

1. FIND THE DY/DX (LONG METHOD)

1. $y = 3x^3 - 6x^2 - x + 7 = f(x)$

$$\frac{dy}{dx} = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$\frac{dy}{dx} 3x^3 - 6x^2 - x + 7 \rightarrow$ constant rule

$$\lim_{h \rightarrow 0} \frac{(3(x+h)^3 - 6(x+h)^2 - (x+h) + 7) - (3x^3 - 6x^2 - x + 7)}{h}$$

$+ 0 \frac{dy}{dx} 3x^3 - 6x^2 - x \rightarrow$ exponent raise to 0 $\lim_{h \rightarrow 0} \frac{(3(x^3 + 3x^2h + 3xh^2 + h^3) - 6(x^2 + 2xh + h^2) - (x+h) + 7) - (3x^3 - 6x^2 - x + 7)}{h}$

$-1 + 0 \frac{dy}{dx} 3x^3 - 6x^2 \rightarrow$ Power Rule and constant multiple

$3(3x^2) - 6(2x) - 1 + 0$

$$\lim_{h \rightarrow 0} \frac{(3x^3 + 9x^2h + 9xh^2 + 3h^3) - (6x^2 + 12xh + 6h^2) - (x+h) + 7 - [3x^3 - 6x^2 - x + 7]}{h}$$

$$\lim_{h \rightarrow 0} \frac{(3x^3 + 9x^2h + 9xh^2 + 3h^3) - (6x^2 + 12xh + 6h^2) - x + 7 - [3x^3 - 6x^2 - x + 7]}{h} \quad h \neq 0$$

$$\lim_{h \rightarrow 0} \frac{(3x^3 + 9x^2h + 0 + 0) - (6x^2 + 12xh + 0) - x + 7 - [3x^3 - 6x^2 - x + 7]}{h}$$

$$\boxed{\frac{dy}{dx} = \lim_{h \rightarrow 0} 9x^2 - 12x - 1}$$

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2) $y = \sqrt{x}$ or $x^{1/2}$

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \frac{dy}{dx} = \lim_{h \rightarrow 0} \frac{(x^{1/2} + h) - x^{1/2}}{h}$$

$$\left(\frac{\sqrt{x+h} - \sqrt{x}}{h} \right)^2$$

$$= \lim_{h \rightarrow 0} \frac{(x^{1/2} + h)^2 - (x^{1/2})^2}{h}$$

$$\frac{(\sqrt{x+h})^2 - (x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{(x + x^{1/2}h + x^{1/2}h + h^2) - (x)}{h}$$

$$\frac{x + 2\sqrt{x}h + h^2 - (x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{x + 2x^{1/2}h - x}{h}$$

$$\rightarrow h \neq 0 \quad x + 2\sqrt{x} + h - (x)$$

$$= \lim_{h \rightarrow 0} \frac{2x^{1/2}h}{h}$$

$$2\sqrt{x} + h \xrightarrow{h \rightarrow 0} 2\sqrt{x} + 0$$

$$\frac{dy}{dx} = \lim_{h \rightarrow 0} 2x^{1/2}$$

$$2\sqrt{x} + 0$$

$$\frac{dy}{dx} = \lim_{h \rightarrow 0} \frac{1}{2} x^{-1/2}$$

or

$$\frac{dy}{dx} = \lim_{h \rightarrow 0} \frac{1}{2\sqrt{x}}$$