Class 5 Problems

Exercises

1. The risk-free rate expressed with continuous compounding is 2.25% to all maturities. An asset has spot price 36, its continuously compounded dividend rate is 80 basis points to all maturities, and its volatility is 40%. Determine the price of the option, along with the Greeks delta, gamma, vega, theta, and rho under Black-Scholes assumptions for the following options on the asset:

(a) A European call option struck at 36 with expiry in 3 months.
(b) A European put option struck at 45 with expiry in 6 months.

2. At a time when the risk-free rate is 5% to all maturities (expressed with continuous compounding), you are short 100 European call options with expiry 3 months and strike 60 on an asset priced at 55. The asset's volatility is 45%, and it pays no dividends.

For the purposes of the below, assume that all assets can be traded in any quantity, and that Black-Scholes assumptions hold; further, assume that the asset's volatility is constant through time.
(a) Suppose you wish to delta-hedge your exposure. How many shares of the underlying must you add to the portfolio in order for it to be delta-neutral?
(b) Suppose you wish to hedge your exposure to changes in the asset's volatility, as well as delta-hedging your exposure. You select a European put option with strike 50 and expiry 6 months for the purposes of vega-hedging. What quantity of this option and underlying must you add to the portfolio in order for it to be delta- and vega-neutral?
(c) What is the gamma of your initial short option position? What is the gamma of the hedged portfolio you constructed in (b)?
(d) You wish to hedge your gamma exposure as well. An additional asset—a European call option struck at 55 with expiry 1 month—is available for this purpose. Create a portfolio hedging the original option exposure which is delta-, vega-, and gamma-neutral.

3. You have several positions related to an asset with spot price 100. The asset pays no dividends and has a volatility of 30% per year; the risk-free zero rate to all maturities is 2%, expressed with continuous compounding.

The assets in question are:

- long 40 shares of the underlying
- short a straddle on 100 shares of underlying with strike 100 expiring in 2 months
- long a strangle on 100 shares of underlying with strikes 90 and 115 expiring in 6 months

(a) Determine the theta, delta, and gamma of this portfolio.
(b) Using Taylor expansions, find an expression that can be used to approximate changes in the value of a portfolio under small changes in time and spot price. To simplify the calculation, this expression should be first-order in time and second-order in spot, with no cross terms.

(c) Use the expression above in (b) to estimate the PL if 1 trading day from now the asset's spot price is 102. (Assume 1 year = 252 trading days.)
(d) Use the Black-Scholes pricing formulas to calculate the portfolio's value, as well as the value after the change described in (c). How would you assess the quality of the approximation from (c) in this particular case?

4. Graph the payoff V_T of the portfolio below versus the terminal spot price S_T. All options have the same expiry:

- long 1 share
- short a put with strike 30
- long 2 puts with strike 40
- long 2 calls with strike 35
- short 4 calls with strike 50

5. A chooser option is a European option with expiry T and strike K and choice time $t < T$. At t, the owner of the option must choose the payoff the option will have—either a vanilla call or put payoff with strike K.

For the purposes of this problem, we consider a chooser option on a non-dividend-paying asset with spot price S_0 at a time when the continuously compounded interest rate r is constant to all maturities.

(a) For which values S_t at the time of the choice should the holder choose a call payoff, and for which a put payoff? Explain your reasoning.
(b) Show that the chooser option on a non-dividend-paying asset is replicated by the portfolio:

- long a call option with strike K and expiry T
- long a put option with strike $Ke^{-r(T-t)}$ and expiry t

Applications
1. You decide to create a portfolio that will provide a piecewise linear payoff in 3 months. To do this, you will use the following instruments:

- risk-free zero-coupon bonds with maturity 3 months
- the asset
- European call options on the asset with expiry 3 months

These assets can be traded in any desired quantity, long or short, including non-integer quantities, and calls can be traded at all strikes, including non-integer strikes.

(a) Show what assets are needed to form a portfolio whose payoff is piecewise linear, with the endpoints as follows:

ST	VT
0	100
20	80
40	180
60	0
80	-10
100	40

(b) In general, given an arbitrary set of pairs $(S_{T,i}, V_{T,i})$, $i = 1, 2, \ldots, N$, with $0 = S_{T,1} < S_{T,2} < \ldots < S_{T,N}$, how would you form a portfolio using the instruments above having a piecewise linear payoff whose endpoints are the provided pairs?