at delivery time, but can only trade a futures contract on a separate variety X with a similar delivery date. The producer is naturally long Y and wishes to take a short position of size β in X:

$$V_\Pi = Y - \beta X$$

We wish to choose the position size to minimize the risk—that is, the variance—of the total portfolio. If ρ is the correlation between the two asset prices, then:

$$Var_\Pi = Var\left[Y - \beta X\right] = VarY + \beta^2 VarX - 2CovY,\beta X$$

$$Var_\Pi = \sigma_Y^2 + \beta^2\sigma_X^2 - 2\beta\rho\sigma_Y\sigma_X$$

To minimize this quantity, we take the derivative with respect to β and set equal to zero:

$$2\beta\sigma_X^2 = 2\rho\sigma_Y\sigma_X$$

$$\beta = \rho\frac{\sigma_Y}{\sigma_X}$$

This is called the minimum-variance hedge ratio. It can easily be obtained from historical realizations of X and Y via ordinary least squares regression.