A High-Order Fast Algorithm Approach for Computing Layer and Volume Potentials

J. Bevan, UIUC

CS 591 Seminar
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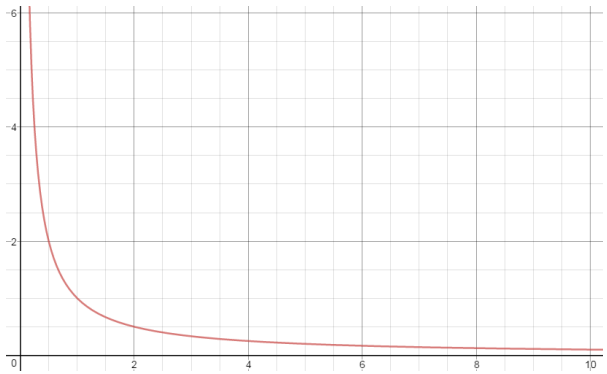
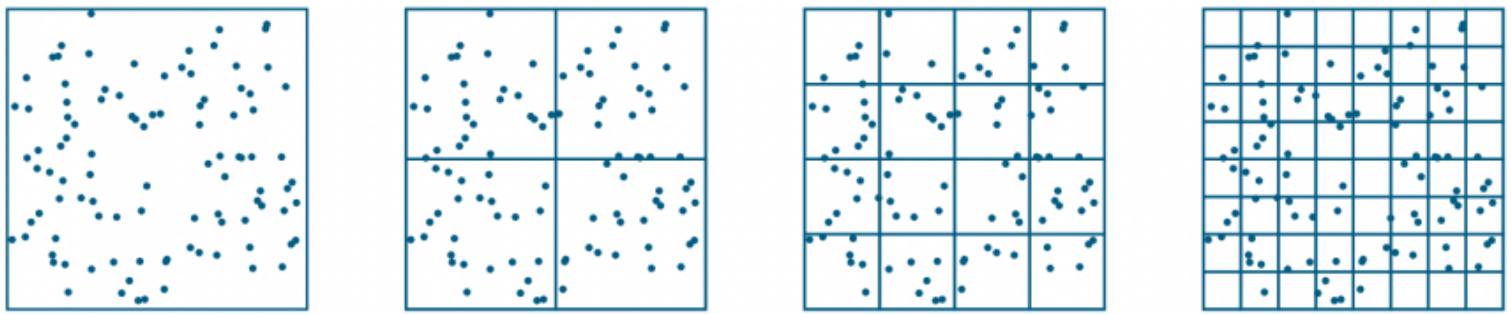
Motivation

- Evaluation of potentials (derived velocity field, electric potential, etc.) is an important physical consideration
- Practical computational im[implementation faces two challenges: $\mathcal{O}(n^2)$ cost and integrable singularities

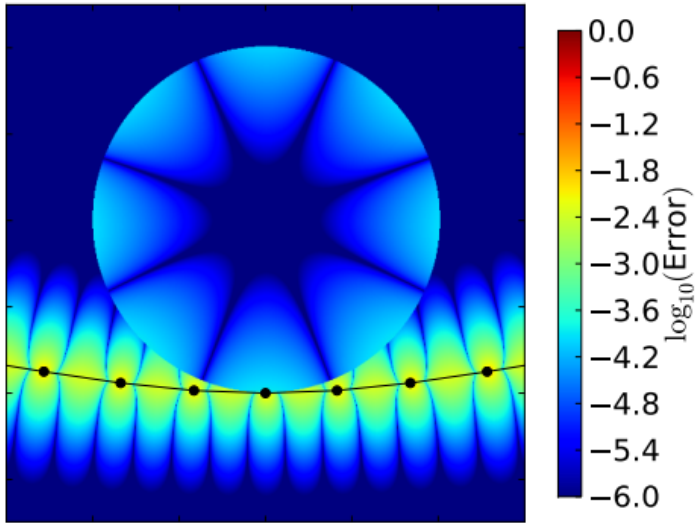




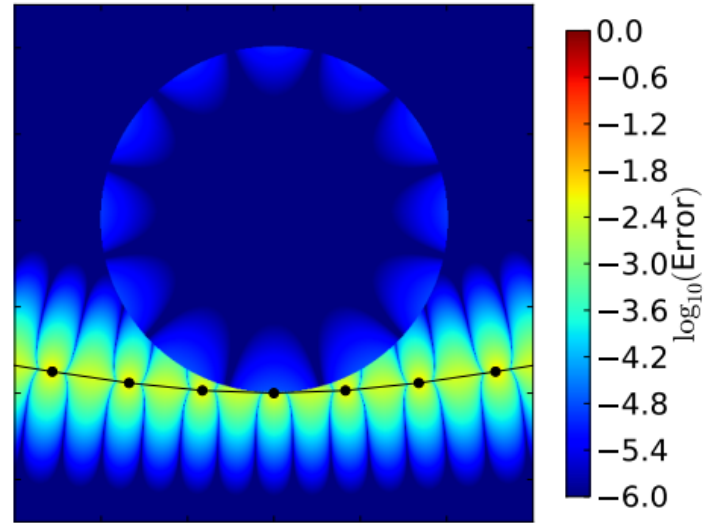
(a) d = 0 (b) d = 1 (c) d = 2 (d) d = 3

[0] https://summerofhpc.prace-ri.eu/

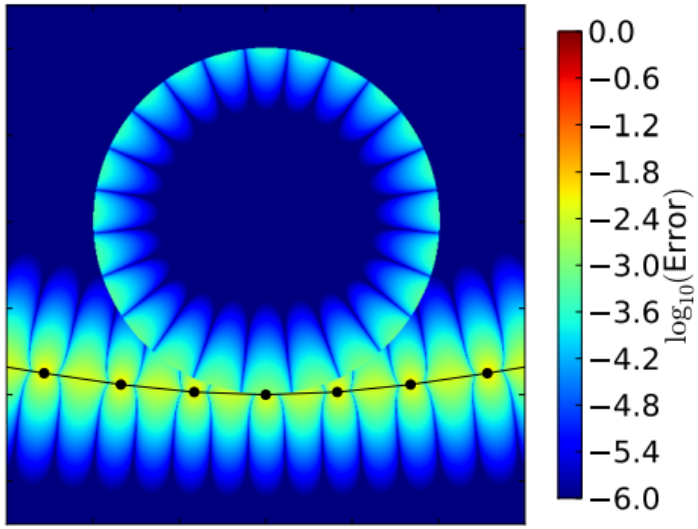
QBX



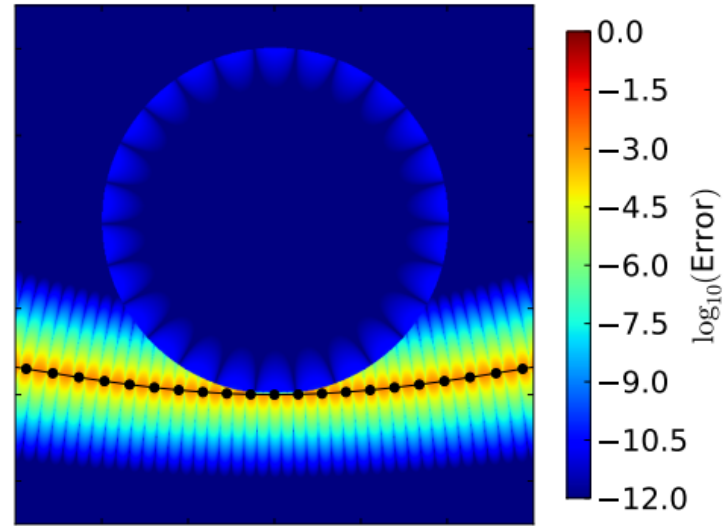
(a) $p = 3$, $N = 80$ quadrature nodes



(b) $p = 6$, $N = 80$ quadrature nodes



(c) $p = 12$, $N = 80$ quadrature nodes



(d) $p = 12$, $N = 240$ quadrature nodes

[0]Klöckner, Andreas, et al. "Quadrature by expansion: A new method for the evaluation of layer potentials." Journal of Computational Physics 252 (2013): 332-349.

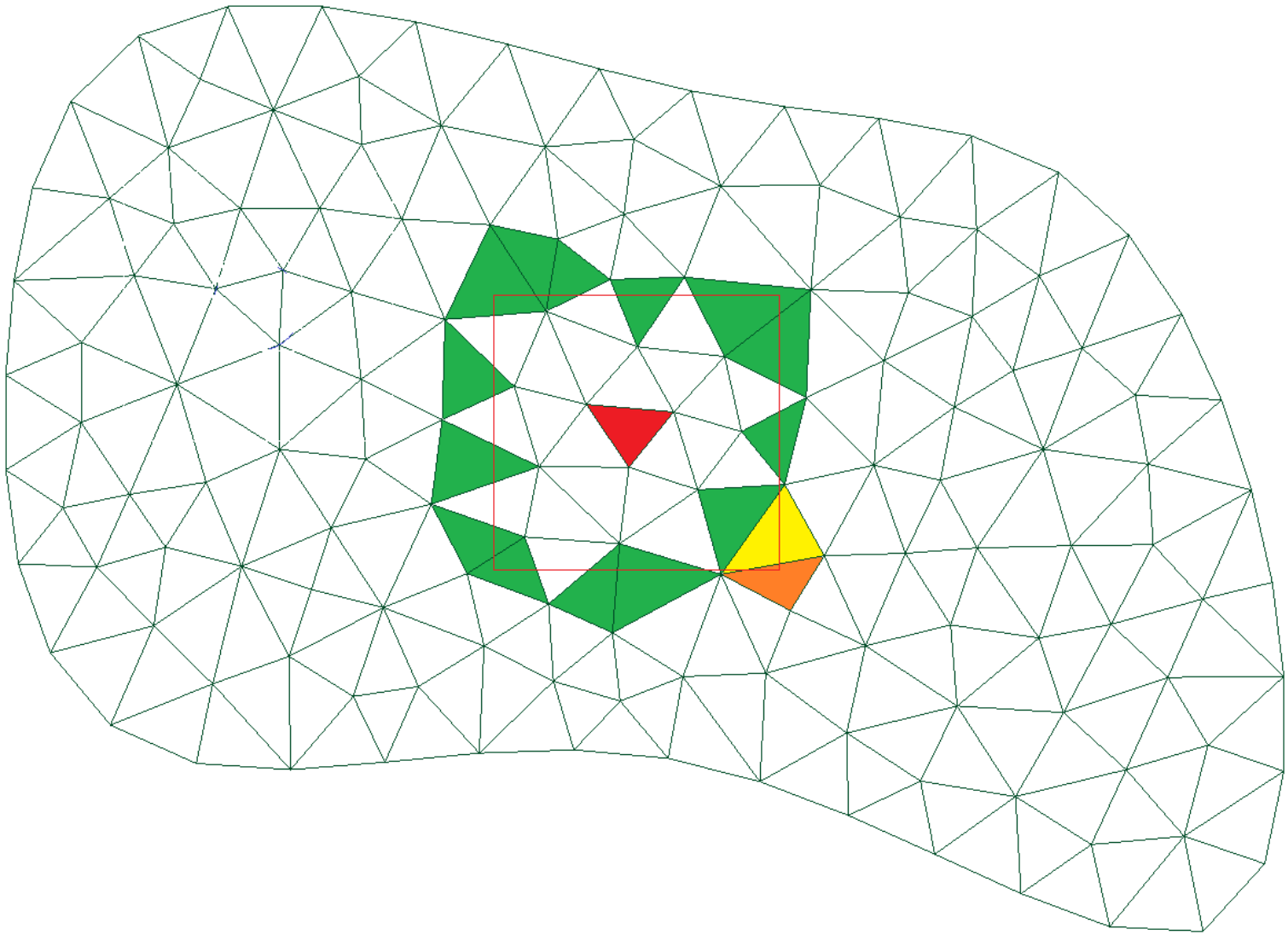
QBX Considerations

- Global vs local approaches involve all source points, or only the "near-field" respectively
- Layer potentials provide physical meaningful off-surface potential
- Volume potentials involve arbitrary choice of "off-volume" potential

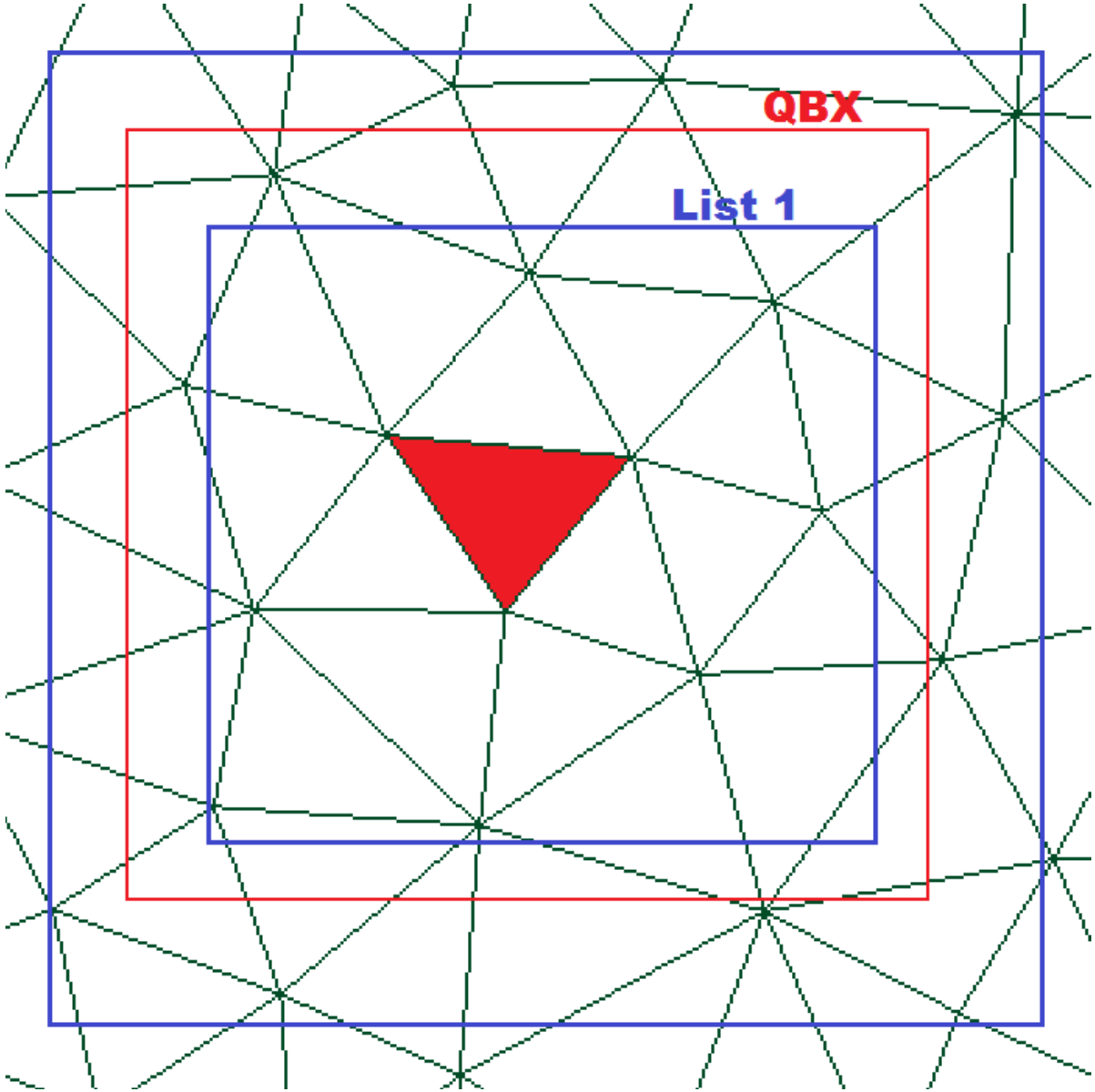
Mesh Interaction

- Given some mesh with a spatially varying blob of "charge", local QBX needs only some of the mesh
- How are varying intersection cases to be handled?
- How conservative/efficient can one be with intersections?

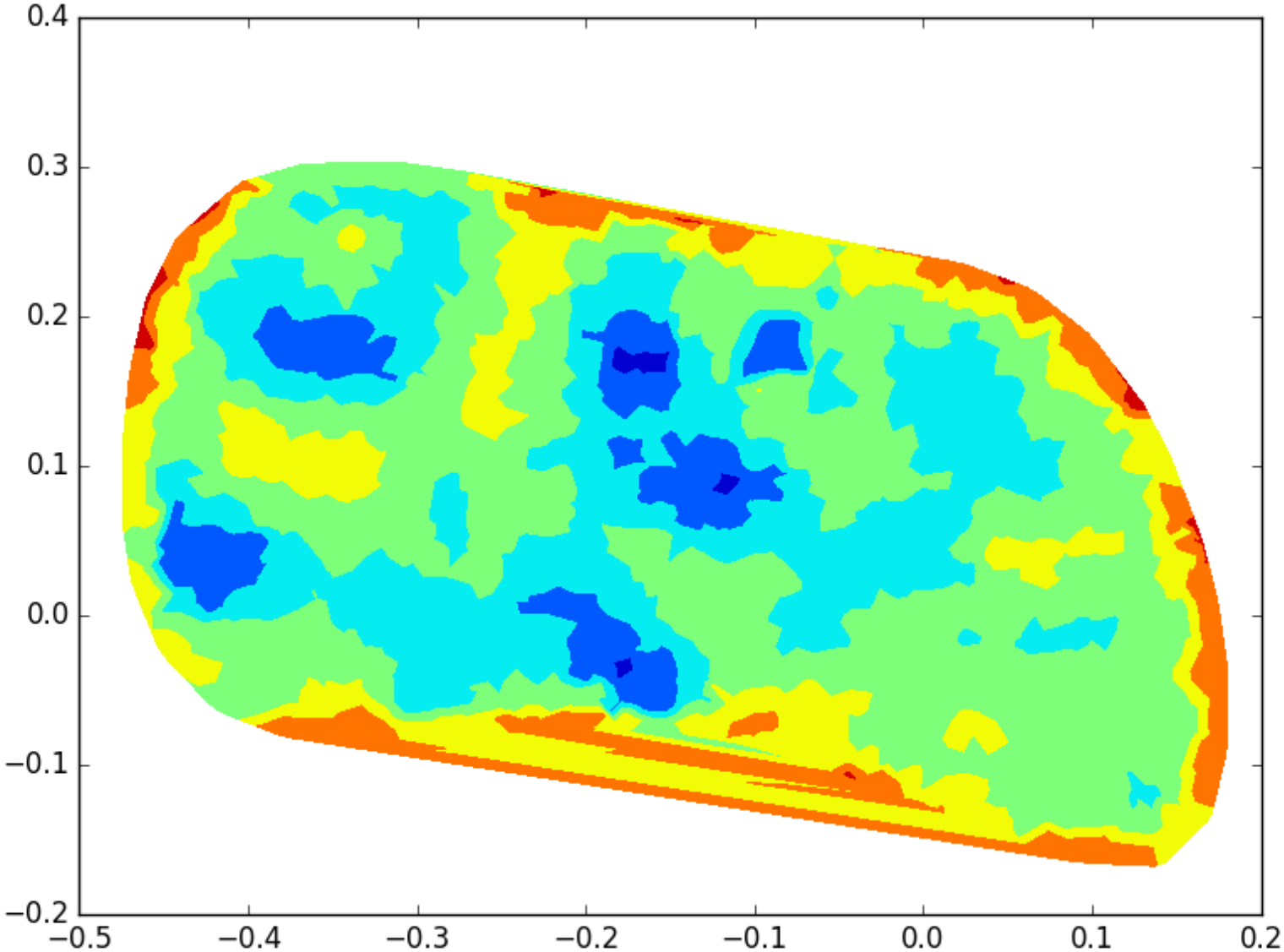
Mesh Interaction



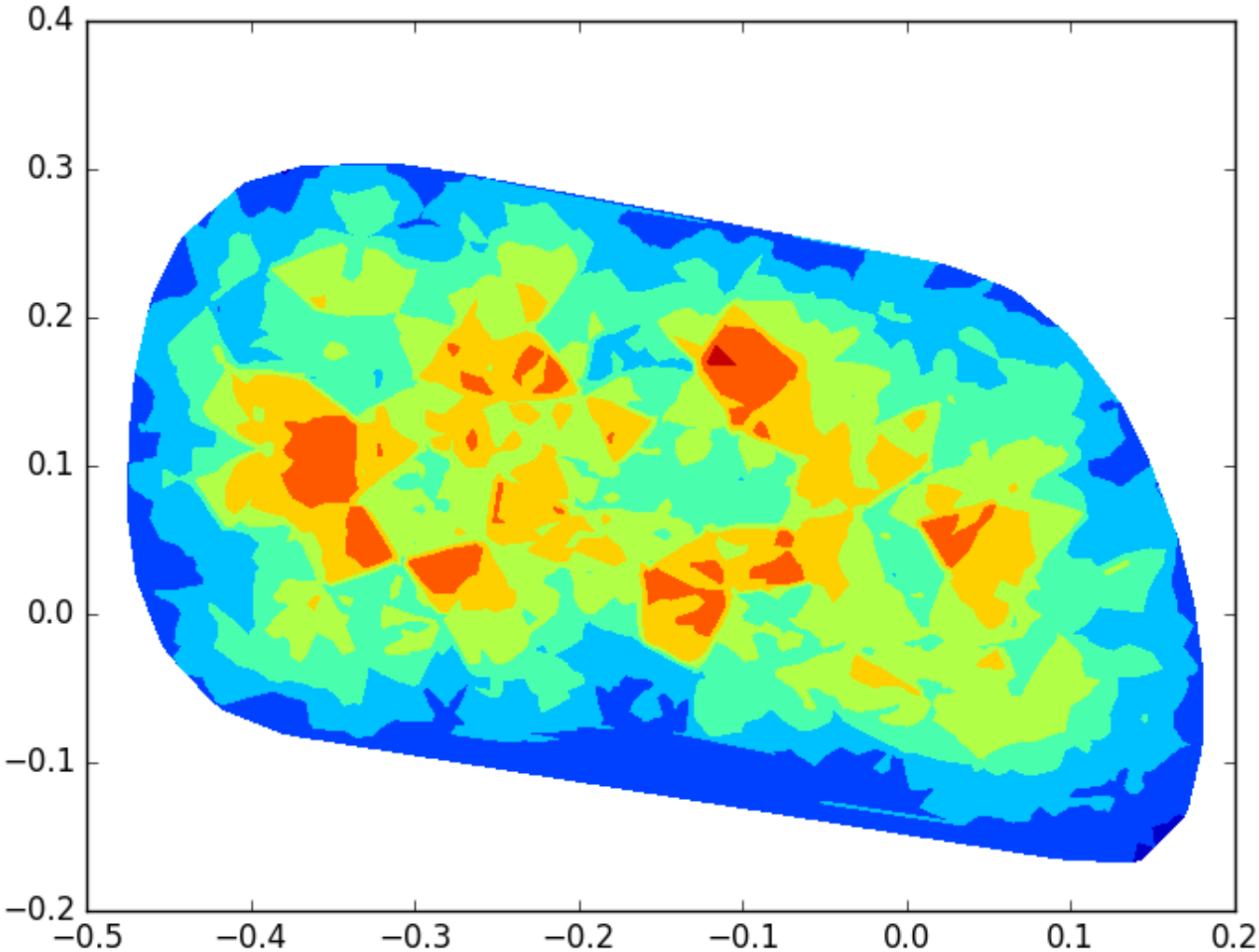
FMM-QBX Interaction



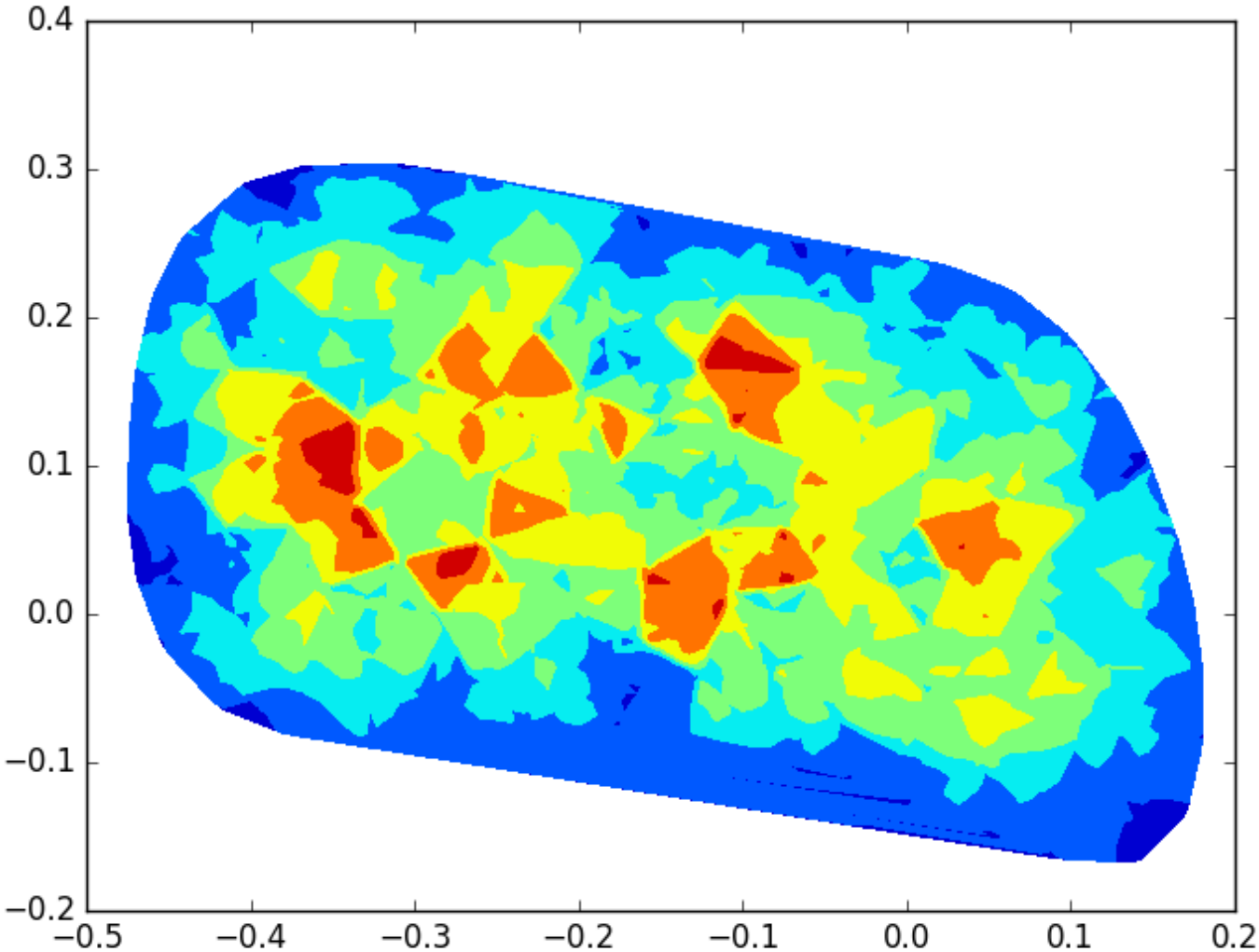
Corrected FMM Contributions



QBX Contributions



QBX-FMM Overlap Corrections



Result of Combinations