Consider the ODE $y' = -5y$ with initial condition $y(0) = 1$. We will solve this ODE numerically using a step size of $h = 0.5$.

Are solutions to this ODE stable? Explain.

Solutions to this ODE are asymptotically stable. Consider Figure 1 which plots solutions of y' for the initial condition $y(0) = 1$ as well as a number of other initial conditions. It is clear that even solutions with perturbations to their initial value converge back to the true solution.

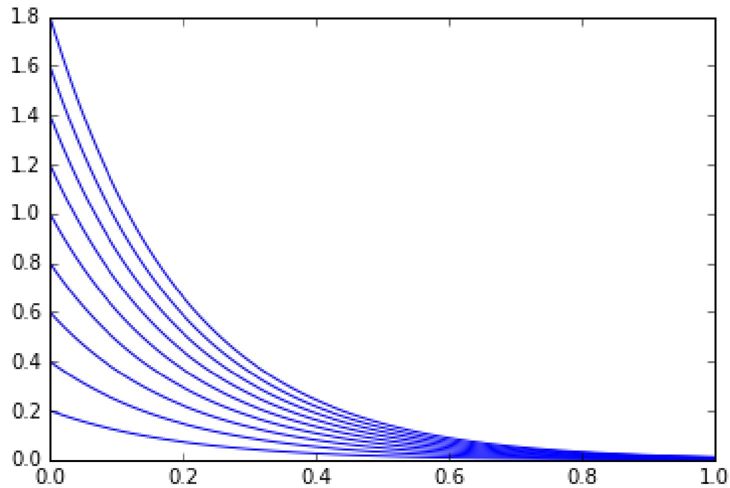

Figure 1: Plot of solutions of y' for differing initial conditions.

Is Euler's method stable for this ODE using this step size? Explain.

Euler's method is not stable for this ODE and step size. Recall that for problems of the type $y' = \lambda y$, with fixed step size h, a stable step size must satisfy $h \leq -\frac{2}{\lambda}$. In this case $\lambda = -5$, so $h \leq -\frac{2}{-5} = 0.4$; we can see that our step size does not satisfy the inequality and therefore Euler's method with this step size will be unstable.

Compute the numerical value for the approximate solution at $t = 0.5$ given by Euler's method. Show your work.

$$y(0.5) \underset{Euler,\, h=0.5}{=} hy'(0) + y(0) = 0.5(-5y(0)) + y(0) = 0.5(-5) + 1 = -2.5 + 1 = -1.5 \quad (1)$$

Is the backward Euler method stable for this ODE using this step size? Explain.

Backward Euler is stable for this ODE using this step size. Recall that for problems of the type $y' = \lambda y$, with fixed step size h, a stable step size must satisfy:

$$|\frac{1}{1 - h\lambda}| \leq 1, \text{ in our case: } |\frac{1}{1 - 0.5(-5)}| = |\frac{1}{3.5}| \leq 1 \quad (2)$$

so backward Euler is stable for this step size.

Compute the numerical value for the approximate solution at $t = 0.5$ given by the backward Euler method. Show your work.

$$y(0.5) \underset{backEuler,\, h=0.5}{=} hy'(0.5) + y(0) = 0.5(-5y(0.5)) + 1 \tag{3}$$

solving for $y(0.5)$:

$$y(0.5) = -2.5y(0.5) + 1 \rightarrow 3.5y(0.5) = 1 \rightarrow y(0.5) = 1/3.5 \approx 0.2857 \tag{4}$$