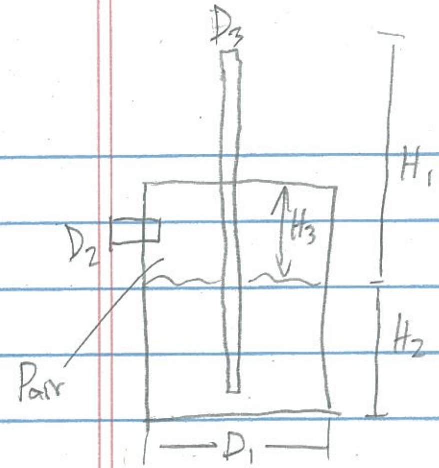

$$U_{exit} = \sqrt{2\left[\frac{(P_{air} - P_{atm})}{\rho_l} - g H_1(t)\right]} \quad (1)$$

$$P_{air} = \rho R T_{air} \quad (2) \qquad \rho_l = 1000\ kg/m^3$$

① From Eqn (1), solving for P_{air} w/ $U_{exit} = 0$:

$$P_{air} = \left[\frac{U_{exit}^2}{2} + g H_1(t)\right]\rho_l + P_{atm}$$

$$\boxed{P_{air} = g \rho_l H_1(t) + P_{atm}}$$

② $\dot{m}_{exit} = \rho_l \dot{V} = \rho_l \pi \frac{D_3^2}{4} U_{exit}$

From Eqn (1)

$$\boxed{\dot{m}_{exit} = \rho_l \pi \frac{D_3^2}{4} \sqrt{2\left[\frac{(P_{air} - P_{atm})}{\rho_l} - g H_1(t)\right]}}$$

③ $V_{air} = \pi D_1^2/4\ H_3(t) \quad (3)$

Imagining control volume around air in pump:

$$\rho_{air}(t) = \frac{m_{air}(t)}{V_{air}(t)} \quad (4)$$

Rate of pressure change related to density change by (2)

(5) $\frac{dP}{dt} = \frac{d\rho}{dt} RT$ where R and T are time invariant

③ cont Applying quotient rule to $\frac{d\rho}{dt}$ w/ relation (4):

$$\frac{d\rho}{dt} = \frac{\frac{dm_a}{dt} \forall_a(t) - m_a(t)\frac{d\forall_a}{dt}}{\forall_a(t)^2} \quad (6)$$

where $\frac{dm_a}{dt} = \dot{m}_{air}$ (7) and $m_a(t) = \rho_{air} \forall_a(t)$ (8)

In general: $\forall_{air} = \forall_{tot} - \forall_e$ so: $\frac{d\forall_{air}}{dt} = -\frac{d\forall_e}{dt}$

but $\forall_e = \frac{m_e}{\rho_e}$ and $\frac{d\rho_e}{dt} = 0$ so: $\frac{d\forall_{air}}{dt} = \frac{-1}{\rho_e}\frac{dm_e}{dt} = \frac{\dot{m}_{exit}}{\rho_e}$ (9)

Substituting (3, 7, 8, 9) into (6), then into (5):

$$\boxed{\frac{dP}{dt} = \frac{4}{\pi D_1^2 H_3(t)}\left[\dot{m}_{air}RT - \frac{P_{air}}{\rho_e}\dot{m}_{exit}\right]} \quad (10)$$

④ From Q_1: $P_{air}(0) = 9.8(1000)0.15 + 101325$
$= 102.8\ kPa$

ICs: $P_{air_0} = 102.8\ kPa$, $H_{1_0} = 0.15\ m$, $H_{3_0} = 0.1\ m$, $\dot{m}_{exit_0} = 0$

Inputs: $\dot{m}_{air}$

Functions: $\dot{m}_{exit}(P_{air}, H_1)$, $\frac{dP}{dt}(H_3, \dot{m}_{exit}, P_{air})$

Pseudocode: ICs

Use $\frac{dP}{dt}$: calc P

Use $\dot{m}_{exit}$: calc H_1, H_3

Loop until $H_3 = 0.5\ m$

(5) Solving answer from problem 2 for P_{air} yields

$$P_{air} = P_{atm} + g H_1(t)\rho_l + \left(\frac{\dot{m}_{exit}}{\rho_l \pi D_3^2}\right)^2 \quad (11)$$

where $\frac{dP_{atm}}{dt} = 0$ and if flow rate is constant:

$\frac{d\dot{m}_{exit}}{dt} = 0$, where by continuity $\dot{\forall}_{air} = \dot{\forall}_l$.

Also:

$H_2 + H_1 = C$ so: $\frac{dH_1}{dt} = \frac{dH_2}{dt}$

$$\frac{dH_2}{dt} = \frac{4}{\pi D_1^2}\frac{d\forall_l}{dt} = \frac{4}{\pi D_1^2 \rho_l}\frac{dm_l}{dt} = \frac{-4\dot{m}_{exit}}{\pi D_1^2 \rho_l} \quad \text{so:}$$

$$\frac{dH_1}{dt} = \frac{4\dot{m}_{exit}}{\pi D_1^2 \rho_l}$$

therefore

$$\boxed{\frac{dP_{air}}{dt} = \frac{4g\dot{m}_{exit}}{\pi D_1^2}}$$

(6) $$\bar{U}(r) = \frac{1}{R}\int_0^R 1 - \frac{r^2}{R^2}\,dr = \frac{R - \frac{R^3}{3R^2}}{R} = 1 - \frac{1}{3} = \frac{2}{3}U_{max}$$

$$\frac{\bar{U}(r)}{U_{max}} = \frac{2/3\,U_{max}}{U_{max}} = \boxed{\frac{2}{3}}$$

7. Temperature constant
Viscous effects negligible
Air treated as ideal gas
In the case of parabolic flow profile: Flow laminar
Weight of air in pump negligible
- If instead of air the pumped gas was more dense, the weight of the gas would tend to help pump the water out.

All cylinders are right cylinders