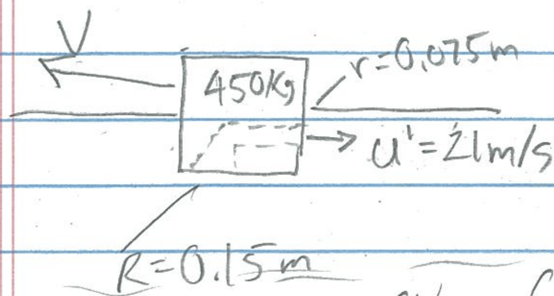
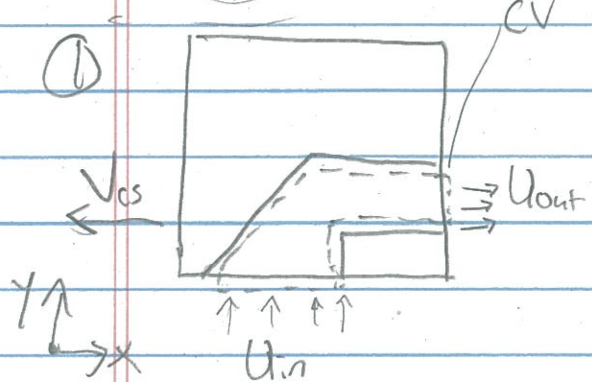
Josh Bevan 22:581 HW #2



$$C_D A = \frac{2D}{\rho V^2}$$

(1)



Cons. of Mass

$$0 = \cancel{\frac{d}{dt}\iiint_{CV} \rho \, d\forall}^{\,0} + \iint_{CS} \rho(\vec{u} - \vec{V}_{cs}) \cdot \vec{n}_{cs} \, dA$$

Steady state

Flow parallel and perp. to CS normal:

$$0 = \iint_{CS} \rho(\vec{u} - \vec{V}_{cs}) \, dA$$

and constant across CS:

$$0 = \rho(\vec{U}_{out} - \vec{V}_{cs}) A_{out} - \rho \vec{U}_{in} A_{in}$$

Treating water as incompressible:

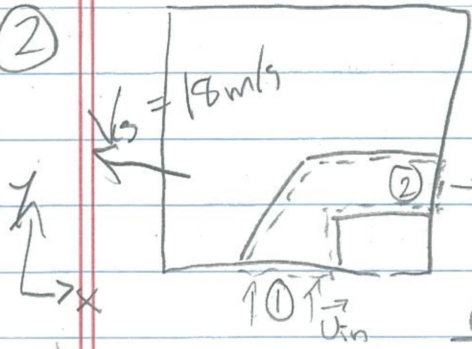
$$0 = (\vec{U}_{out} - \vec{V}_{cs}) A_{out} - U_{in} A_{in} \rightarrow U_{in} = \frac{A_{out}}{A_{in}} (\vec{U}_{out} - \vec{V}_{cs})$$

with $u' = u_{out} - V_{cs}$ so: $U_{in} = \frac{A_{out}}{A_{in}} u'$

$$\dot{m} = \rho A_{in} u_{in} = \rho A_{out} u' = 1000 \pi (0.075)^2 21$$

$$\boxed{\dot{m} = 371 \text{ kg/s}}$$

(2)



Assuming constant pressure and negligible shear and body forces:

$$\frac{d\vec{M}}{dt} = \underbrace{\frac{\partial}{\partial t}\iiint_{CV} \rho \vec{u}\, d\forall}_{\to 0 \text{ steady state}} + \iint_{CS} \rho \vec{u}\left[(\vec{u} - \vec{V}_{cs}) \cdot \hat{n}\right] dA$$

$$= \sum \vec{F}_{ext}$$

being only concerned with momentum in $\vec{x}$

$$\frac{d\vec{M}_x}{dt} = \iint_{CS} \rho \vec{u}_x \left[(\vec{u} - \vec{V}_{cs}) \cdot \hat{n}\right] dA = \sum F_x$$

with $\vec{u}$ and $\vec{u}_x$ being constant across the CS and $\vec{u} - \vec{V}_{cs}$ being parallel and perp. across the CS:

$$-\rho \vec{u}_{x①} \vec{u}_{in} A_{in} + \rho \vec{u}_{x②}(\vec{u}_{out} - \vec{V}_{cs}) A_{out} = \sum F_x$$

with constant density (incompressible) and $\vec{u}_{x①} = 0$ (fluid starts at rest) and $\vec{u}_{x②} = \vec{u}' + \vec{V}_{cs}$

$\vec{u}_{x_2} = 21 + {}^{-}18 = 3$ m/s

$$\rho \vec{u}_{x②} u' A_{out} = F_D = 1000(3)21\pi(0.075)^2 = \boxed{1113 N}$$

③ $C_D A = \frac{2D}{\rho V^2}$ where from Prob. 2 $D = 1113\,N$

and $\rho = 1000\ kg/m^3$

$V = 18\,m/s$

$$C_D A = \frac{2(1113)}{1000\,(18)^2} = \boxed{6.87 \times 10^{-3}\ m^2}$$

④ $D = \frac{1}{2} C_D A \rho V^2$

For the accelerating jet-ski: thrust = Drag + Reaction

Thrust $\rho \vec{u}_{x_0}\, u' A_{out} = \frac{1}{2} C_D A \rho \vec{V}^2 + m_{jetski}\, \vec{a}$

$m\vec{a} = \rho\left[(u'^2 + u'V)A_{out} - \frac{1}{2} C_D A V^2\right]$ Reaction

$a = \frac{\text{Reaction}}{m}$ Acceleration

$v(t+\Delta t) = v(t) + a\,\Delta t$ Velocity

$x(t+\Delta t) = x(t) + v\,\Delta t$ Distance

ICs

$V_0 = 0\ m/s$

$X_0 = 0\ m$

$C_D A$ = constant

Loop while $V < 16\ m/s$

Calc Thrust

Calc Accel

Calc Velocity, use Accel

Calc Distance, use Velocity