Given the three data points (-1,1), (0,0), (1,1), determine the interpolating polynomial of degree two:

Using the monomial basis:

In general:

$$\begin{bmatrix} 1 & t_1 & t_1^2 \\ 1 & t_2 & t_2^2 \\ 1 & t_3 & t_3^2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

with $p(t) = x_1 + x_2 t + x_3 t^2$

For our data:

$$\begin{bmatrix} 1 & -1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

which yields $x = [0 \ 0 \ 1]^T$

So our interpolating polynomial is simply $p(t) = t^2$.

Using the Lagrange basis:

Using the Newton basis: