At what point does f attain a minimum?

Gradient:

$$\nabla f(\mathbf{x}) = \begin{bmatrix} 2x_1^3 - 2x_1x_2 + x_1 - 1 \\ -x_1^2 + x_2 \end{bmatrix},$$

Hessian:

$$\mathbf{H}(\mathbf{x}) = \begin{bmatrix} 6x_1^2 - 2x_2 + 1 & -2x_1 \\ -2x_1 & 1 \end{bmatrix}.$$

There is a critical point at $\mathbf{x}^* = \begin{bmatrix} 1 & 1 \end{bmatrix}$ which is a local minimum because $\mathbf{H}_f(\mathbf{x}^*)$ is positive definite.

Perform one iteration of Newton's method for minimizing f using starting point $x_0 = \begin{bmatrix} 2 & 2 \end{bmatrix}^T$.

Form linear system using starting point:

$$\begin{bmatrix} 21 & -4 \\ -4 & 1 \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = \begin{bmatrix} -9 \\ 2 \end{bmatrix}$$

Solving yields $\mathbf{s} = \begin{bmatrix} -0.2 & 1.2 \end{bmatrix}^T$, so next step's value is $\begin{bmatrix} 1.8 & 3.2 \end{bmatrix}^T$.

In what sense is this a good step? In what sense is this a bad step?

The step is good insofar as it has reduced the value of the function $f(\mathbf{x}_1) = 0.32$, down from 2.5. However it's bad because we are actually *farther* away from the correct solution moving from $\begin{bmatrix} 2 & 2 \end{bmatrix}^T$ to $\begin{bmatrix} 1.8 & 3.2 \end{bmatrix}^T$, away from the correct value of $\begin{bmatrix} 1 & 1 \end{bmatrix}^T$.