1 Rehabilitation Engineering

§1.6 *Gait analysis and control*

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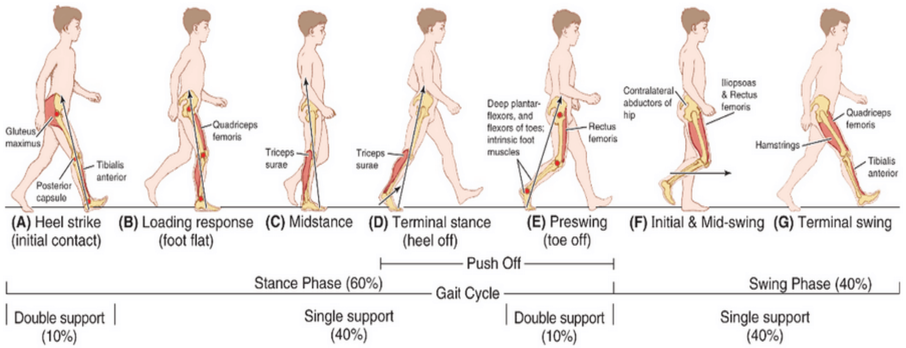


Gait analysis in the lab: tools

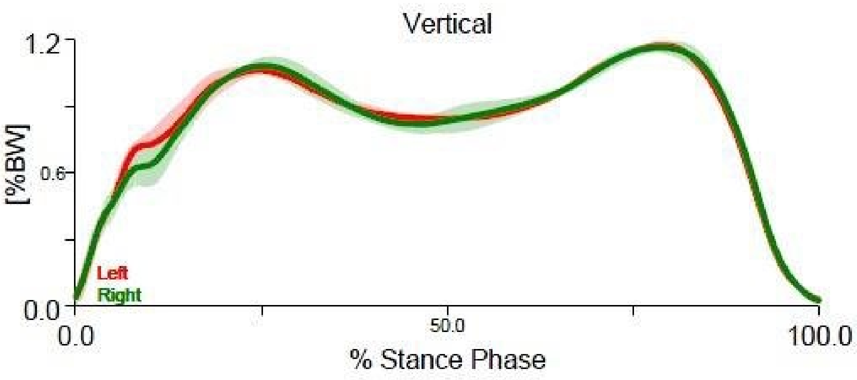
Optoelectronic Cameras
(N≥2)

Force plate(s)

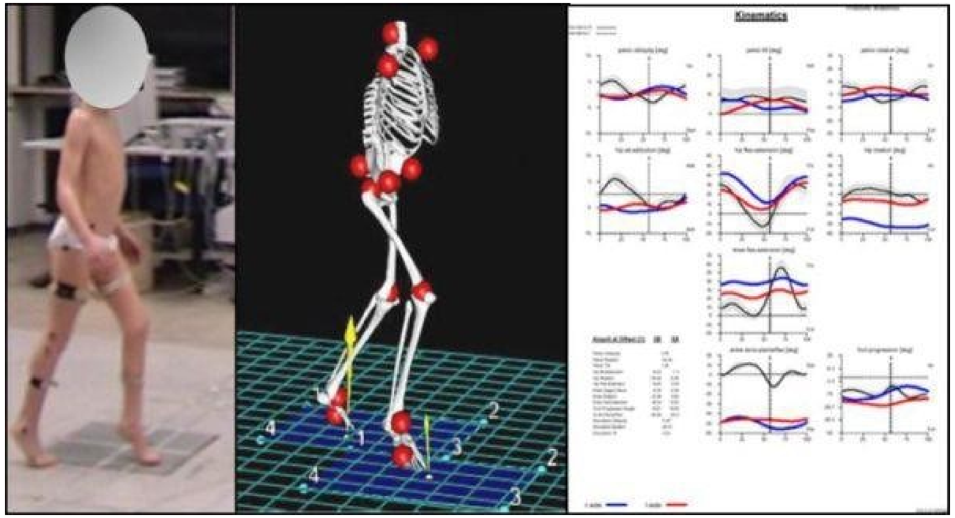
Surface EMG

Pressure plate/math

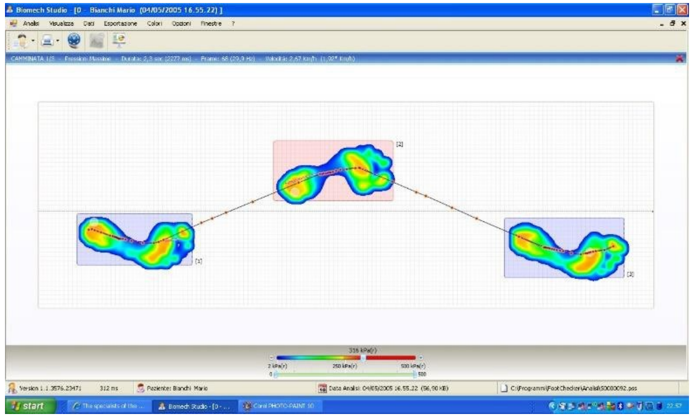
Gait analysis in the lab: outcomes

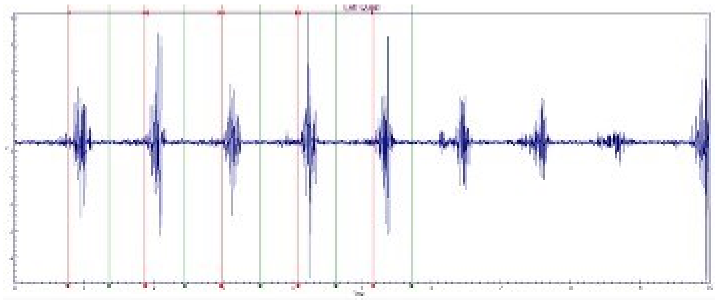


Kinematic report
(joint angles)

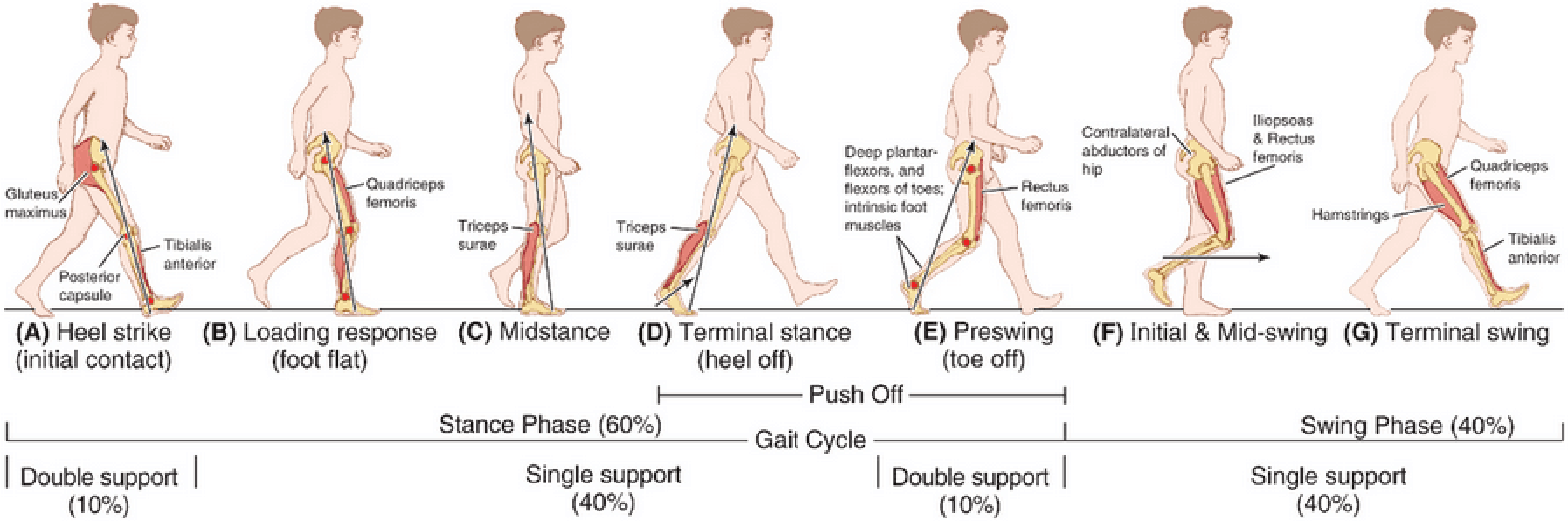


Dynamic report
(forces,torques)





Gait analysis: basic phases and events

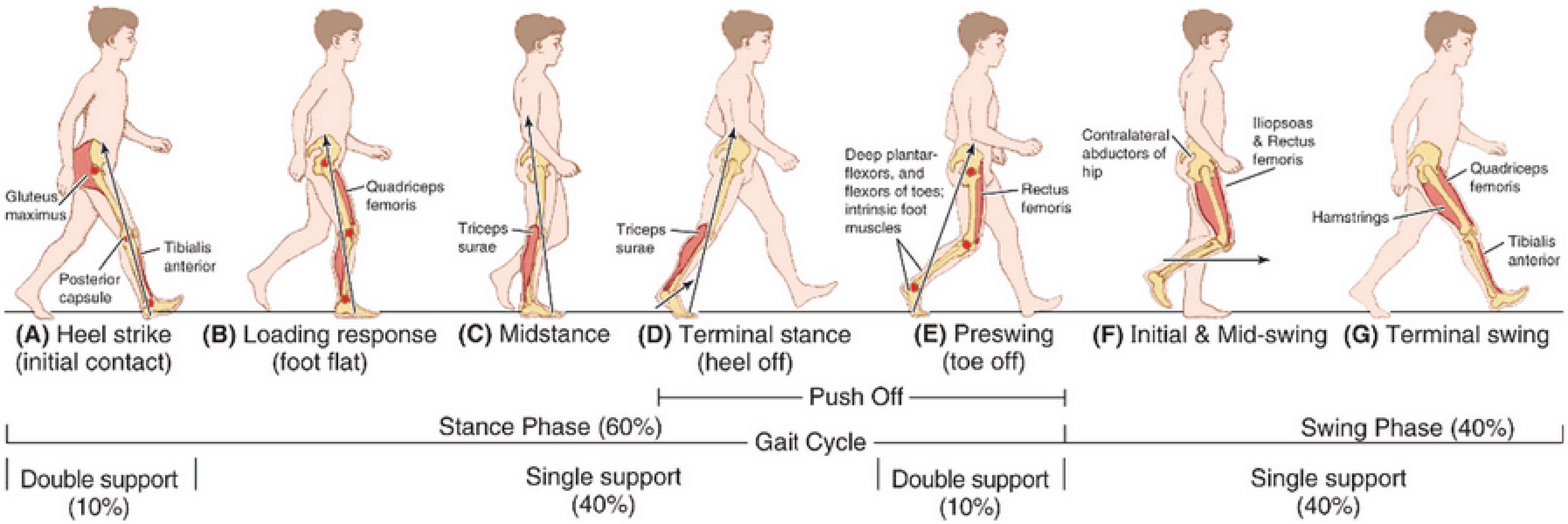


Source: epomedicine.com

Stride period or cycle time (s): The walking cycle (stride) is defined as the time interval between two successive instants of contact with the ground of the same foot (usually the contact of the heel).

Step period (s): The half stride refers to the interval between the initial contact of a foot and the initial contact of the contralateral foot. It usually refers to the foot that moves anteriorly (anterior step).

Gait analysis: basic phases and events



Source: *epomedicine.com*

Heel strike, HS; heel contact, HC; initial contact, IC; foot strike, FS: The instant of the walk in which the foot comes into contact with the ground.

Foot flat, FF: The instant in which the foot is completely resting on the ground.

Heel rise, HR; heel off, HO: The instant in which the heel rises from the ground.

Toe off, TO: The instant in which the foot is detached from the ground.

Gait clinimetrics: spatio-temporal parameters

Time parameters	Normal values
Cadence [cycles/min]	(~55 strides/min)
Stride duration [s]	(~1.1 s)
Step duration [s]	(~0.55 s)
Double-stance duration %	(20-30 %)
Spatial parameters	
Gait speed [m/s]	(~1.2 m/s)
Stride length [m]	(~80-90% *h*)
Step length [m]	(~40-45% *h*)
Foot clearance [m]	

h – subject's height

Exam Calendar

Written test

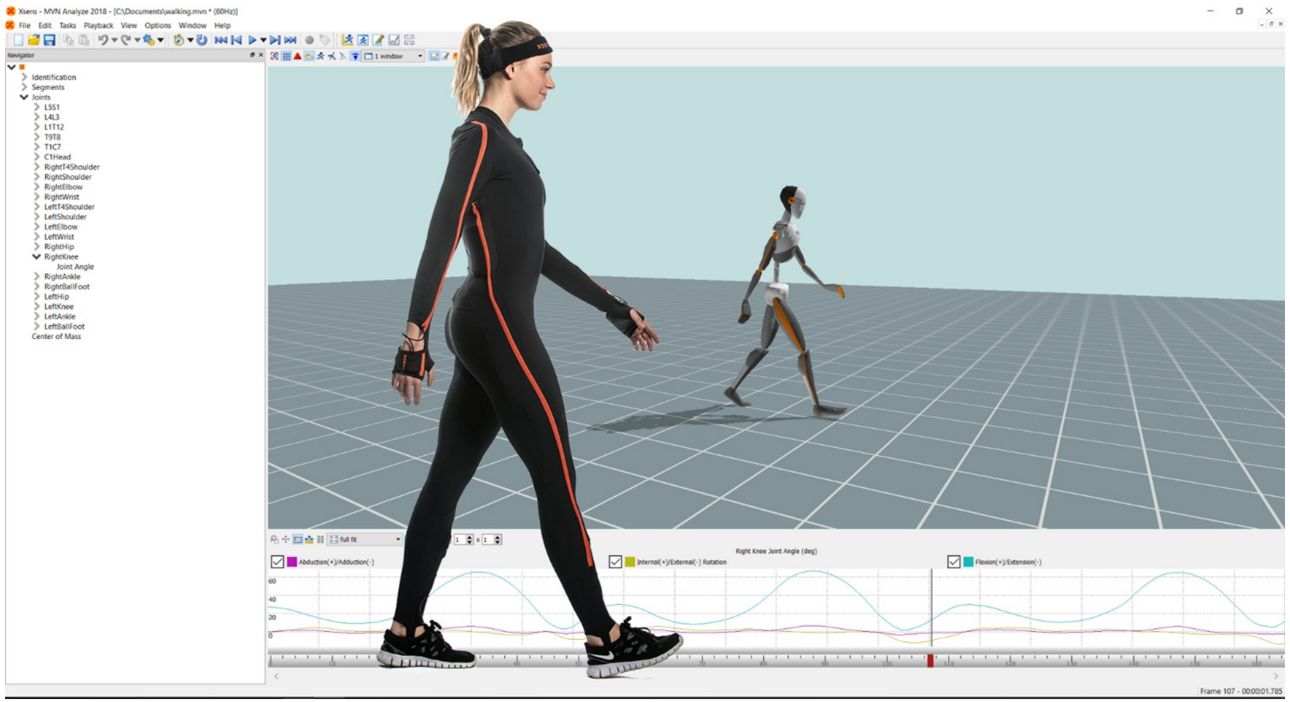
	Attività formative	Descrizione	Data e ora	Luogo	Tipo	Stato
	93932 - Ageing And Rehabilitation Engineering (Cds. 9266 - Ce) 34811 - Bioingegneria Della Riabilitazione Lm (Cds. 9243 - Ce) 34811 - Bioingegneria Della Riabilitazione Lm (Cds. 8198 - Ce)	Written test	11/01/2024 14:00	Aula 2.3 - Ex-Zuccherificio – Edificio 1, Ingresso Via Machiavelli - Via dell’Università, 50, Cesena - Piano terra (livello 2)	Scritto	> Pubblicata
	93932 - Ageing And Rehabilitation Engineering (Cds. 9266 - Ce) 34811 - Bioingegneria Della Riabilitazione Lm (Cds. 8198 - Ce) 34811 - Bioingegneria Della Riabilitazione Lm (Cds. 9243 - Ce)	Written test	01/02/2024 14:00	Aula 2.3 - Ex-Zuccherificio – Edificio 1, Ingresso Via Machiavelli - Via dell’Università, 50, Cesena - Piano terra (livello 2)	Scritto	> Pubblicata
	93932 - Ageing And Rehabilitation Engineering (Cds. 9266 - Ce) 34811 - Bioingegneria Della Riabilitazione Lm (Cds. 9243 - Ce) 34811 - Bioingegneria Della Riabilitazione Lm (Cds. 8198 - Ce)	Written test	15/02/2024 14:00	Aula 2.3 - Ex-Zuccherificio – Edificio 1, Ingresso Via Machiavelli - Via dell’Università, 50, Cesena - Piano terra (livello 2)	Scritto	> Pubblicata

Oral presentations

30	93932 - Ageing And Rehabilitation Engineering (Cds. 9266 - Ce) 34811 - Bioingegneria Della Riabilitazione Lm (Cds. 8198 - Ce) 34811 - Bioingegneria Della Riabilitazione Lm (Cds. 9243 - Ce)	Team projects presentations	08/02/2024 10:00	Vedi note - LIB-M	Orale	> Pubblicata

All students who worked on the projects in the current academic year and shall present on this day must register here.

Wearable inertial sensors

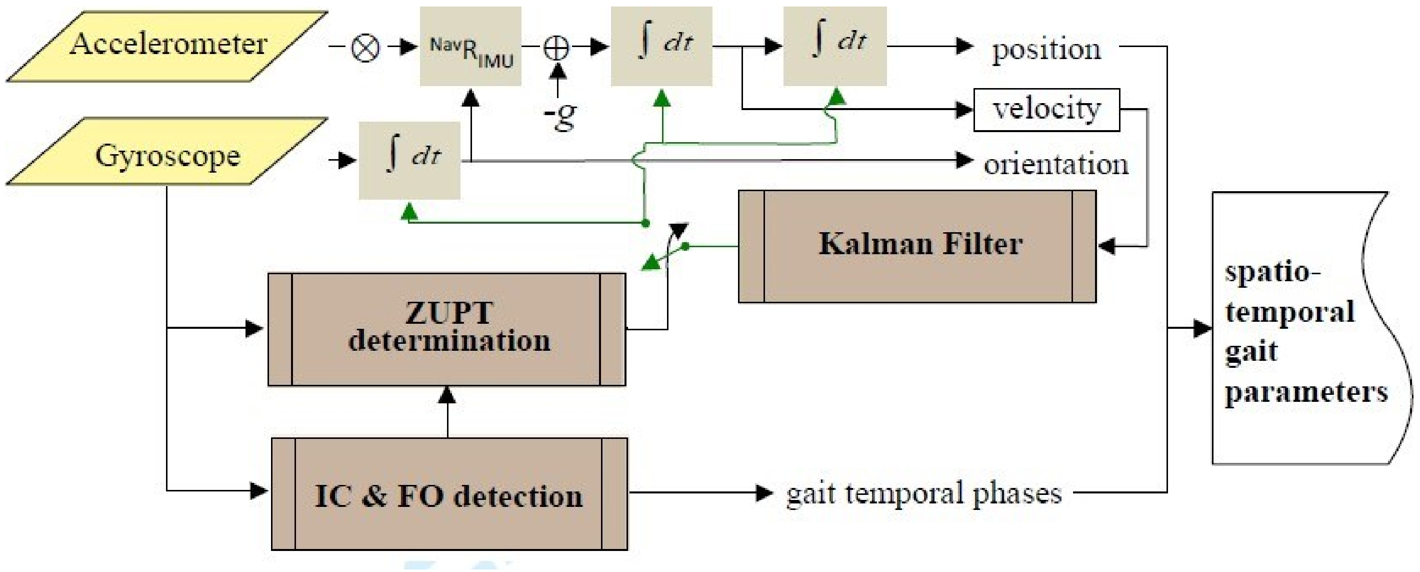


IMU, MIMU

(Inertial Measurement Units, Magneto-Inertial Measurement Units)

Inertial platforms and the need of sensor fusion algorithms

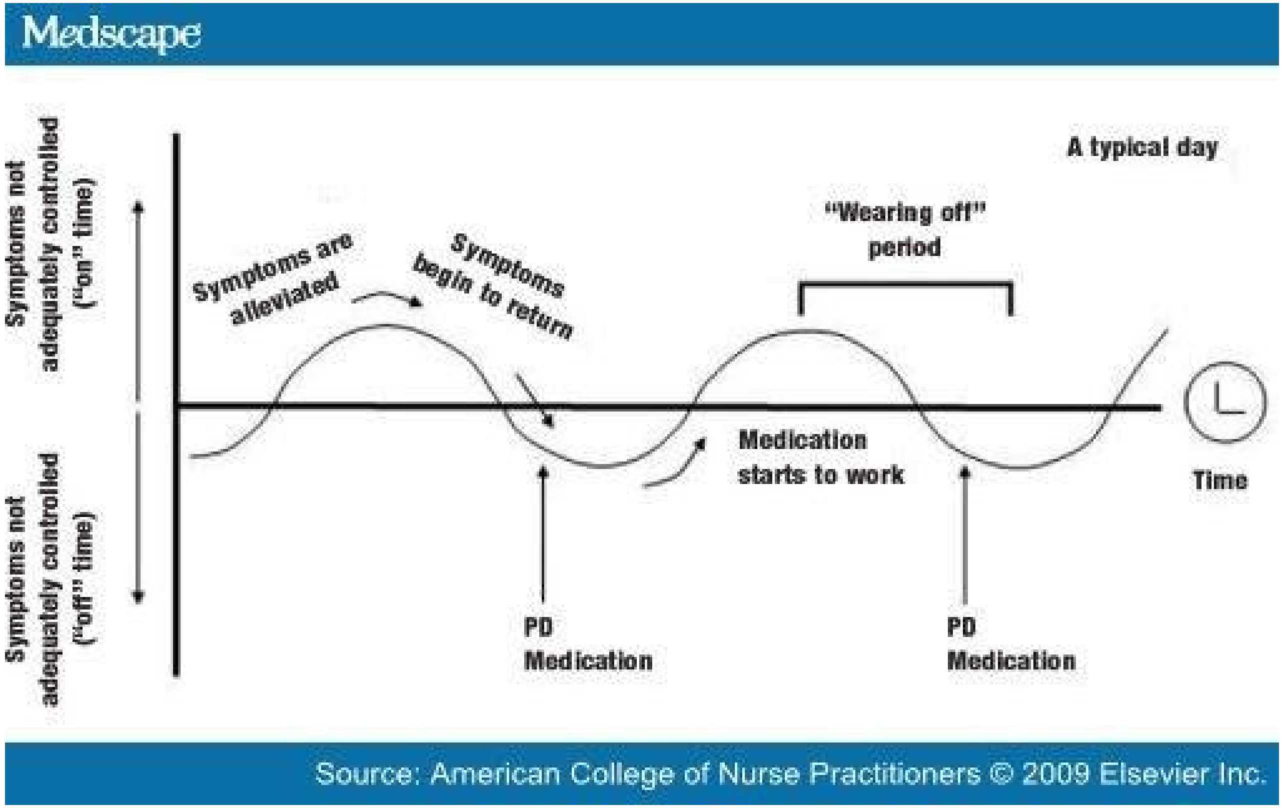
- It is only with similar platforms, and possibly with the use of a multiplicity of IMUs, that it is possible to accurately measure the **spatial** gait parameters (Length, width, height of the step, foot clearance, etc.)



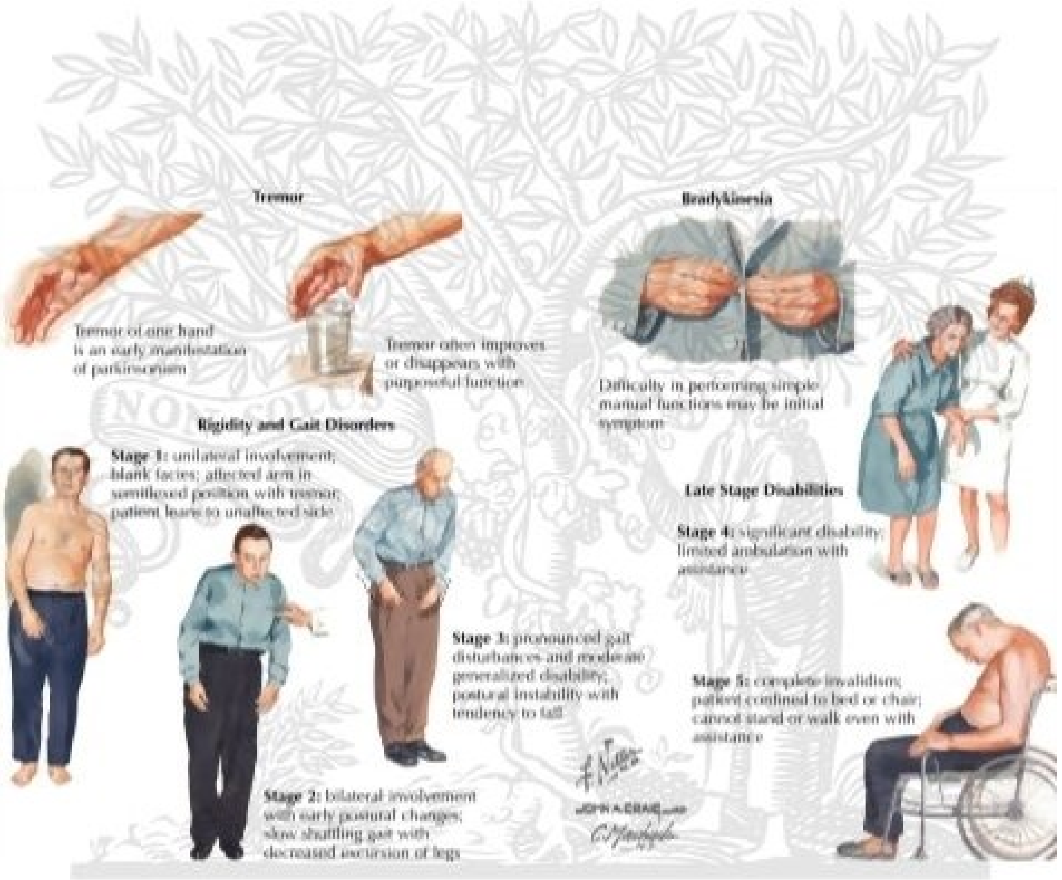
Ferrari et al., 2015

Time is an important variable

The main (loco)motor disorders **fluctuate** strongly over time, also in relation to, e.g., the timing of a therapy or a rehabilitation intervention

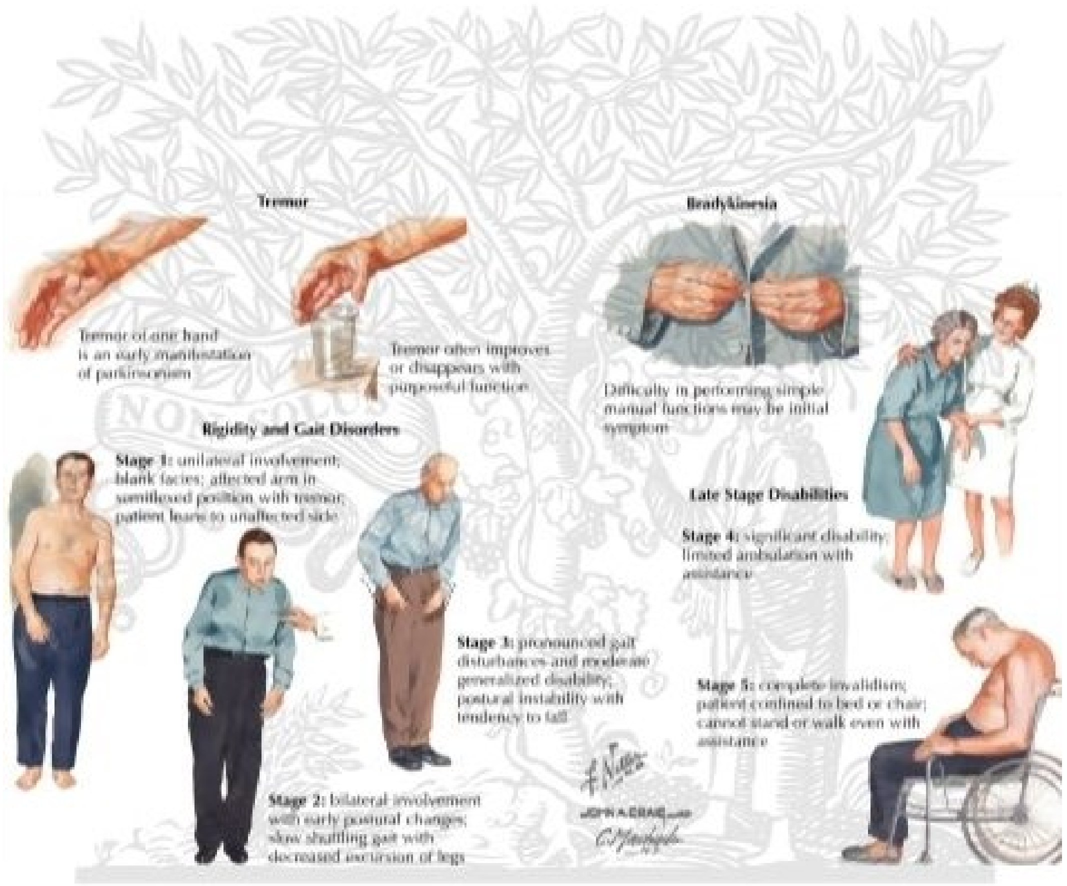


Example: Parkinson’s Disease



Parkinson’s disease (PD) is a neurological disorder with **progressive disability** over time. PD is caused by deficiency of the dopamine neurotransmitter in the brain. The areas of the brain particularly affected are the *basal ganglia*, a group of nuclei involved in movement control. Current therapy for PD is based on augmentation or replacement of dopamine.

Example: Parkinson’s Disease



The disease’s consequences can be observed in walking, daily life movements such as transitions, movement initiations and turns. Drug therapy eventually causes motor complications (motor fluctuations, dyskinesia, loss of balance).

In the past two decades, there has been evidence from research studies demonstrating the **ability of motor learning in PD** and **improvements as a result of cueing and training**.

Time is an important variable

The main (loco)motor disorders **fluctuate** strongly over time, also in relation to, e.g., the timing of a therapy or a rehabilitation intervention

Courtesy: Jeff Hausdorff, TASMC, Tel Aviv

Contextual factors

The main (loco)motor disorders **fluctuate** strongly over time, also in relation to, e.g., the timing of a therapy or a rehabilitation intervention and are often **context-sensitive**

Courtesy: Alice Nieuwboer, KU Leuven

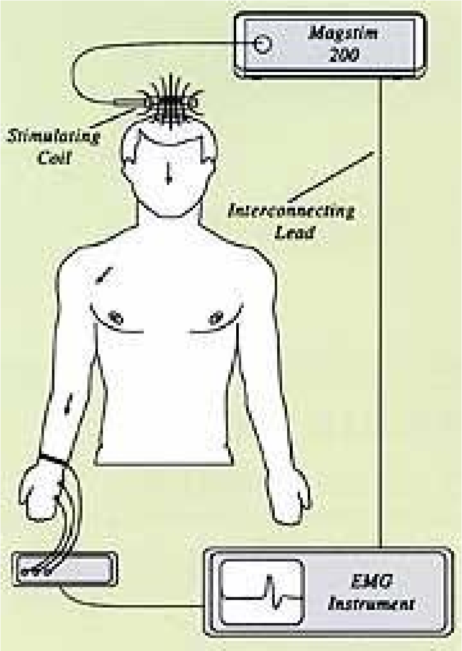
Other gait measures

- (A)simmetry
- Smoothness
- Coordination
- Variability
- Stability

Asymmetry: causes

- Anatomical or functional dysmetry of the lower limbs or pelvis
- Hemiplegia
- Asymmetry in corticomotor input

(Palmer et al, Clin Neurophysiol, 2016)



Note: Here it is actually measured in the lower limb, in the soleus

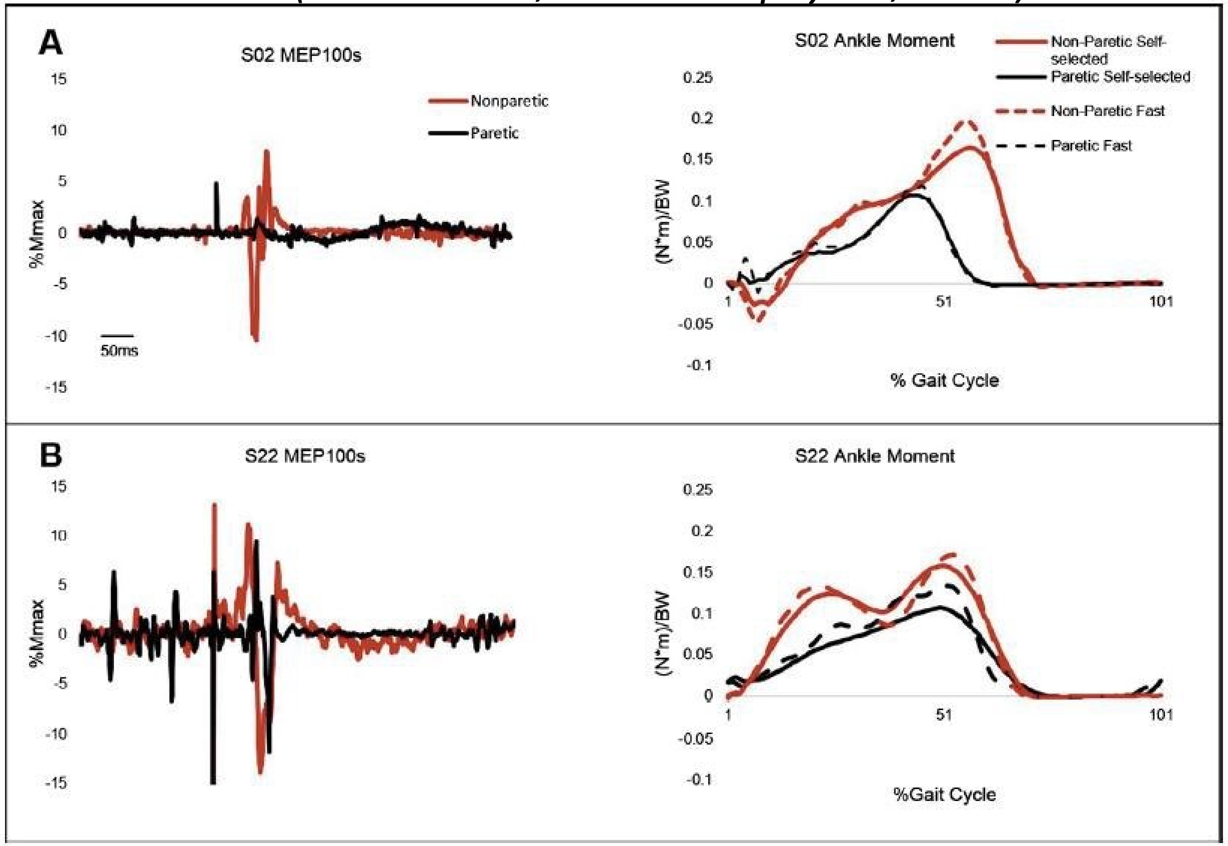


Fig. 1. Raw soleus MEP_{100} data (left column) and average plantarflexion moments over all strides for paretic and nonparetic legs (right column) are shown for a participant with poor (A) and good (B) corticomotor symmetry.

Asymmetry: causes

- Anatomical or functional dysmetry of the lower limbs or pelvis
- Hemiplegia
- Asymmetry in corticomotor input

(Palmer et al, Clin Neurophysiol, 2016)

Corticomotor asymmetry

Asymmetry in the plantarflexion moment

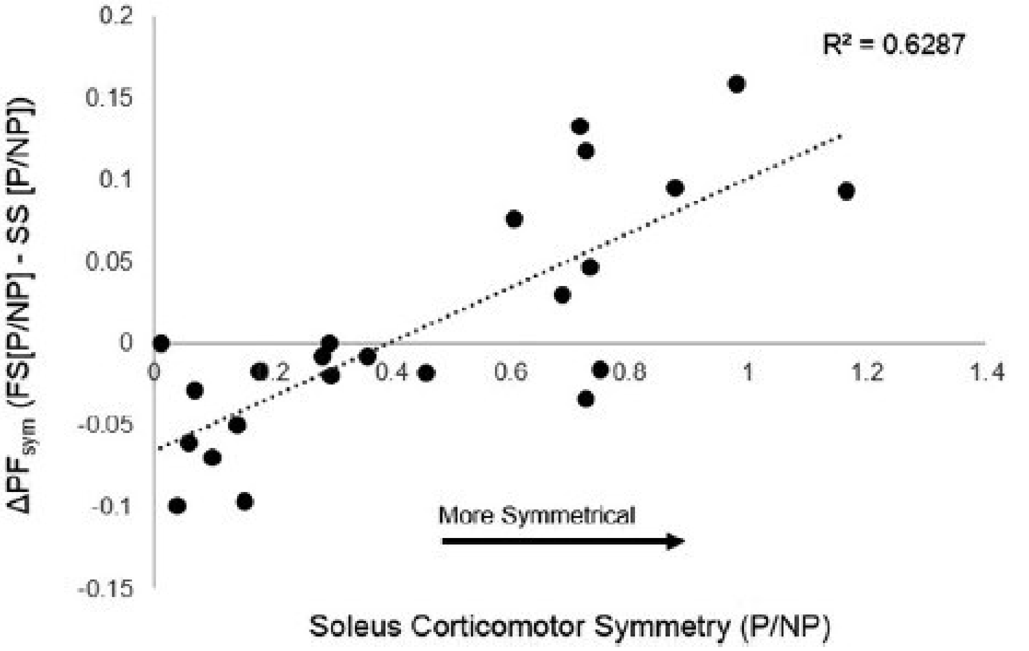


Fig. 2. The relationship between corticomotor symmetry (paretic (P) MEP_{100}/nonparetic (NP) MEP_{100}) and change in plantarflexion moment symmetry (ΔPF_{sym}) (paretic plantarflexion moment/nonparetic plantarflexion moment) from self-selected (SS) to fastest walking (FS) walking speed ($F(1,21) = 35.56$, $p < 0.001$, $R^2 = 0.629$) ($n = 23$).

Asymmetry: causes

- Asymmetrical movement patterns are common in patients with **unilateral weakness** or **pain**. Individuals with unilateral lower limb musculoskeletal pathologies such as **osteoarthritis**, or after procedures such as **total joint arthroplasty** or **anterior cruciate ligament reconstruction**, preferentially unload the affected side and shift the weight to the non-affected side during sit-to-stand and squat tasks.
- These asymmetries are particularly concerning in patients before and after total joint arthroplasty because weight-bearing asymmetry is related to worse functional performance. **Restoring movement symmetry is an important component of rehabilitation** for patients after total joint arthroplasty; however, methods to quantify inter-limb differences in loading during functional tasks are not always available or feasible in clinical settings.

(Abujaber et al, Gait & Posture, 2015)

Asymmetry measures

Global Gait Asymmetry Index

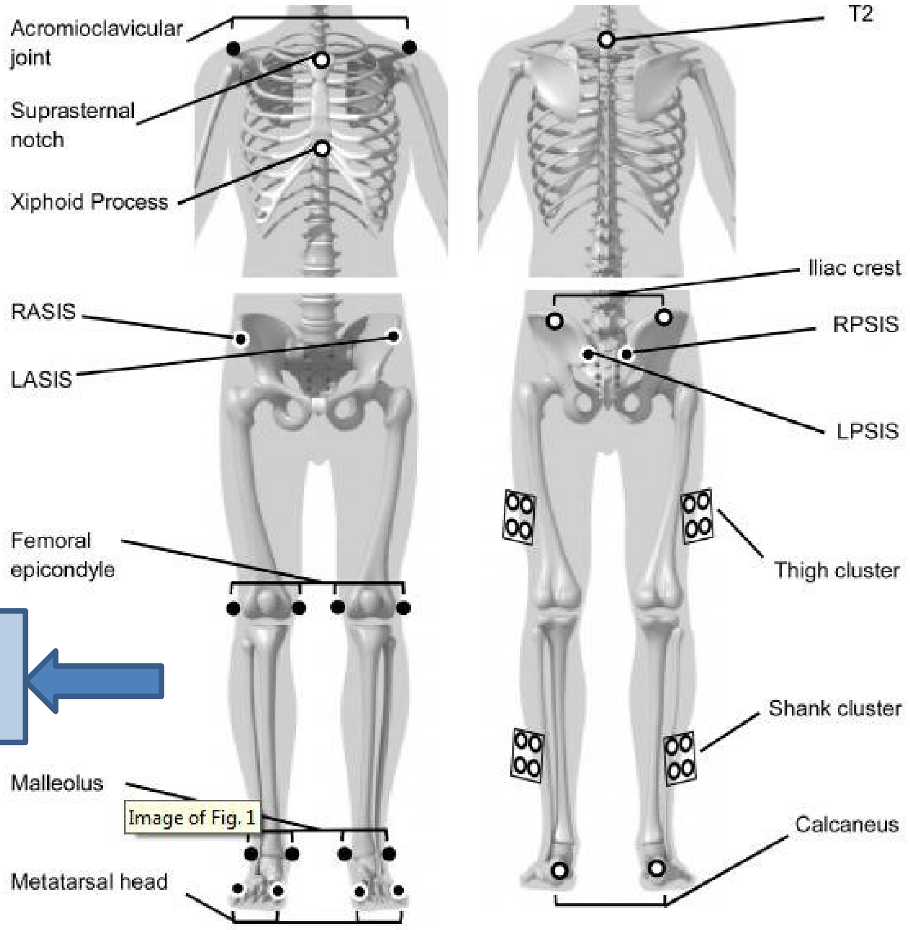
(Cabral et al, J Appl Biomech, 2016)

All three components of these angles (15 waveforms) were used to calculate the GGA index, according to the equation below:

$$GGA = \sum_{i=1}^{15} \sqrt{\Sigma_{i=0}^{100} [l_{i,t} - r_{i,t}]^2}$$

where i are the angular variables, and $l_{i,t}$ and $r_{i,t}$ are the values obtained for the left and right sides of each angular variable, respectively, at t (each percentage of the time normalised gait cycle).

15 joint angles ← 8-segment model ← 18+6 markers



Symmetry measures

Harmonic Ratio

(Gage, 1964)

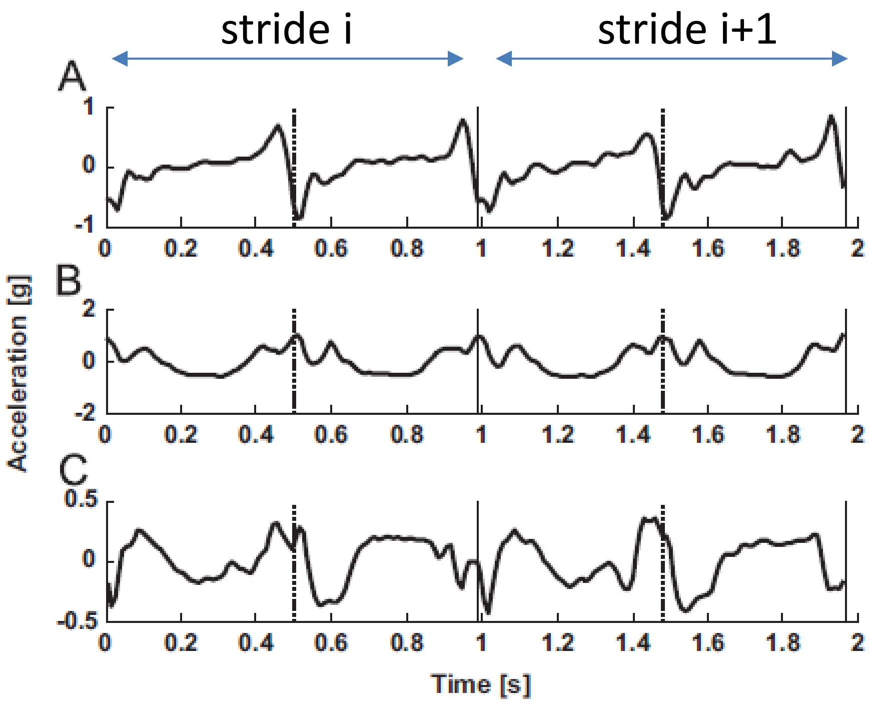


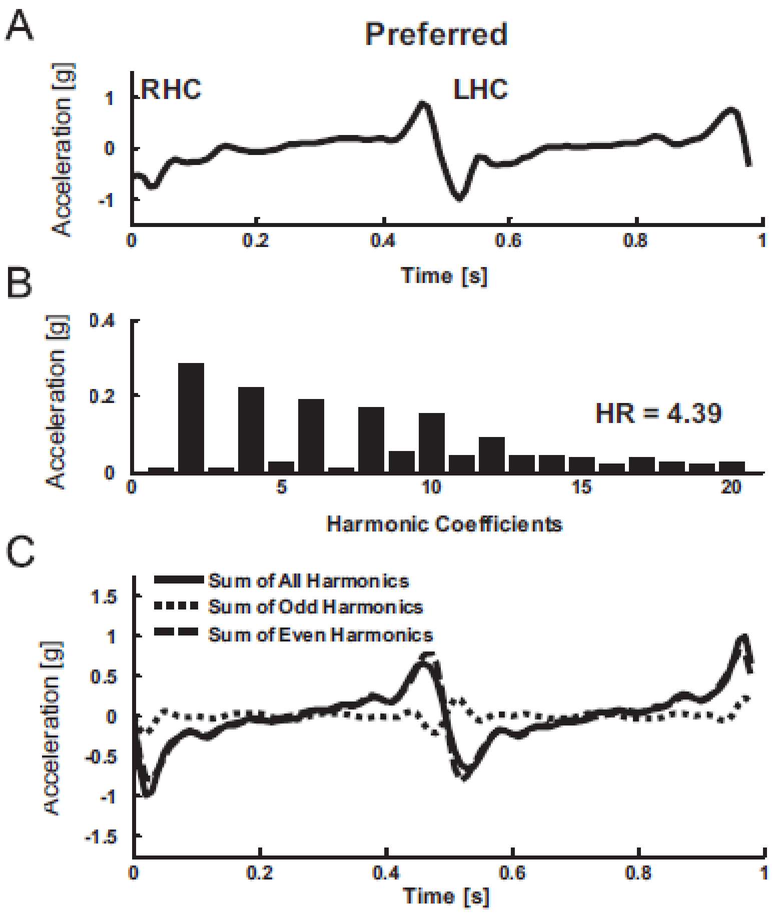
Fig. 1. The L3/L4 acceleration signal for (A) anterioposter, (B) vertical and (C) mediolateral directions with right (vertical solid) and left (vertical dotted) heel contacts. *Note*: the gravitational component has been removed.

The **harmonic ratio** (HR) is a measure used to quantify the smoothness of walking. In gait research, HR is most commonly extracted from **trunk accelerations** in the anteroposterior (AP), vertical (VT), and mediolateral (ML) directions. The **VT and AP accelerations are biphasic within a stride** due to the right and left steps. In contrast, the **ML accelerations are monophasic** (Fig. 1). The HR quantifies the harmonic composition of these accelerations for a given stride, where *a high HR is interpreted as greater walking smoothness*. The HR calculated from trunk accelerations, unlike typical spatiotemporal parameters, is a summary measure of whole-body movement.

Symmetry measures

Harmonic Ratio

(Gage, 1964)



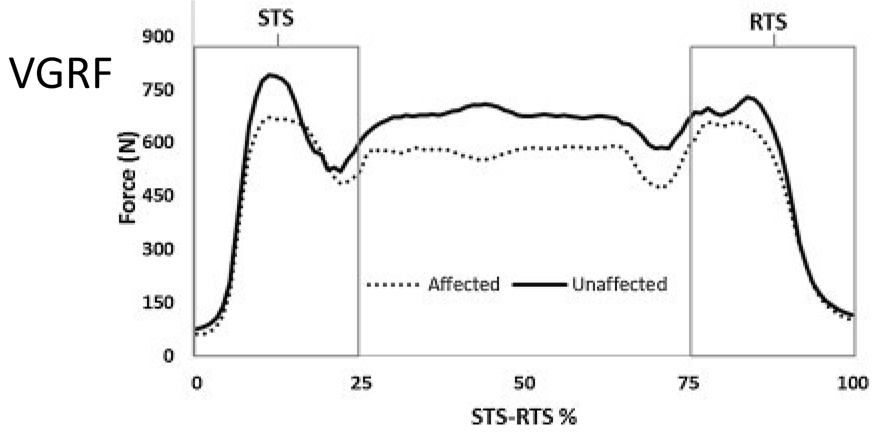
The HR method is based upon harmonic theory to examine the symmetry within a stride by exploiting the periodicity of the signal (Gage, 1964; Smidt et al., 1971). The measured accelerations for each stride are analyzed in the frequency domain through a well-established technique of Fourier analysis based on the stride frequency. **The HR is then defined from the DFT as a ratio of the sum of the amplitudes of the even harmonics (n=2,4,6,…) to the sum of the amplitudes of the odd harmonics (n=1,3,5,…).**

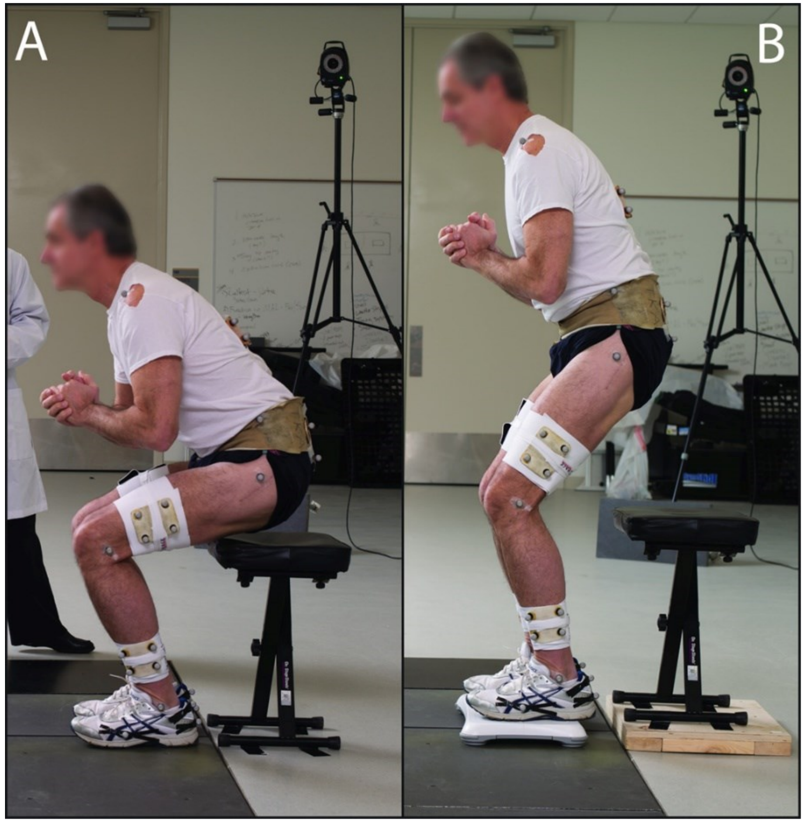
Symmetry measures

Gait Posture. 2015 February ; 41(2): 676–682. doi:10.1016/j.gaitpost.2015.01.023.

Validity Of The Nintendo Wii Balance Board To Assess Weight Bearing Asymmetry During Sit-To-Stand And Return-To-Sit Task

Sumayeh Abujaber[a,d], Gregory Gillispie[b], Adam Marmon[a], and Joseph Zeni Jr[a,c]





Peak VGRF under each limb and the symmetry ratio were calculated and used in this analysis. Peak VGRF during both the STS and RTS phases were calculated in Newtons (N). Interlimb force symmetry was calculated using the ***symmetry ratio****, which was defined as the* ***(peak force of the affected limb/peak force of the unaffected limb) × 100****. This value was expressed as a percentage where a value of 100 implies perfect symmetry between limbs. Values less than 100 indicate greater force on the unaffected limb and values greater than 100 indicate greater force on the affected limb.*

Smoothness measures

Balasubramanian *et al. Journal of NeuroEngineering and Rehabilitation* (2015) 12:112
DOI 10.1186/10.1186/s12984-015-0090-9

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METHODOLOGY **Open Access**

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On the analysis of movement smoothness

Sivakumar Balasubramanian[1*], Alejandro Melendez-Calderon[2,3], Agnes Roby-Brami[4] and Etienne Burdet[5]

Abstract

Quantitative measures of smoothness play an important role in the assessment of sensorimotor impairment and motor learning. Traditionally, movement smoothness has been computed mainly for discrete movements, in particular arm, reaching and circle drawing, using kinematic data. There are currently very few studies investigating smoothness of rhythmic movements, and there is no systematic way of analysing the smoothness of such movements. There is also very little work on the smoothness of other movement related variables such as force, impedance etc. In this context, this paper presents the first step towards a unified framework for the analysis of smoothness of arbitrary movements and using various data. It starts with a systematic definition of movement smoothness and the different factors that influence smoothness, followed by a review of existing methods for quantifying the smoothness of discrete movements. A method is then introduced to analyse the smoothness of rhythmic movements by generalising the techniques developed for discrete movements. We finally propose recommendations for analysing smoothness of any general sensorimotor behaviour.

Keywords: Smoothness, Sensorimotor assessment, Discrete movements, Rhythmic movements

Sources of variability in gait

"Variability in quantitative gait data arises from many potential sources, including

- *natural temporal dynamics of neuromotor control,*
- *pathologies of the neurological or musculoskeletal systems,*
- *the effects of aging, as well as*
- *variations in the external environment, assistive devices, instrumentation or data collection methodologies.*

*In light of this variability, unidimensional, cycle-based gait variables such as stride period should be viewed as **random variables** and prototypical single-cycle kinematic or kinetic curves ought to be considered as **random functions of time**."*

Chau et al., JNER, 2005

Gait variability analysis

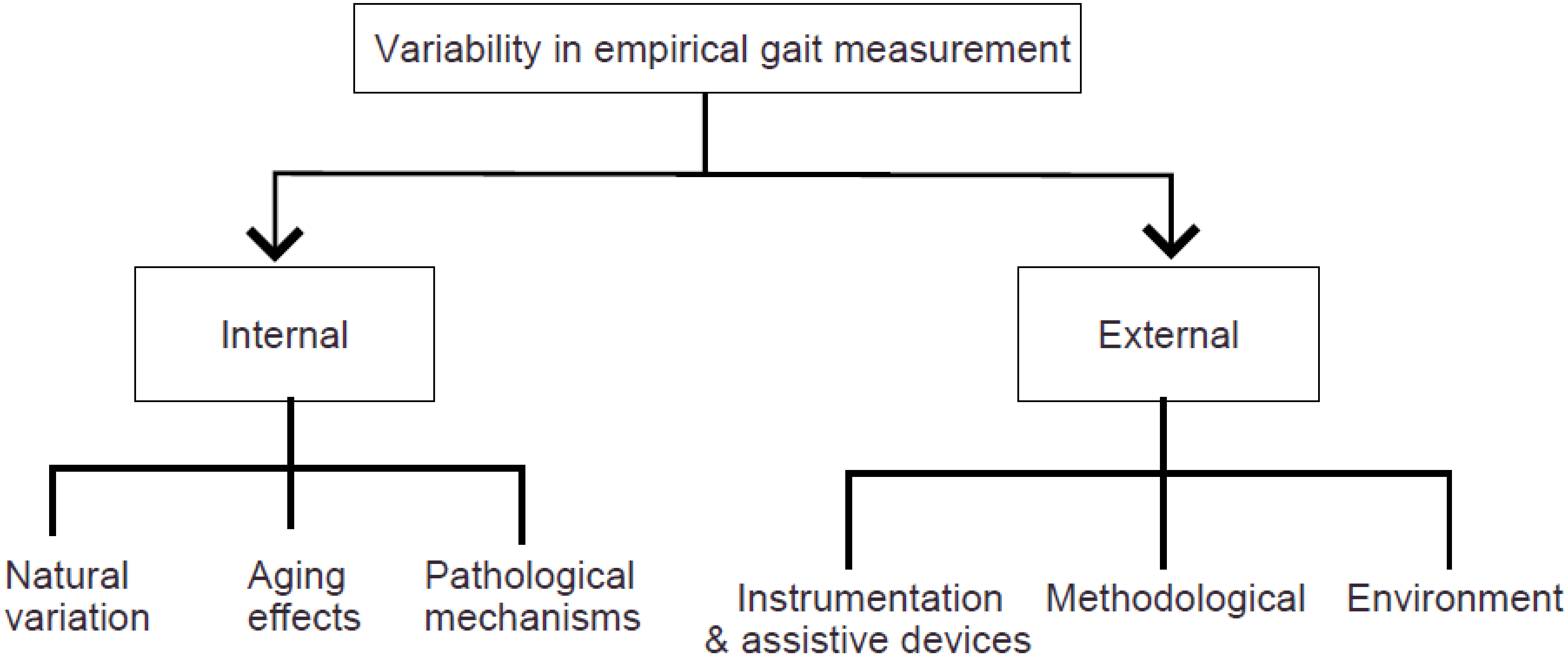


Figure 1
Sources of variability in empirical gait measurements.

Chau et al., JNER, 2005

Gait variability analysis: common metrics

$SD(\mathbf{X})$	Standard Deviation
$CV(\mathbf{X}) = \dfrac{\sqrt{1/N\sum_{i=1}^{N}(x_i - \bar{X})^2}}{\bar{X}}$	Coefficient of Variation
$IQR(X) = x_{0.75} - x_{0.25}$	Interquartile Range
$MAD(X) = \mathrm{med}\,(\lvert X - \mathrm{med}(X)\rvert)$	Median Absolute Deviation

Gait variability analysis: common metrics

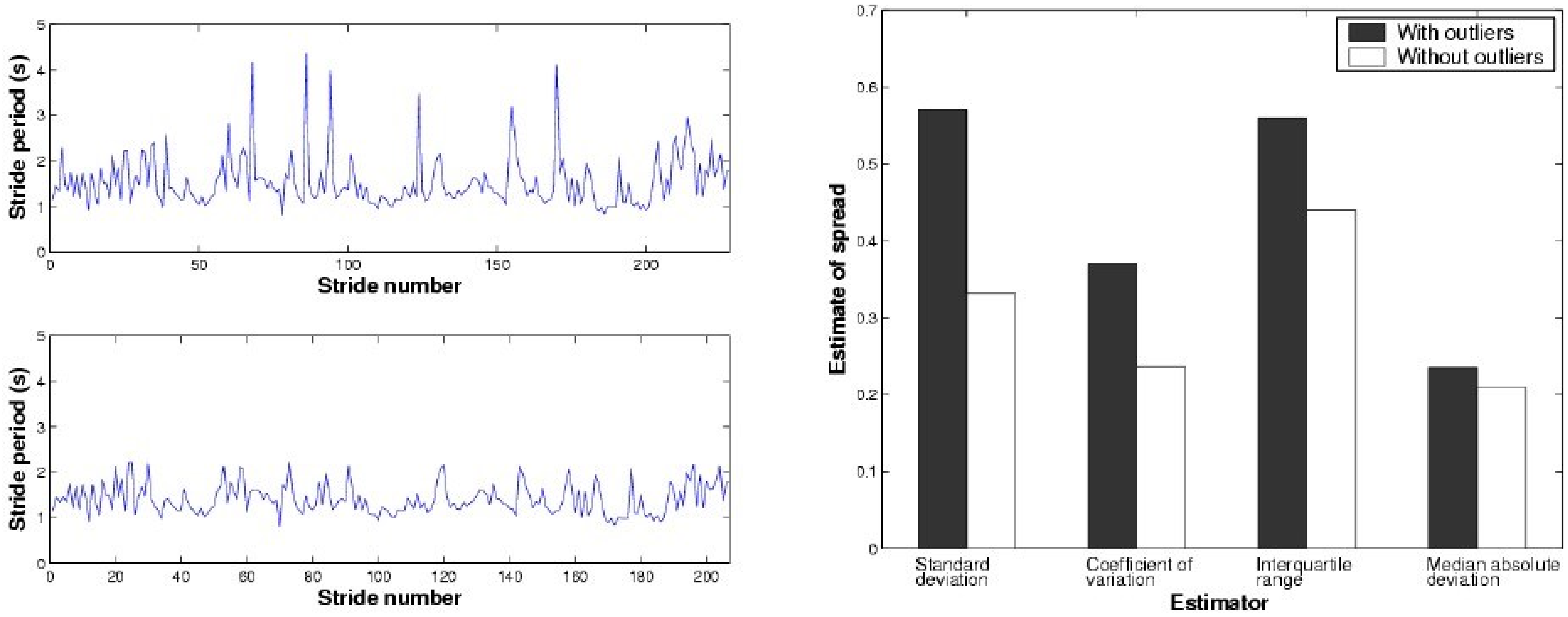
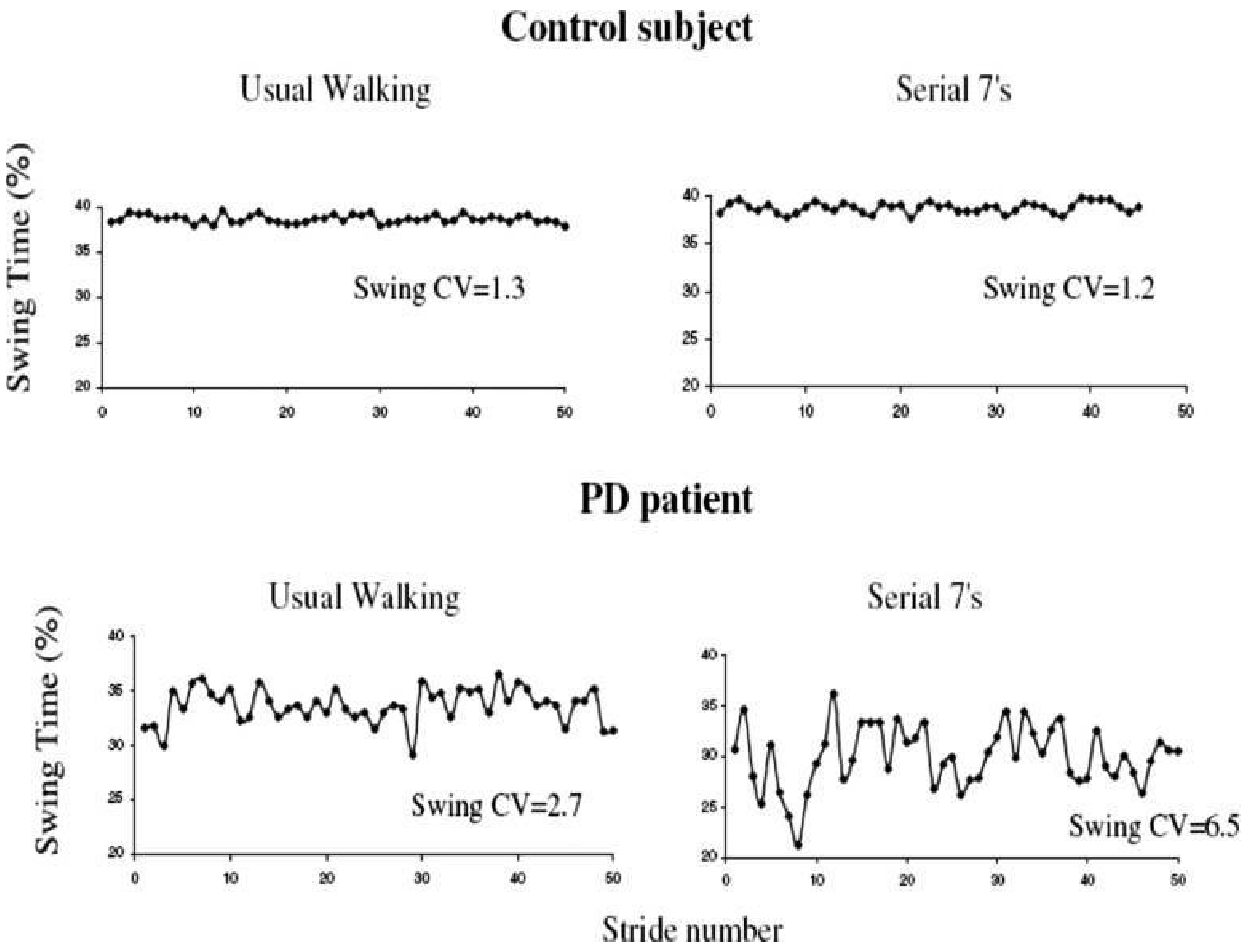
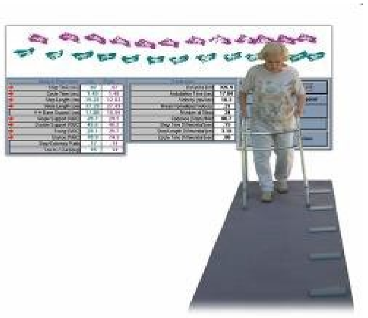


Figure 2
Robust vs. non-robust estimators of parameter spread. The left pane shows a sequence of stride periods with outliers (top) and after removal of outliers (bottom). The right pane is a bar graph showing the values of four different spread estimators before and after outlier removal.

The four estimators have different sensitivity to the presence of outliers.
The latter two are more robust.

Gait variability is increased in Parkinson's Disease





(Hausdorff, Chaos, 2009)

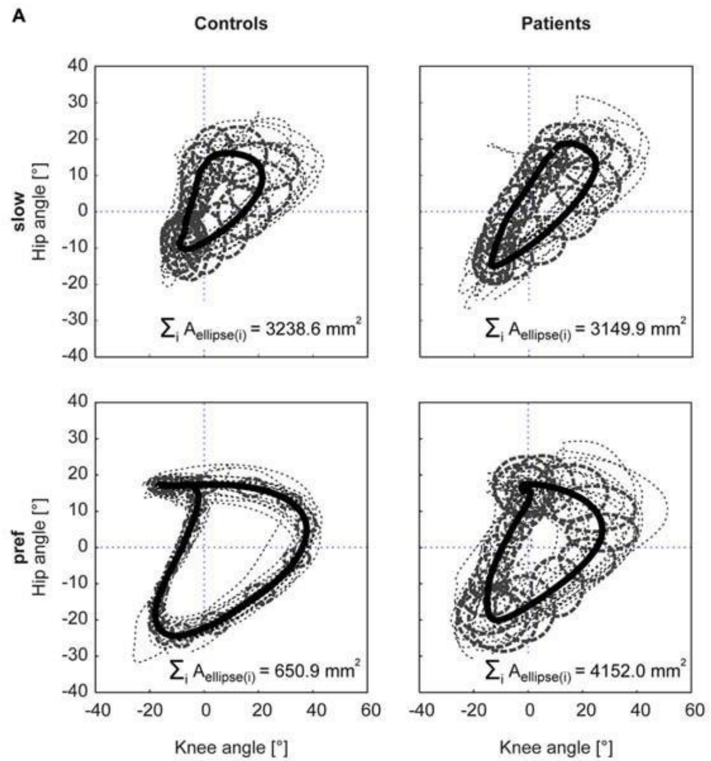
Stability measures

The human locomotion control system is, to all intents and purposes, a **dynamic, non-linear and complex system**. Numerous tools have been proposed to verify its stability starting from the biomechanical manifestation of gait (Hamacher et al., 2011).

In general, **stability is the ability of a system to cope with a perturbation** (Nayfeh & Balachandran, 1995), therefore an appropriate characterization of motor stability should be based on the quantification of a subject's ability to react to small perturbations (natural or artificial) that normally intervene during the execution of any motor task with a cyclic structure such as walking (Pecoraro et al., 2006).

Stability measures

In a repetitive task such as walking all the biomechanical variables have a cyclical and quasi-periodic behaviour which can be investigated with a non-linear stability analysis. For example, reporting the temporal evolution of the knee flexion angle with respect to the hip angle, an **orbit** will be obtained, which varies dynamically over time, but will almost always maintain the same trend. In dynamics, the set of variables describing this orbit (two or more) is called the **state space**.



Stability measures

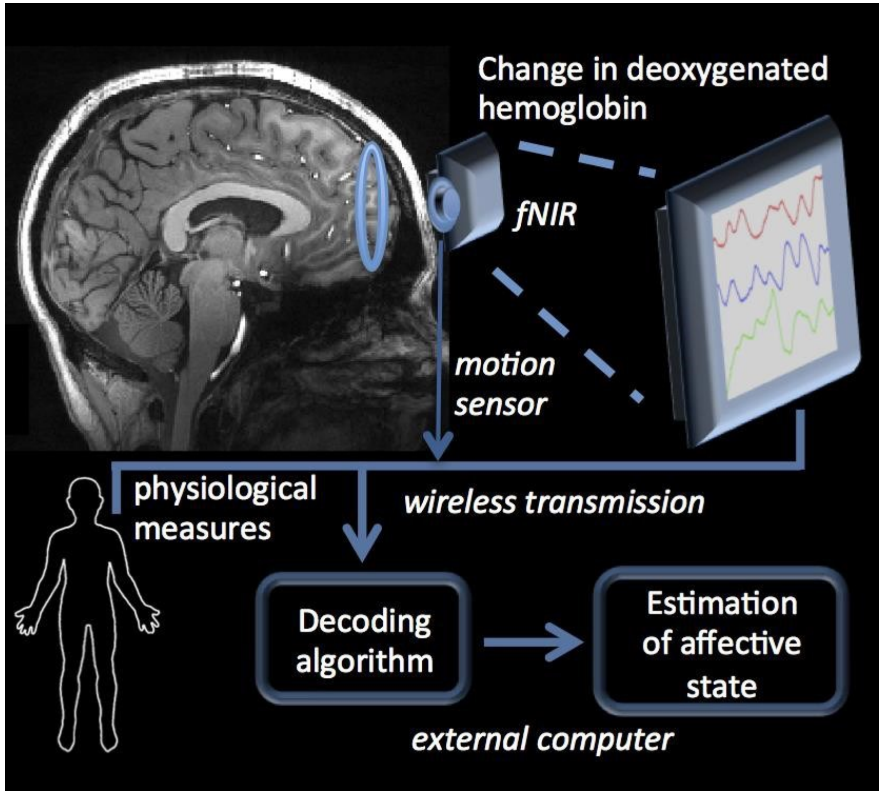
The locomotion pattern will force those variables to remain around a fixed orbit, in a sort of behaviour like a **limit cycle**. If a perturbation occurs during the motor task, the orbit will tend to deviate from the reference one: **in case of stability, the orbit will then tend to return to the reference one, in case of instability to diverge**.

Local stability analysis	→	Lyapounov Exponents
Orbital stability analysis	→	Floquet Multipliers

Multidimensional measurement systems

Cognitive domain measurement systems (e.g., fNIRS, EEG)

Motor domain measurement systems (e.g., MIMU)



Clinical applications

- Analysis of gait, balance, preparatory postural adjustments
- Instrumenting clinical rating scales and standardized tests (e.g., **i-**miniBesTest, **i-**TUG)
- Evaluation of the outcome of a treatment or therapy (quantification of tests of dexterity or proficiency)
- Evaluation of motor activity (ADL, detection of critical events such as freezing of gait and falls): quantity and quality of movements, motor behaviour
- Longitudinal assessment of a disease progression or a rehabilitation intervention
- Implementation of an assisted rehabilitation intervention

Feature selection on gait analysis parameters

Table 3. Item Loadings for the Five-Factor Rotated Solution and Communalities (Varimax Rotation)

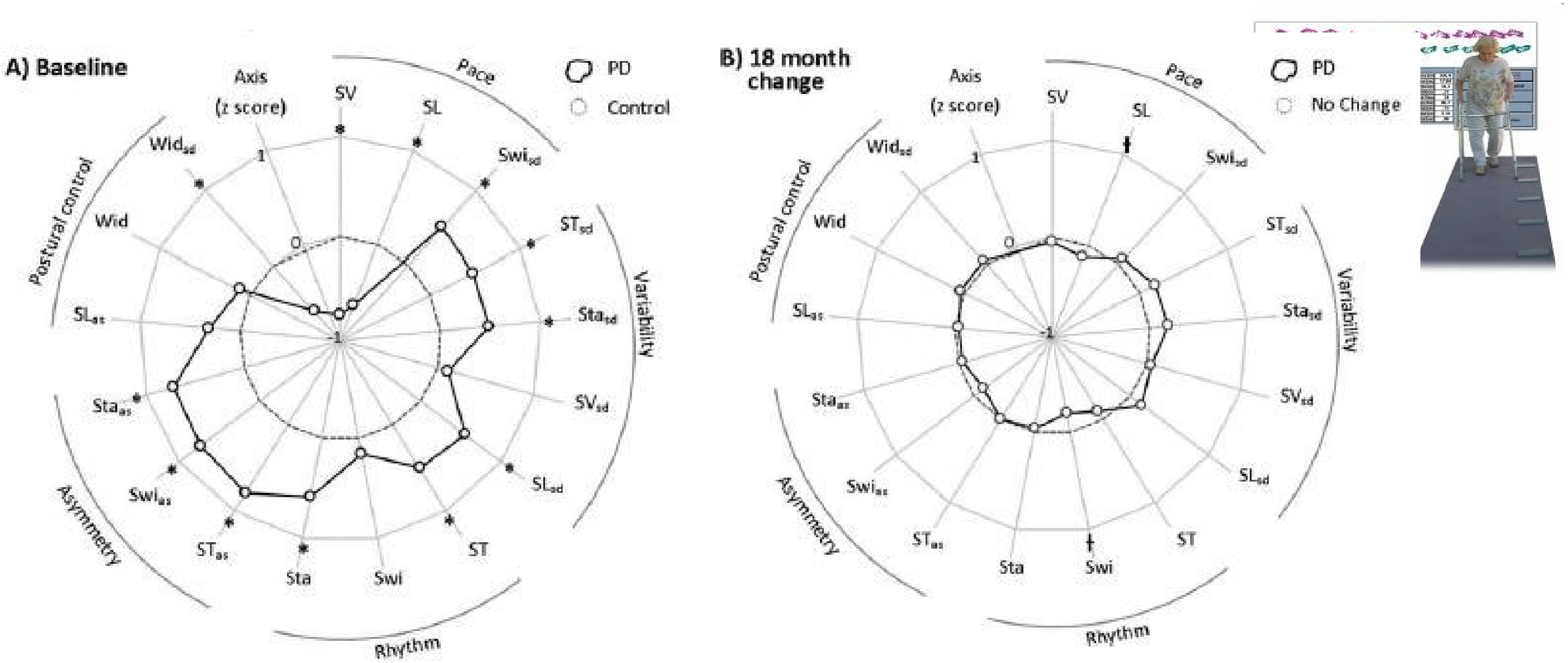
Item	Pace	Rhythm	Asymmetry	Variability	Postural Control
Pace					
Step velocity (m/s)	**−0.866**	−0.355	−0.178	0.019	−0.117
Step length (m)	**−0.915**	0.184	−0.120	0.015	−.043
Step time variability (ms)	**0.711**	0.347	0.150	0.483	0.093
Step swing time variability (ms)	**0.618**	0.390	0.244	0.386	0.135
Step stance time variability (ms)	**0.749**	0.302	0.098	0.461	0.059
Rhythm					
Step time (ms)	0.303	**0.906**	0.191	0.027	0.143
Step swing time (ms)	−0.078	**0.912**	0.114	−0.007	−0.004
Step stance time (ms)	0.426	**0.807**	0.202	0.037	0.189
Asymmetry					
Step time asymmetry (ms)	0.155	0.178	**0.670**	−0.044	0.247
Step swing asymmetry (ms)	0.105	0.114	**0.934**	0.062	−0.016
Step stance asymmetry (ms)	0.133	0.099	**0.916**	0.058	−0.036
Variability (*SD*)					
Step velocity variability (m/s)	0.148	−0.215	−0.054	**0.879**	−0.026
Step length variability (m)	0.181	0.118	0.117	**0.799**	0.257
Step width variability (m)	−0.426	0.368	−0.037	**0.476**	−0.013
Postural control					
Step width (m)	0.063	0.023	−0.001	0.275	**0.656**
Step length asymmetry (m)	0.081	0.129	0.105	−0.070	**0.788**
%Variance	**22.5**	**19.3**	**15.1**	**14.5**	**8.0**

Note: Relevant item loadings in bold.

Lord et al., 2012

16 gait features → Factor analysis → 5 Factors

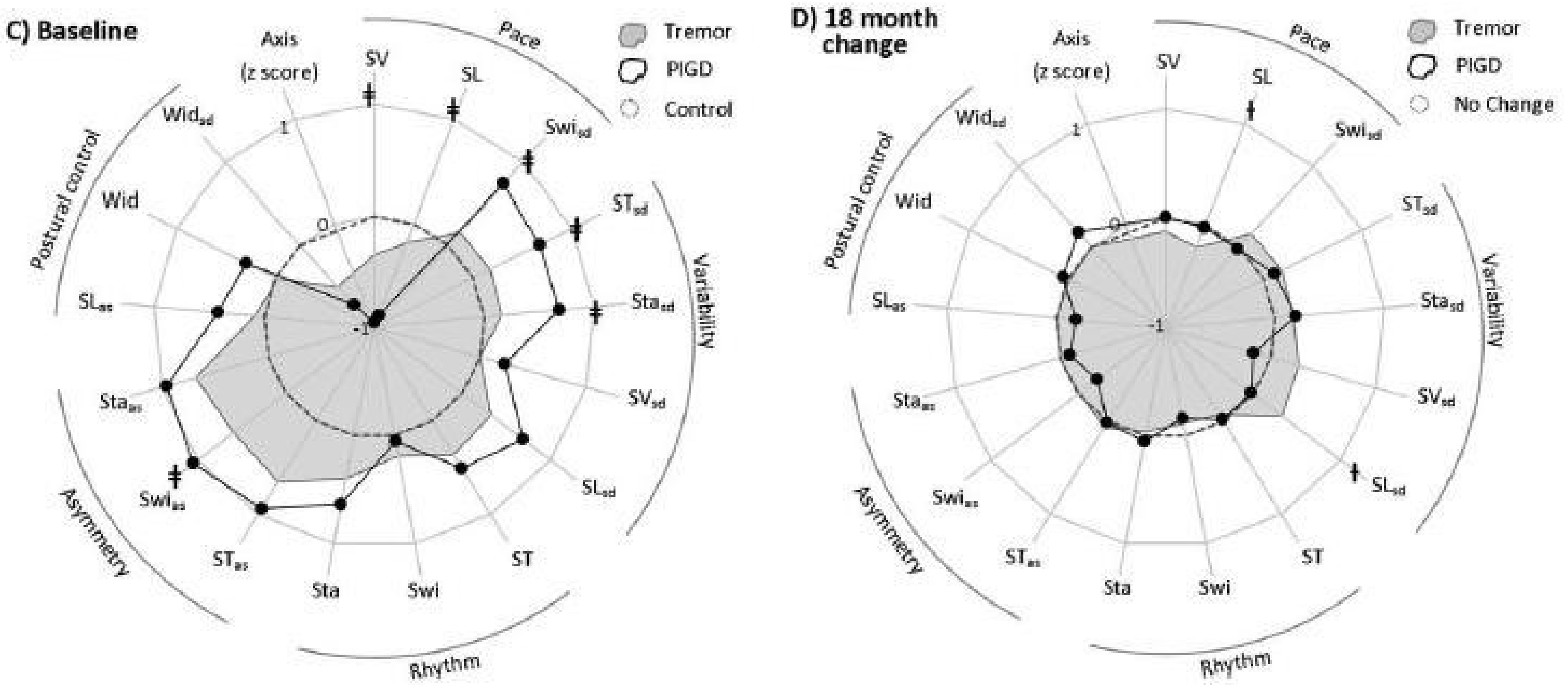
Analyzing progression of a disease (PD)



“Pace and rhythm deteriorated over 18 months in people with PD, with other gait domains remaining stable.”

Galna et al., Mov. Disord., 2015

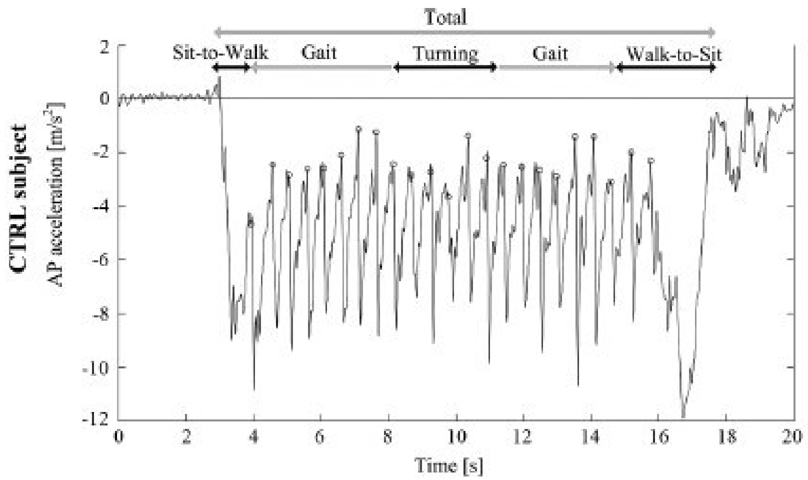
Analyzing progression of a disease (PD)

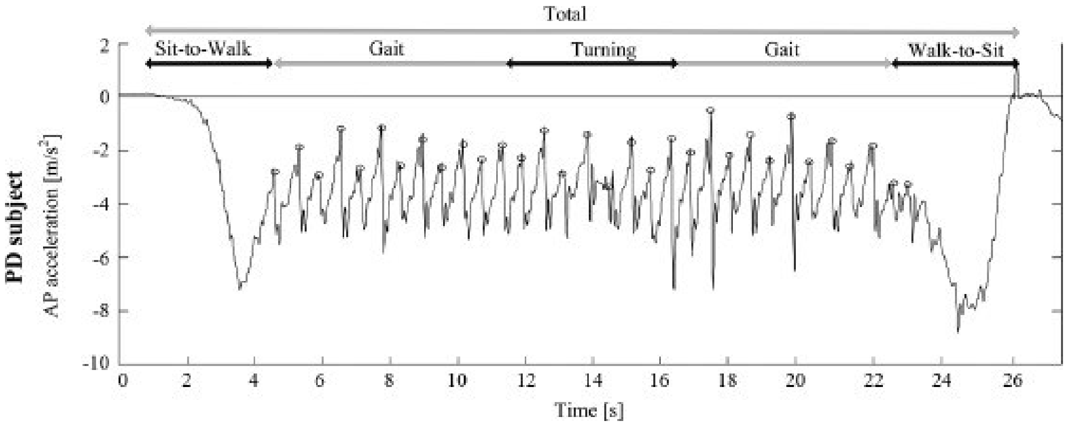


"People with the PIGD phenotype had more impaired gait at baseline compared with a TD phenotype, which was most evident in temporal characteristics. In contrast, pace and variability deteriorated over the subsequent 18 months in the TD only."

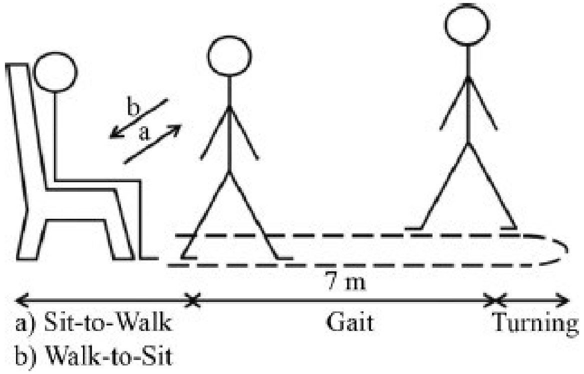
Galna et al., Mov. Disord., 2015

Instrumenting standardized tests: i-TUG





1 wearable IMU (McRoberts Dynaport Hybrid)



37 iTUG features → Feature selection → Classifier

Palmerini et al., IEEE Trans Neural Sys Rehab Eng, 2013

Instrumenting standardized tests: i-TUG

TABLE I
INSTRUMENTED TIMED UP AND GO MEASURES: COMPONENTS THEY WERE EXTRACTED FROM, DESCRIPTIONS, MEASUREMENT UNITS (M.U.), AND CONSIDERED DIRECTIONS

MEASURE	COMPONENTS	DESCRIPTION	M.U.	DIRECTIONS
Duration	Total, Sit-to-Walk, Gait, Turning, Walk-to-Sit	Duration of each iTUG component.	[s]	NA
RMS	Sit-to-Walk, Gait, Turning, Walk-to-Sit	Root mean square of the acceleration (a) during the considered component: $RMS = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(a(i) - mean(a))^2}$	[m/s^2]	AP, ML, V
NJS	Sit-to-Walk, Gait, Turning, Walk-to-Sit	Normalized jerk score of the acceleration: $NJS = \sqrt{\frac{T^5}{2}\int_{Tstart}^{Tend}(\dot{a})^2\,dt}$ where T is the duration ($Tend$-$Tstart$) of the considered component and a is the acceleration measured in m/s^2. The NJS during Gait is computed for each step (i.e., between two consecutive heel strikes), then normalized to the step duration, and then averaged across all steps: $Gait\ NJS = \frac{1}{N}\sum_{i=1}^{N}\left(\sqrt{\frac{(hs_{i+1} - hs_i)^5}{2}\int_{hs_i}^{hs_{i+1}}(\dot{a})^2\,dt}\right)$ where hs_i denotes the time of the i^{th} heel strike and a is the acceleration (m/s^2).	[m]	AP, ML, V
Tstep	Gait, Turning	Mean value of the step duration, computed as the time distance between two consecutive heel strikes.	[s]	NA
Tstep STD	Gait	Standard deviation (STD) of the step duration, computed as the time distance between two consecutive heel strikes.	[s]	NA
Tstep CV	Gait	Coefficient of variation (CV) of the step duration, computed as the time distance between two consecutive heel strikes. $Tstep\ CV = 100 \cdot \frac{Tstep\ STD}{Tstep}$	[%]	NA

Palmerini et al., IEEE Trans Neural Sys Rehab Eng, 2013

Scale cliniche strumentate: iTUG

Phase	Gait	Mean value of the phase denoted in degrees. The i^{th} phase (φ_i), measures the step time with respect to the stride time assigning 360° to each stride (gait cycle): $$\varphi_i = 360^\circ \frac{hs_{Si} - hs_{Li}}{hs_{L(i+1)} - hs_{Li}}$$ Phase is the average of φ_i, where hs_{Li} and hs_{Si} denote the time of the i^{th} heel strike of the legs with the long and short step times, respectively.	[deg]	NA
Phase STD	Gait	Standard deviation (STD) of the phase.	[deg]	NA
Phase CV	Gait	Coefficient of variation (CV) of the phase.	[%]	NA
PCI	Gait	Phase coordination index (PCI). PCI measures gait coordination (i.e., the accuracy and consistency of the phase generation). $$PCI = Phase\ CV + 100 \cdot \frac{\frac{1}{N}\sum_{i=1}^{N} \lvert \varphi_i - 180^\circ \rvert}{180^\circ}$$	[%]	NA
HR	Gait	Stride frequency is used as the fundamental frequency of the periodic acceleration signals during steady state walking; the fundamental period of such signals is a multiple of the stride duration. The coefficients of the first 10 even harmonics and the first 10 odd harmonics are computed by using a finite Fourier series; then the harmonic ratio (HR) is calculated by dividing the sum of the amplitudes of the in phase harmonics by the sum of the amplitudes of the out of phase harmonics: $$HR = \frac{\sum_{i=1}^{10} eh_i}{\sum_{i=1}^{10} oh_i}$$ for AP and V directions $$HR = \frac{\sum_{i=1}^{10} oh_i}{\sum_{i=1}^{10} eh_i}$$ for ML direction where eh_i and oh_i denote the coefficient of the i^{th} even and odd harmonic.	[-]	AP, ML, V

NA = NOT APPLICABLE; AP = ANTERO-POSTERIOR; ML = MEDIO-LATERAL; V = VERTICAL, [-] = UNITLESS

Palmerini et al., IEEE Trans Neural Sys Rehab Eng, 2013

Instrumenting standardized tests: i-TUG

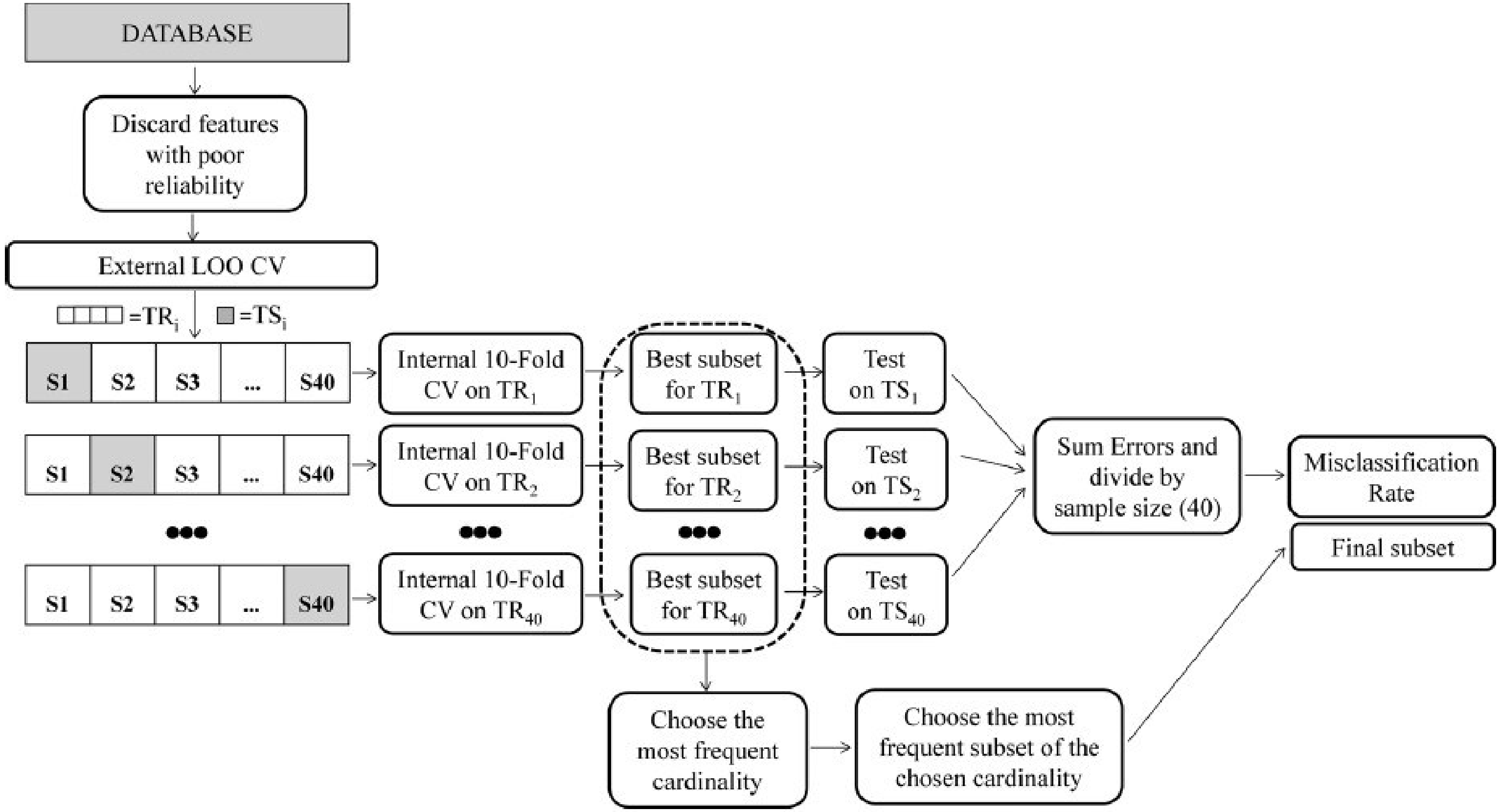


Fig. 3. Explanation of the feature selection procedure; $\mathrm{TR_i} = \mathrm{i^{th}}$ training set, $\mathrm{TS_i}$ = ith testing set.

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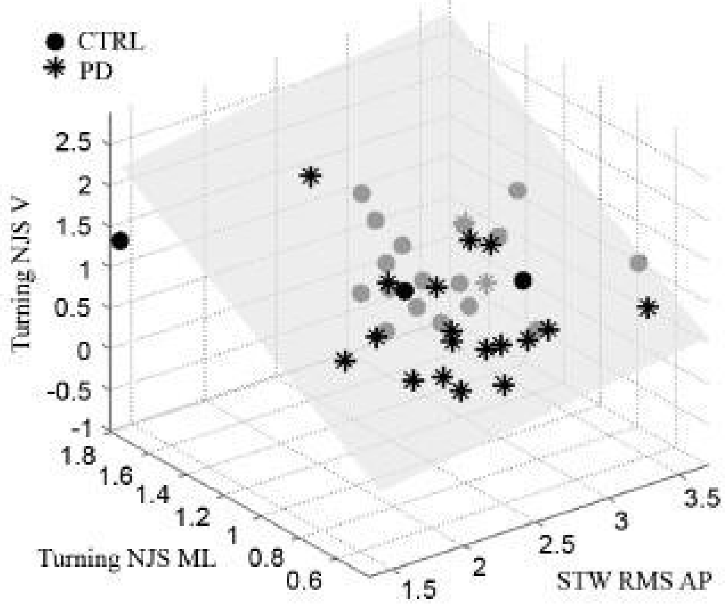
Instrumenting standardized tests: i-TUG

TABLE III

FOR EACH CLASSIFIER THE FINAL SELECTED SUBSET IS REPORTED WITH CORRESPONDING MISCLASSIFICATION RATE

CLASS	FINAL SUBSETS Measure			MISCLASSIFICATION RATE % [CI]
LDA	STW RMS AP	Turning NJS ML	Turning NJS V	22.5 [12.3-37.5]
QDA	Gait NJS AP	Gait HR V		27.5 [16.1-42.8]
MC	Gait NJS AP	Gait HR V	Turning RMS AP	37.5 [24.2-53]

CI = 95% CONFIDENCE INTERVAL; LDA = LINEAR DISCRIMINANT ANALYSIS; QDA = QUADRATIC DISCRIMINANT ANALYSIS; MC = MAHALANOBIS CLASSIFIER



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From gait analysis to the design of exoskeletons

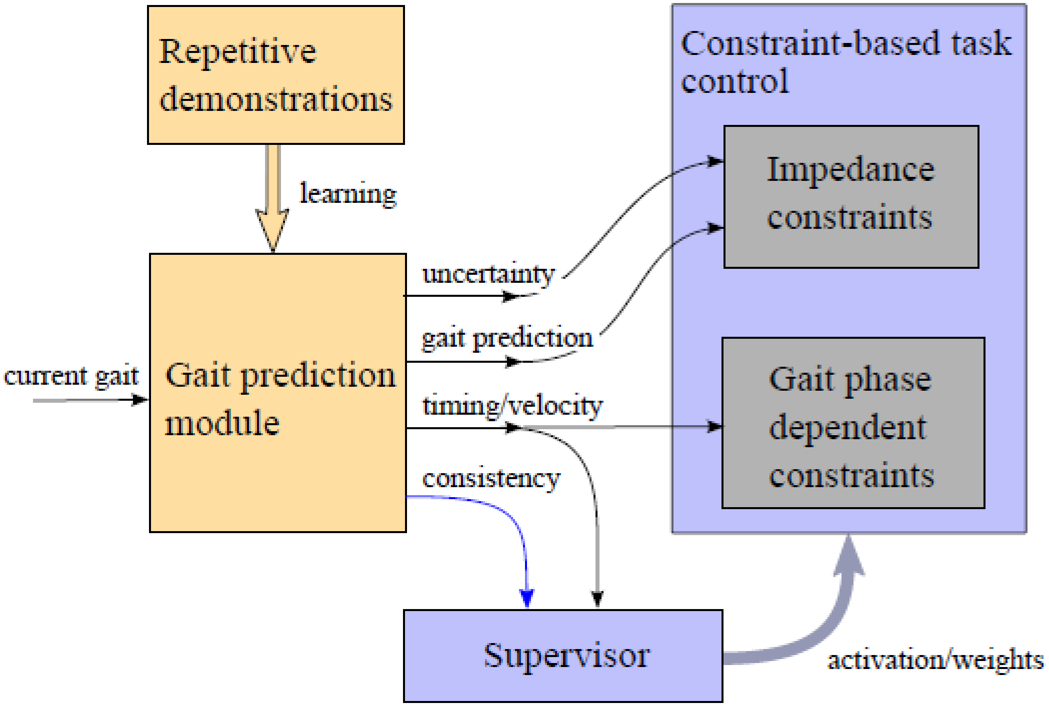

Fig. 1. The use of a gait prediction module for the control of a lower-limb exoskeleton. The lightly colored blocks are the main topic of this paper.

Aertbelien and De Schutter, BioRob, 2014

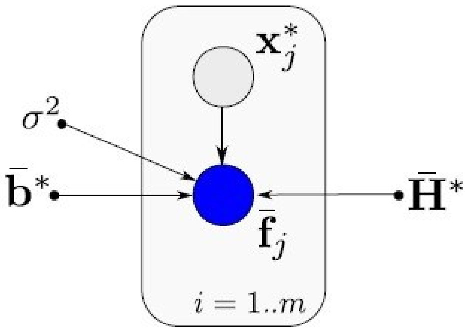

Fig. 3. Probabilistic graphical model (Bayesian network) with continuous variables. See [13] for the graphical notation.

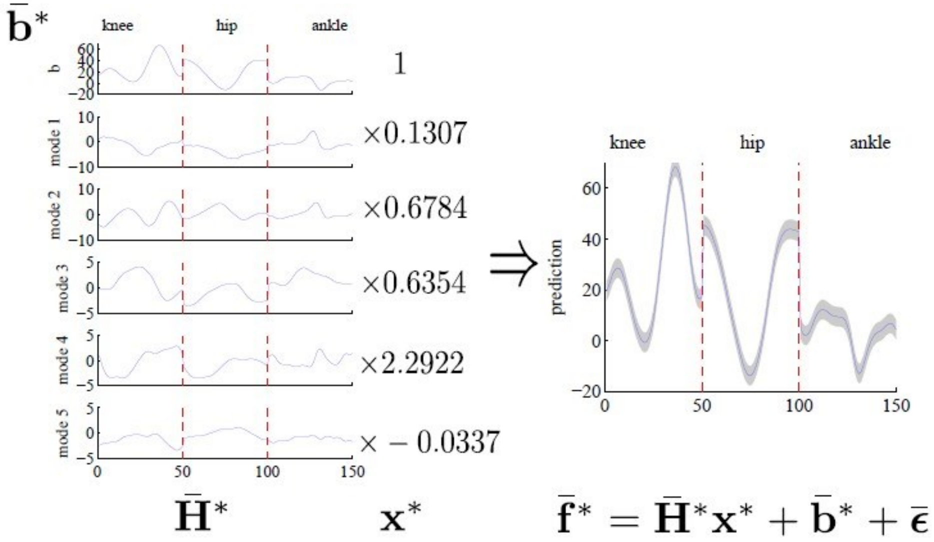

Fig. 4. The latent variable model describes a gait with a limited number of parameters.

Aertbelien and De Schutter, BioRob, 2014

3D Bipedal Robotic Walking: Models, Feedback Control, and Open Problems ⋆

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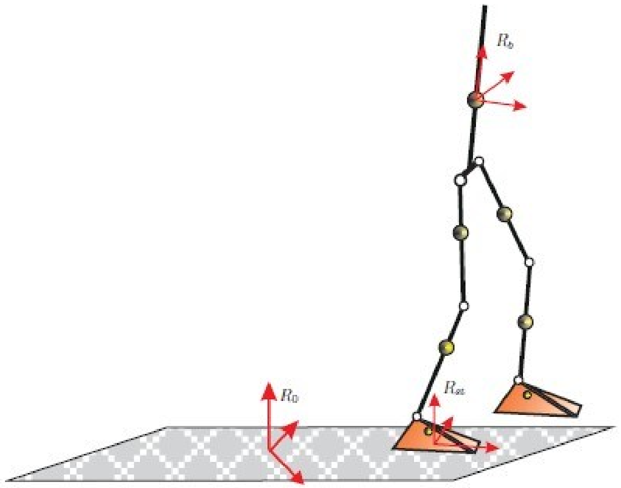
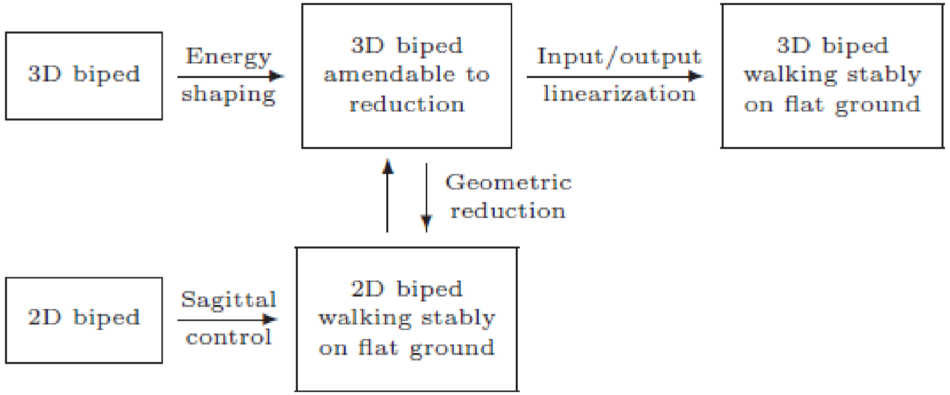


Fig. 2. A frame R_b is attached to the body. The position and orientation of the robot are expressed with respect to a fixed inertial frame R_0. A frame R_{st} attached to the stance foot is useful for expressing the contact conditions.

Fig. 10. Proposed scheme for obtaining walking



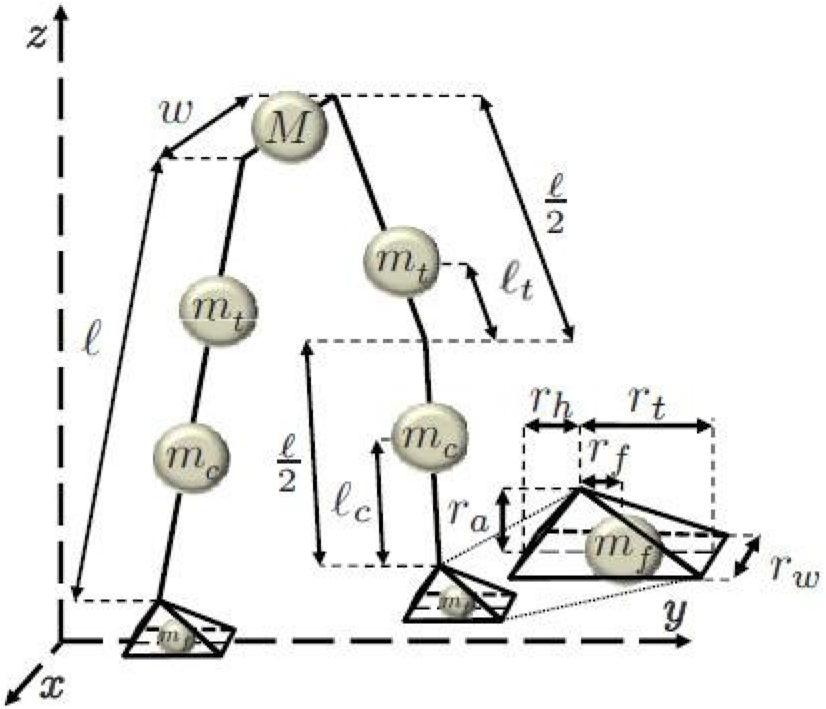


Fig. 12. Configuration of bipedal model