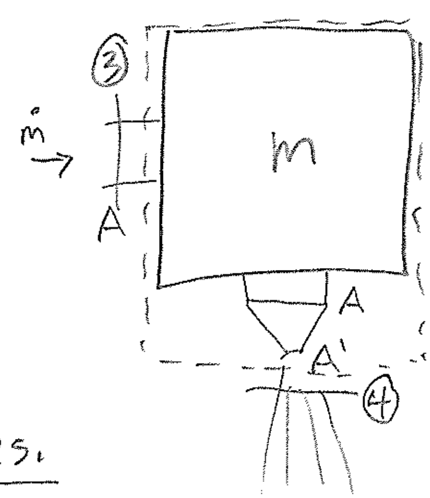
Josh Bevan HW3 - 22.581

1

	V	P	h	A
1	0	0	0	∞
2	V	P_2	0	A
3	V	P_3	h	A
4	V'	0	h	A'

CV



$m = 125\,kg$

$h = 3\,m$

Pump Power

$$Power = \dot{m}\left[\frac{\Delta P}{\rho} + \frac{1}{2}\Delta V^2 + g\Delta h\right]$$

Diff Pres.

$$P_2 - P_1 = P_3 + \rho g h - 0$$

Mass Flow Rate

$$\dot{m} = \rho V A$$

Continuity

$$VA = V'A'$$

Assuming:
- Steady State
- Inviscid
- Inlet Pressure balanced by Pipe support force

Conservation of Momentum

$$\frac{dM}{dt} = 0 = F_{ap} = \iint_{cs} \rho\vec{u}\left(\vec{u} - \vec{V}_{cs}\right)\cdot\hat{n}\, dA$$

For constant $\vec{u}$ parallel to $\hat{n}$

$$F_{ap} = \rho\vec{u}\,\vec{u}A = \rho V A \Delta V$$

$$\boxed{F_{ap} = \dot{m}(V' - V)}$$

2 $V(F, \rho, A, A')$:

$$F = \dot{m}(V' - V)$$
$$= \rho V A (V' - V)$$
$$= \rho V A \left(V\left(\frac{A}{A'}\right) - V\right) \rightarrow V = \sqrt{\frac{F}{\rho}\frac{1}{A\left(\frac{A}{A'} - 1\right)}}$$

$P_2(V, \rho, A, A', h, g)$: Streamline between 3 and 4

$$P_3 + \frac{1}{2}\rho V^2 + \rho g h = \cancel{P_4}^{\,0} + \frac{1}{2}\rho V'^2 + \rho g h$$

$$V' = V\frac{A}{A'}$$

$$P_3 = \frac{1}{2}\rho V^2\left(\left(\frac{A}{A'}\right)^2 - 1\right)$$

$$P_2 = P_3 + \rho g h$$

(2) cont

For pump $\Delta h = 0$ $\Delta V = V - 0 = V$ $\Delta P = P_2 - 0 = P_2$ so

$$\text{Power} = \dot{m}\left[\frac{P_2}{\rho} + \frac{1}{2}V^2\right]$$

$$= \rho V A\left[\frac{1}{2}V^2\left(\left(\frac{A}{A'}\right)^2 - 1\right) + gh + \frac{1}{2}V^2\right]$$

with $\frac{1}{2}V^2\left(\left(\frac{A}{A'}\right)^2 - 1\right)\rho V A = \frac{FV}{2}\frac{\rho}{\rho}\frac{A}{A}\frac{1}{\left(\frac{A}{A'} - 1\right)}\left(\frac{A}{A'} + 1\right)\left(\frac{A}{A'} - 1\right)$

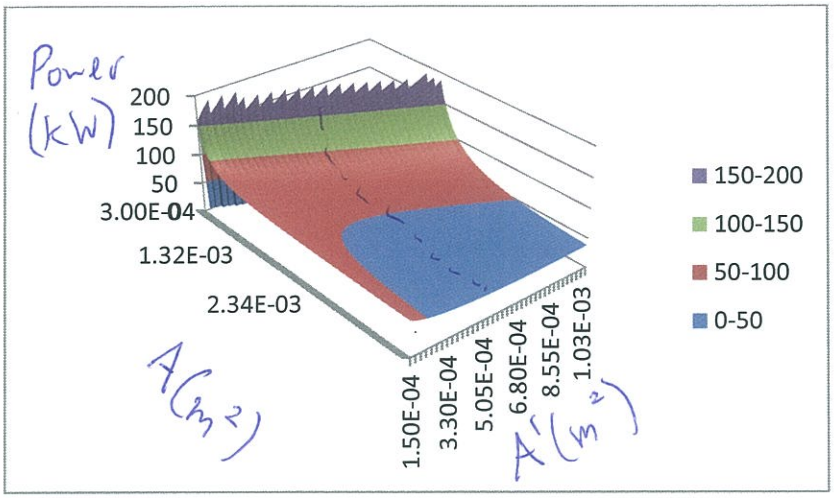
$$= \frac{FV}{2}\left(\frac{A}{A'} + 1\right)$$

and $\frac{1}{2}V^2 \rho V A = \frac{FV}{2}\frac{\rho}{\rho}\frac{A}{A}\frac{1}{\left(\frac{A}{A'} - 1\right)} = \frac{FV}{2}\left(\frac{A'}{A - A'}\right)$

$$\text{Power} = V\left[\frac{F}{2}\left(\frac{A}{A'} + 1 + \frac{A'}{A - A'}\right) + \rho g h A\right]$$

$$\text{Power} = \sqrt{\frac{mg}{\rho}\frac{1}{A\left(\frac{A}{A'} - 1\right)}}\left[\frac{mg}{2}\left(\frac{A}{A'} + \frac{A'}{A - A'} + 1\right) + \rho g h A\right]$$

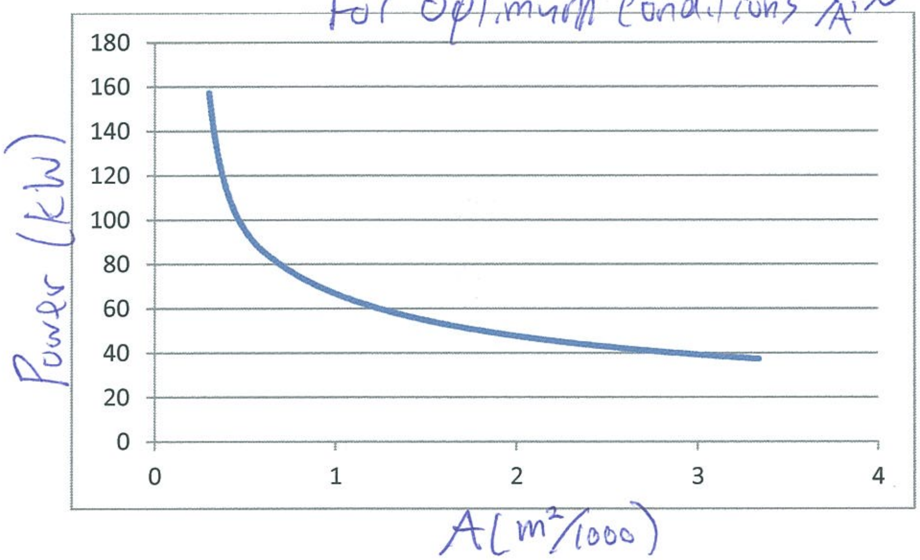
3



Plot of Pump Power vs Inlet and Outlet areas

- Dotted line indicates rough region of optimality

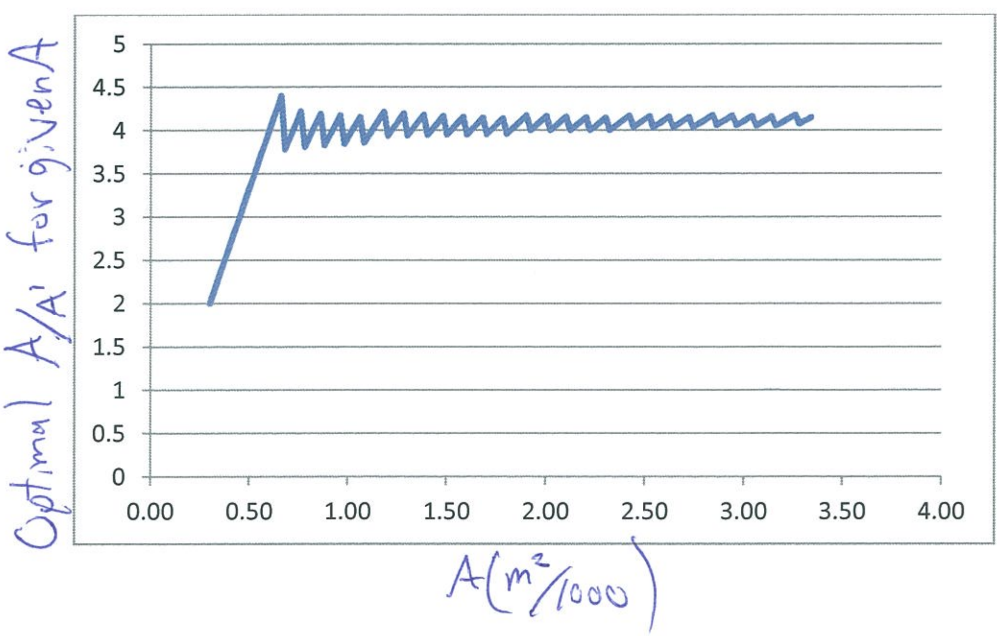
For optimum conditions $A/A' \approx 4$



Power necessary for particular inlet area

Decreases asymptotically, larger A only benefits so much for particular A'

$\boxed{\text{Minimum Power} \approx 40\,kW}$



For a given A the A/A' ratio with lowest necessary power was plotted.

Appears to converge to $A/A' = 4$ for all A!