Use induction and the result from the previous part to show that applying the operator

$$D^{-1}\phi(x) := \int_0^x \phi(y)dy$$

n **times is the same as the single application of the following operator**

$$\underbrace{(D^{-1} \circ D^{-1} \circ \cdots \circ D^{-1})}_{n \textbf{ times}} \phi = D^{-n}\phi(x) = \frac{1}{(n-1)!}\int_0^x (x-y)^{n-1}\phi(y)dy$$

Base case: For $n = 1$ the proposed statement holds:

$$\frac{1}{(n-1)!}\int_0^x (x-y)^{n-1}\phi(y)dy = \int_0^x \phi(y)dy$$

$$\frac{1}{(1-1)!}\int_0^x (x-y)^{1-1}\phi(y)dy = \int_0^x \phi(y)dy$$

$$\frac{1}{1}\int_0^x 1\,\phi(y)dy = \int_0^x \phi(y)dy$$

Inductive step: Assume the proposed statement holds for n, then for $n+1$

$$D^{-(n+1)}\phi(x) = (D^{-1} \circ D^{-n})\phi(x) = \int_0^x \frac{1}{(n-1)!}\int_0^z (z-y)^{n-1}\phi(y)\,dy\,dz.$$

Examining the domain of integration we can make an appropriate change of order of integration to obtain

$$D^{-(n+1)}\phi(x) = \frac{1}{(n-1)!}\int_0^z \int_y^x (z-y)^{n-1}\phi(y)\,dz\,dy$$

and then pull out $\phi(y)$ from the inner integral

$$D^{-(n+1)}\phi(x) = \frac{1}{(n-1)!}\int_0^z \phi(y)\int_y^x (z-y)^{n-1}\,dz\,dy.$$

Now we can evaluate the inner integral to obtain

$$D^{-(n+1)}\phi(x) = \frac{1}{(n-1)!}\int_0^z \phi(y)\frac{1}{n}(x-y)^n\,dy.$$

Rearranging yields

$$D^{-(n+1)}\phi(x) = \frac{1}{n!}\int_0^z (x-y)^n\phi(y)\,dy.$$

and shows that the inductive hypothesis holds.