Does convergence improve or deteriorate as the number of levels increases?

A problem with $n = 2^{10} - 1$ was chosen, therefore the coarsest level had $kmax = 10$. The problem was solved with varying size V-cycles going to depth $1 < kmin < kmax$, for all possible $kmin$. Figure 1 plots the instantaneous convergence factor at iteration 10. As the depth of the V-cycle increases the convergence factor decreases by a moderate amount in this case, from 0.42 to 0.47. This should be unsurprising considering the fewer levels down we go in the V-cycle, the closer the problem directly solved at the coarsest $\mathbf{A}$ is to the original $\mathbf{A}$. Interestingly, the convergence factor is relatively insensitive to the depth of the V-cycle past a certain point; this is convenient in that we are more free to choose an “optimal” depth without sacrificing convergence factor (assuming our “optimal” depth falls within the insensitive region, which we might expect).

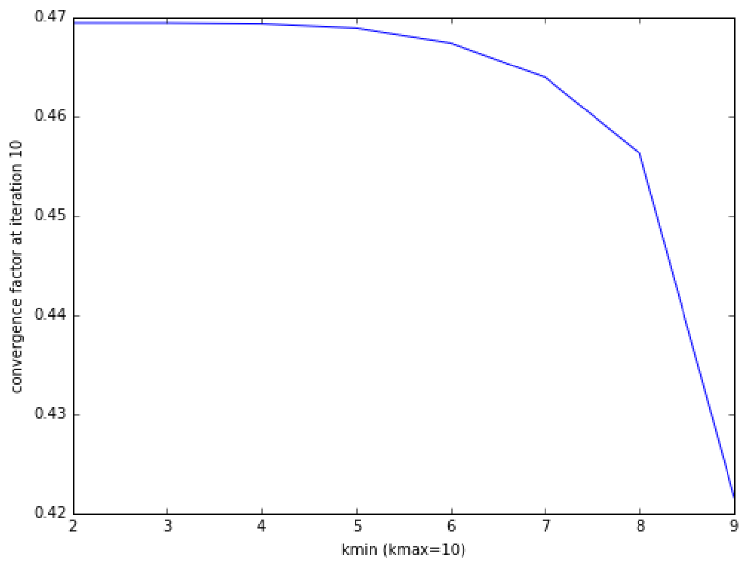

Figure 1: Convergence factor at iteration 10 for various kmin (kmax=10).

Does the cost increase or decrease?

If we only considered the number of iterations performed for a given depth, then it would appear that a more shallow V-cycle is superior as shown in Figure 2. The residual for the two-level method at a particular iteration will be lower than any of the deeper V-cycles. Of course the cost of the direct solve in the two-level method is likely to be more expensive than going deeper in the V-cycle and a direct solve of a smaller system.

To account for this, each $kmin$'s V-cycle was allowed to iterate for a fixed time rather than fixed number of iterations. If we again plot residual vs iteration number for each, we have the unhelpful Figure 3 plot. Instead it is more useful to plot the time evolution of the residual for each choice of $kmin$, as shown in Figure 4. Here it is much more clear the relative cost of each choice of $kmin$. As the depth of our V-cycle increases the cost per iteration increases, but the cost per equivalent reduction in residual actually decreases (to a point). Given a fixed time budget the two-level method is worse than the three-level method, which in turn is worse than all the deeper V-cycles.

Also important to note is that a $kmin = 6$ is actually the lowest time cost per reduction in residual, even better than the deeper V-cycles (lower values of $kmin$). Considering the convergence factor doesn't change much from $kmin = 6$ and lower, the relative cost difference is due nearly entirely to the ratio of the cost of a direct solve versus a deeper V-cycle.

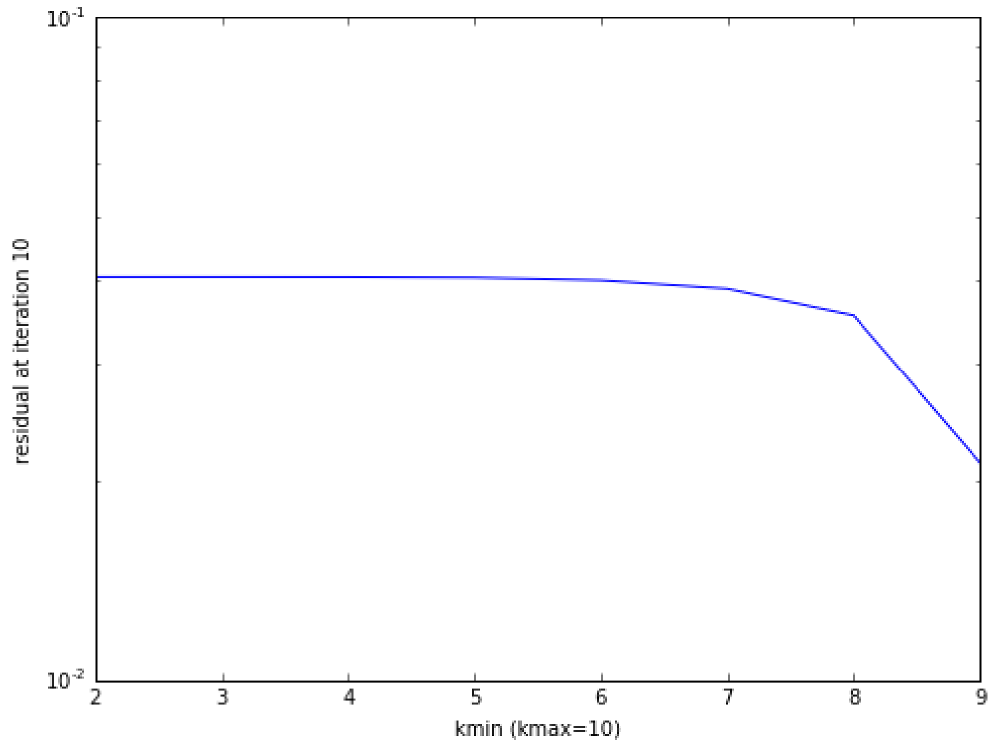

Figure 2: Log plot of residual for various kmin (kmax=10) at iteration 10 for all runs.

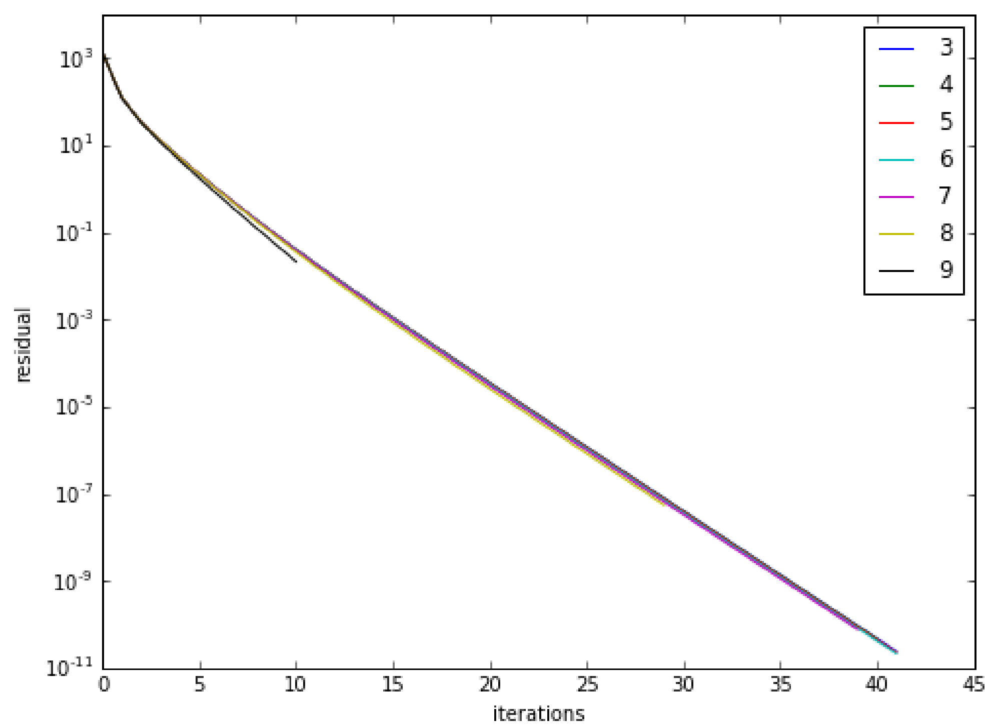

Figure 3: Log plot of residual for various kmin (kmax=10), all of which have been allowed to run for 60 seconds.

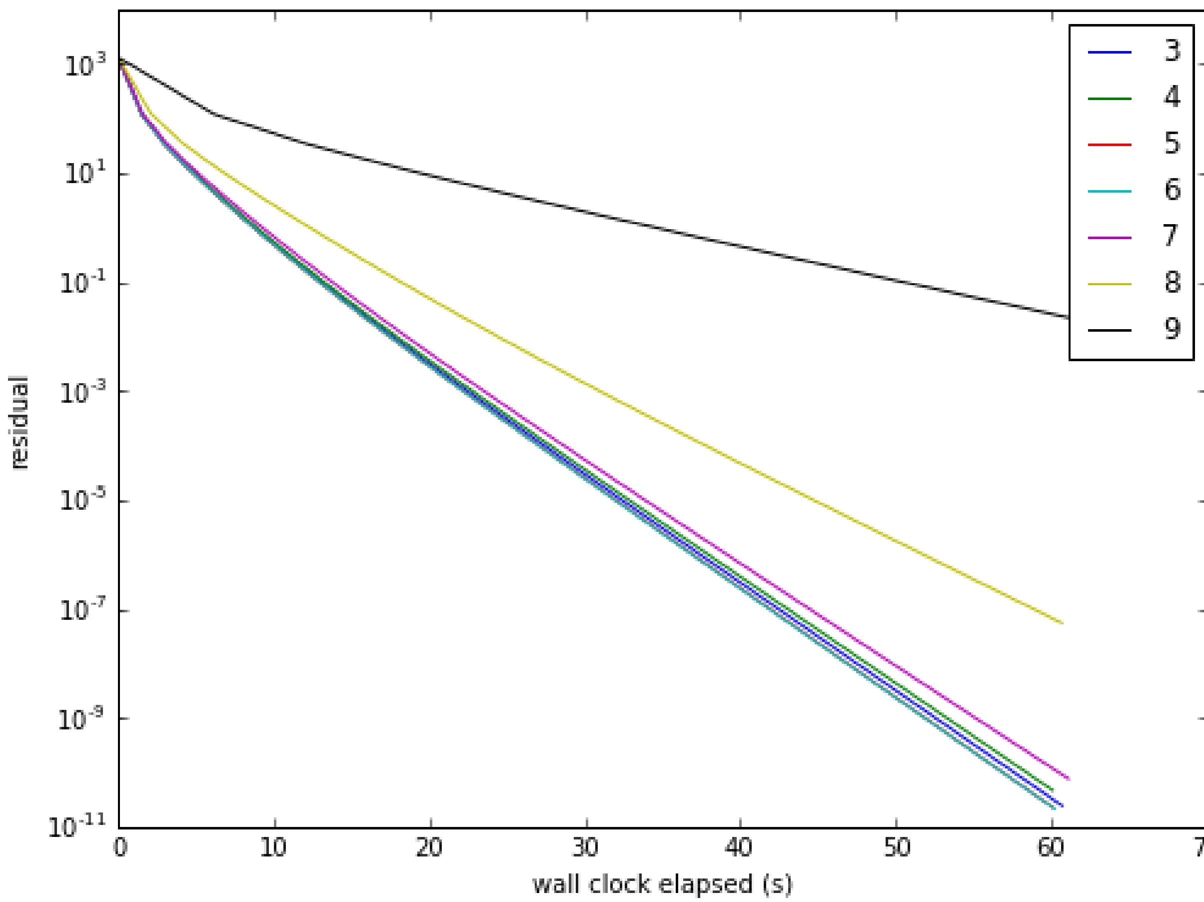

Figure 4: Wall clock time evolution of log of residual for various kmin (kmax=10).