1 Rehabilitation Engineering

§1.5 *Balance control – part III*

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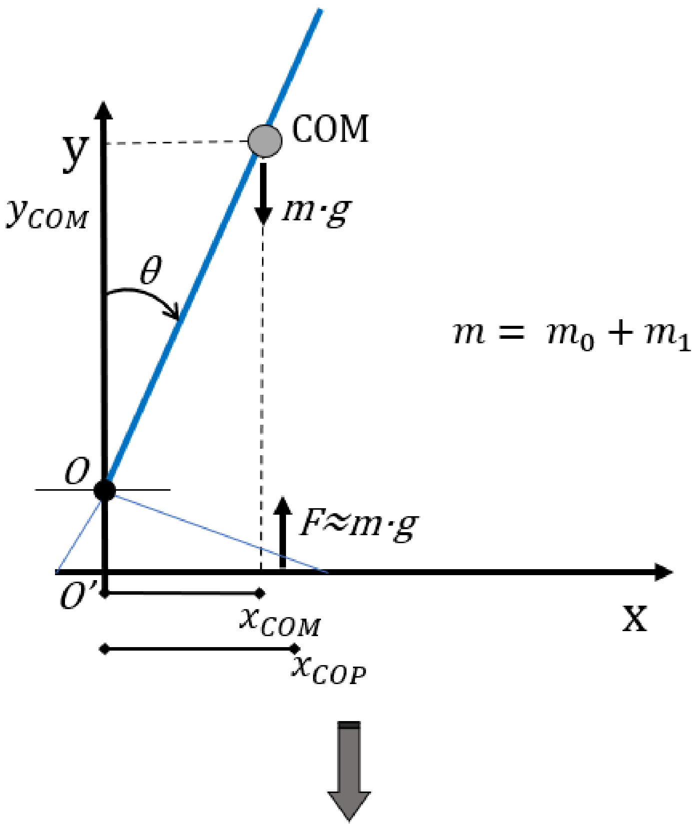
1. Biomechanics of the quiet standing body modelled as an inverted pendulum

Dynamic Equilibrium Equations (Euler – Newton)

$$\begin{cases} R = \sum_i F_i = m \cdot a_{COM} \\ M_0 = \sum_j M_j + \sum_i F_i \cdot b_i = J_0 \cdot \dot{\omega} \end{cases}$$

where J_O is the moment of inertia of the body against rotations around an axis passing through point O, center of the ankle joint.



Free Body Diagram

HP: Balance assessed through a Force Platform

Segment ∅: Feet

- Since feet are not moving by assumption, we shall use the Static Equilibrium equations.

$$\begin{cases} \boldsymbol{R} = 0 \\ \boldsymbol{M_0} = 0 \end{cases}$$

where $\boldsymbol{R}$ is the resulting vector of all the applied forces and $\boldsymbol{M_0}$ is the resulting vector of all the applied torques about the center of rotation O. If we then decompose the vector equations into the corresponding scalar equations (which are three as the degrees of freedom of the rigid body in the 2D plane), we get (**Eqs. [1]**):

Horizontal Translation

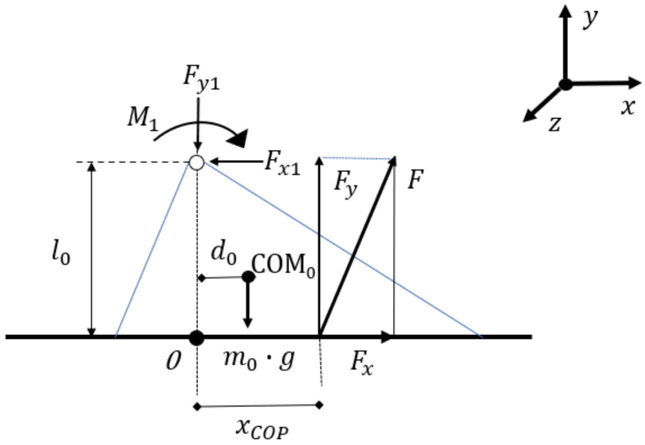
$F_{x_1} = F_x$

Vertical Translation

$F_{y_1} = F_y - m_0 \cdot g$

Rotation about the z axis

$M_1 = F_y \cdot x_{COP} + F_x \cdot l_0 - m_0 \cdot g \cdot d_0$



Segment 1: Body

- Since the rest of the body (but the feet) is free to move by assumption, we shall use the Dynamic Equilibrium equations (**Eqs. [2]**):

Horizontal Translation

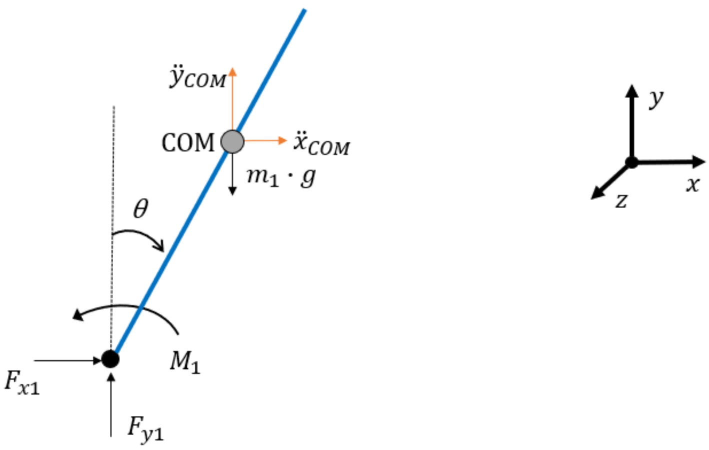
$m_1 \cdot \ddot{x}_{COM} = F_{x_1}$

Vertical Translation

$m_1 \cdot \ddot{y}_{COM} = F_{y_1} - m_1 \cdot g$

Rotation about z axis

$J_O \cdot \ddot{\theta} = m_1 \cdot g \cdot d \cdot \sin\theta - M_1$



Eqs (1) + (2)

$\Downarrow$

$$\begin{cases} m_1 \cdot \ddot{x}_{COM} = F_x \\ m_1 \cdot \ddot{y}_{COM} = F_y - m_0 \cdot g - m_1 \cdot g = F_y - (m_0 + m_1) \cdot g \\ J_O \cdot \ddot{\theta} = m_1 \cdot g \cdot d \cdot \sin\theta - F_y \cdot x_{COP} - F_x \cdot l_0 + m_0 \cdot g \cdot d_0 \end{cases}$$

Comments:

- $m_1 \cdot \ddot{x}_{COM} = F_x \;\Rightarrow\; x_{COM} = \iint \frac{F_x}{m_1} dt^2$

  Estimation of x_{COM} from force platform (FP) measurements in the *time domain*. (need the knowledge of initial conditions, $x_{COM}(0)$ & $\dot{x}_{COM}(0)$+ numerical integration of a noisy signal + drift $\Rightarrow$ high risk of numerical instability
- $m_1 \cdot \ddot{y}_{COM} = F_y - (m_0 + m_1) \cdot g = F_y - m \cdot g$

where $F_y \approx m \cdot g \;\; \Rightarrow \;\; \ddot{y}_{COM} \approx 0$
which simply tells us that the vertical acceleration of COM is negligible (hence, not very useful)

- $J_0 \cdot \ddot{\theta} = \underbrace{m_1 \cdot g \cdot d \cdot \sin\theta + m_0 \cdot g \cdot d_0}_{\text{Destabilizing terms due to the gravitational field}} - \underbrace{(F_y \cdot x_{COP} + F_x \cdot l_0)}_{\text{Stabilizing terms controlled by (the brain through) muscle activation}}$

1.1 COP-COM relationship

After canceling out negligible terms, the last equation simplifies to:

$$J_0 \cdot \ddot{\theta} = m \cdot g \cdot d \cdot \sin\theta - m \cdot g \cdot x_{COP} = m \cdot g \cdot (x_{COM} - x_{COP})$$

In fact:

- $m_0 \ll m_1 \Rightarrow \; m_1 \approx m$
- $F_x \ll F_y, \;\; l_0 \approx x_{COP}$
- $d \cdot \sin\theta = x_{COM}$

Under the small angle approximation:

$$x_{COM} = d \cdot \theta$$

$$\ddot{x}_{COM} = d \cdot \ddot{\theta}$$

$$\ddot{\theta} = \frac{\ddot{x}_{COM}}{d}$$

Hence:

$$\ddot{x}_{COM} = m \cdot g \cdot \frac{d}{Jo}(x_{COM} - x_{COP}) = k \cdot (x_{COM} - x_{COP}) \qquad (3)$$

where $k = \frac{mgd}{Jo}$ [s^{-2}] is an **anthropometric parameter** (function of body mass, height, mass distribution, posture).

- *Time domain*

From Eq. (3) we get:

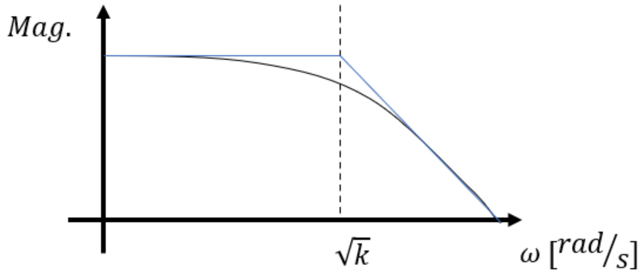
$$x_{COP} = -\frac{\ddot{x}_{COM}}{k} + x_{COM}$$

$$= -\frac{F_x}{m} \cdot \frac{J_0}{m \cdot g \cdot d} + x_{COM}$$

$$= x_{COM} - \frac{J_0}{m^2 \cdot g \cdot d} \cdot F_x = x_{COM} - \frac{F_x}{m \cdot k}$$

where the second term, driven by F_x, is an high-frequency content (noisy) signal measured with a 6-D force platform, superimposed to a low-frequency content (x_{COM}). This equation provides a relationship between x_{COM} and x_{COP} in the *time domain*, in the range of validity of the inverted pendulum model.

- *Frequency domain*



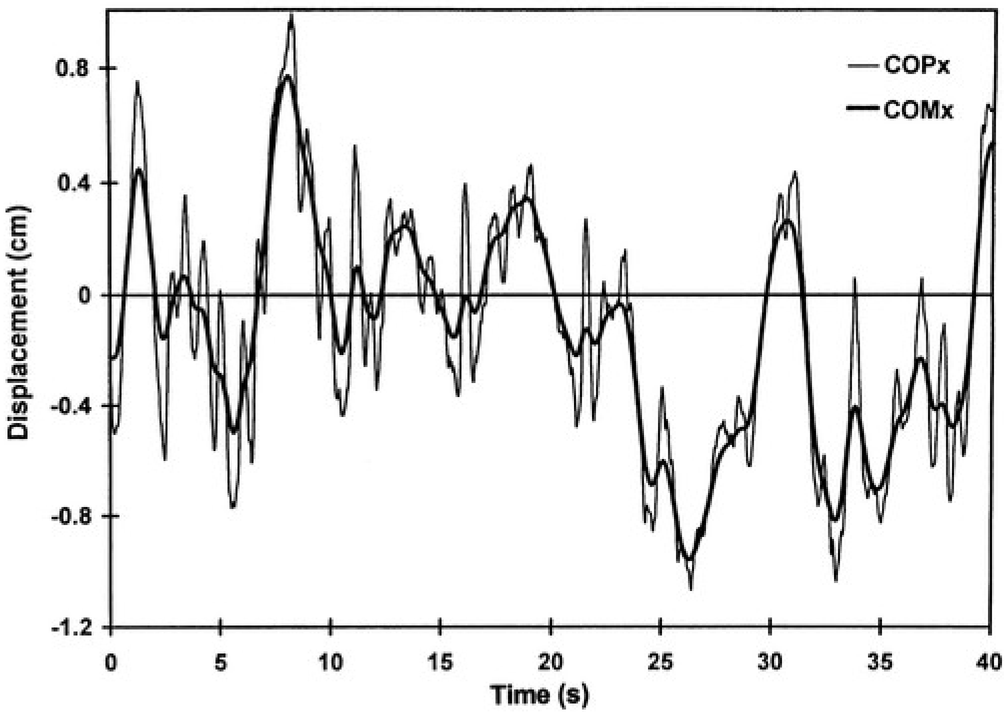
If we Laplace-transform Eq.3 we obtain:

$$s^2 \cdot x_{COM}(s) = k \cdot [x_{COM}(s) - x_{COP}(s)]$$

$$x_{COM}(s) = \frac{k}{k - s^2} \cdot x_{COP}(s)$$

Poles: $\pm\sqrt{k} = \pm\sqrt{\frac{m \cdot g \cdot d}{J_0}}$; double pole in the Fourier domain.

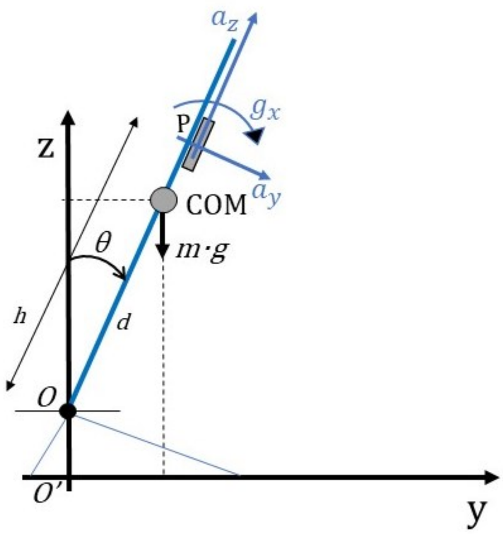
This equation tells us that x_{COM} is a low–pass filtered version of x_{COP}, where the filter is an *anthropometric filter* since the cut-off frequency is a function of the anthropometric properties of the swaying body only.

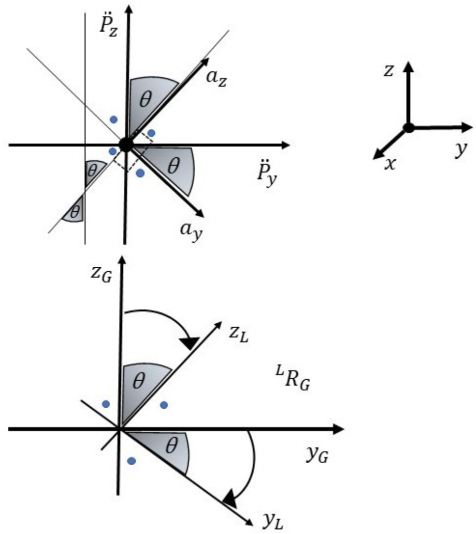


Typical 40-s record from a subject standing quietly. Center-of-pressure (COP) and center-of-mass (COM) in anterior/posterior direction (A/P) direction show the COP to "track" the COM almost in phase and to be oscillating on either side of the COM. You can see the low-pass effect of the anthropometric filter (from Winter, David A., Aftab E. Patla, Francois Prince, Milad Ishac, and Krystyna Gielo-Perczak. Stiffness control of balance in quiet standing. J. Neurophysiol. 80: 1211–1221, 1998)

2. IMU-based posture analysis

Let's assume we want to measure sway in quiet standing through an IMU worn on the lower trunk. Its center P is at a distance h from the ankle joint O. In the case of a 3-D IMU, but with body sway taking place in the sagittal (y-z) plane only, the sensitive axes of the IMU relevant to the problem are a_y, a_z, and g_x (see figure below). Consider θ positive clockwise.





G – global reference frame
L – local reference frame

Gyroscope

$$g_x = \dot{\theta}$$

$$y_{COM}(t) = d \cdot \sin\theta \approx d \cdot \theta = d \cdot \int g_x \cdot dt$$

numerical integration with noise and drift

Accelerometer

$$P = (0, P_y, P_z) = (0, h \cdot \sin\theta\,, h \cdot \cos\theta), h\ cost$$

$$\dot{P} = (0, h \cdot \dot{\theta} \cdot \cos\theta\,, -h \cdot \dot{\theta} \cdot \sin\theta)$$

$$\ddot{P} = (0, h \cdot (\ddot{\theta} \cdot \cos\theta - \dot{\theta}^2 \cdot \sin\theta), -h \cdot (\ddot{\theta} \cdot \sin\theta + \dot{\theta}^2 \cdot \cos\theta))$$

$${}^{G}a = {}^{G}\begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} = {}^{G}\begin{bmatrix} \ddot{P}_x \\ \ddot{P}_y \\ \ddot{P}_z \end{bmatrix} + {}^{G}\begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix}$$

$${}^{L}a = {}^{L}\begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} = {}^{L}R_G \times {}^{G}a == {}^{G}R_L^{\ -1} \times {}^{G}a = {}^{G}R_L^{\ T} \times {}^{G}a =$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\theta & -sin\theta \\ 0 & sin\vartheta & cos\theta \end{bmatrix} \times \begin{bmatrix} 0 \\ h \cdot (\ddot{\theta} \cdot \cos\theta - \dot{\theta}^2 \cdot \sin\theta) \\ -h \cdot (\ddot{\theta} \cdot \sin\theta + \dot{\theta}^2 \cdot \cos\theta) + g \end{bmatrix} =$$

$$= \begin{bmatrix} 0 \\ h\cdot\ddot{\theta}\cdot\cos^2\theta - h\cdot\dot{\theta}^2\cdot sin\vartheta cos\vartheta + h\cdot\ddot{\theta}\cdot\sin^2\theta + h\cdot\dot{\theta}^2\cdot sin\vartheta cos\vartheta - g\cdot\sin\theta \\ h\cdot\ddot{\theta}\cdot sin\vartheta cos\vartheta - h\cdot\dot{\theta}^2\cdot sin^2\theta - h\cdot\ddot{\theta}\cdot sin\vartheta cos\vartheta - h\cdot\dot{\theta}^2\cdot\cos^2\theta + g\cdot\cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ h\ddot{\theta} - g sin\theta \\ -h\dot{\theta}^2 + g cos\vartheta \end{bmatrix}$$

which, in the case of small angles, takes to the following relationships for the antero-posterior and the vertical projections of the acceleration vector, respectively:

$$\begin{cases} a_y = h\cdot\ddot{\theta} - g\cdot\vartheta \\ a_z = -h\cdot\dot{\theta}^2 + g \end{cases}$$

- *Linearized block diagram of the antero-posterior accelerometer*

$a_y(s) = (h\cdot s^2 - \mathrm{g})\cdot\theta(s)$

$$\boxed{\frac{a_y(s)}{\theta(s)} = h\cdot s^2 - \mathrm{g}}$$