Lab 1

Solving simple computational problems in functional style

Objective: Getting acquainted with IDE and functional programming language, solving simple iterative tasks in functional style.

Task 1: Write a program in a functional language that will print the table of values of some elementary function given by the table below. It should print function values calculated using built-in functions and by calculating Taylor series. Values should be printed in 10 different point, evenly distributed in the given interval [a,b], which is selected to guarantee convergence of the Taylor series.

Taylor series should be calculated in two ways: using "dumb" scheme by calculating each series value according to its formula, and using more effective scheme, when next item in series is calculated based on the previous one. The number of items to add should be determined dynamically according to the value of the next item (stop adding when the value is sufficiently small).

As a result, you should produce the following table:

x	Taylor series, dumb	# iterations	Taylor series, smart	# iterations	Value of a function
0.00	…		0.0	…	
0.05			0.0008 …		
…	…		…	…	
0.50	…		…	…	

(Note: to print a table, you can use printf F# function, which is similar to the function with the same name from the C library)

Please take function and Taylor series from Table 1 according to your own number in the class list.

Task 2: Develop a function to solve transcendental algebraic equations numerically, using three different methods: iterations, dichotomy and Newthon's. All equations should be specified as functional parameters, and methods to some equations should be sufficiently generic.

Please apply three procedures for three methods to three consecutive equations in the table 2, starting from the row specified by your personal number from the class list. You should produce a table with 9 solutions.

About the authors

This lab is based on the course project and labs for the Computer Science and Programming course taught for students of Applied Mathematics faculty in Moscow Aviation Technical University. Tasks were intitially developed by Prof. V.E. Zaytsev together with co-authors of the course, and adopted to Functional Programming course by Dmitry Sosnikov. All Taylor series and equations were developed by Assoc.Prof. I.P.Soprunenko.

Table 1

Functions and Corresponding Taylor Series

#	Series	a	b	Function
1	$\frac{x}{9} - \frac{x^3}{9^2} + \ldots + (-1)^n \frac{x^{2n+1}}{9^{n+1}}$	-1.0	1.0	$\frac{x}{9 + x^2}$
2	$2(\frac{x}{1} + \frac{x^3}{3} + \ldots + \frac{x^{2n+1}}{2n+1})$	0.0	0.5	$\ln \frac{1+x}{1-x}$
3	$x - \frac{5}{2}x^2 + \ldots + \frac{(-1)^{n+1} \cdot 2^n - 1}{n} x^n$	-0.2	0.3	$\ln(1 + x - 2x^2)$
4	$\ln 2 + \frac{x}{2} - \frac{x^2}{2^3} + \ldots + (-1)^{n-1} \frac{x^n}{n \cdot 2^n}$	-1.0	1.0	$\ln(2 + x)$
5	$-\frac{4x^2}{2} + \frac{16x^4}{24} + \ldots + (-1)^n \frac{(2x)^{2n}}{(2n)!}$	0.0	0.5	$2(\cos^2 x - 1)$
6	$x + \frac{x^3}{3!} + \ldots + \frac{x^{2n-1}}{(2n-1)!}$	0.0	1.0	$\text{sh}\, x$
7	$3x + 8x^2 + \ldots + n \cdot (n+2)x^n$	0.0	0.5	$\frac{x(3-x)}{(1-x)^3}$
8	$-\frac{1}{5} - \frac{2x}{5^2} - \frac{4x^2}{5^3} - \ldots - \frac{2^{n-1}x^{n-1}}{5^n}$	0.0	2.0	$\frac{1}{2x-5}$
9	$-(1+\frac{2}{3}) - (1+\frac{2}{3^2})x - \ldots - (1+\frac{2}{3^{n+1}})x^n$	0.0	0.5	$\frac{3x-5}{x^2 - 4x + 3}$
10	$\frac{2x^2}{2!} - \frac{2^3 x^4}{4!} + \ldots + (-1)^{n-1} \frac{2^{2n-1} x^{2n}}{(2n)!}$	0.0	1.0	$\sin^2 x$
11	$1 - \frac{3}{2}x^2 + \ldots + (-1)^n \frac{2n^2+1}{(2n)!} x^{2n}$	0.1	0.6	$(1 - \frac{x^2}{2}) \cos x - \frac{x}{2} \sin x$
12	$1 + \frac{\ln 3}{1!}x + \frac{\ln^2 3}{2!}x^2 + \ldots + \frac{\ln^n 3}{n!}x^n$	0.0	1.0	3^x
13	$x - \frac{x^3}{3!} + \ldots + (-1)^n \frac{x^{2n+1}}{(2n+1)!}$	0.0	1.0	$\sin x$
14	$-3 - 4x - 5x^2 - \ldots - (n+3)x^n$	0.1	0.6	$\frac{2x-3}{(x-1)^2}$

15	$1-\frac{x^2}{2!}+\ldots+(-1)^n\frac{x^{2n}}{(2n)!}$	0.0	1.0	$\cos x$
16	$1+3x^2+\ldots+\frac{2n+1}{n!}x^{2n}$	0.0	1.0	$(1+2x^2)e^{x^2}$
17	$\frac{x-1}{x+1}+\frac{1}{3}(\frac{x-1}{x+1})^3+\ldots+\frac{1}{2n+1}(\frac{x-1}{x+1})^{2n+1}$	0.2	0.7	$\frac{1}{2}\ln x$
18	$\frac{x^3}{3}-\frac{x^5}{15}+\ldots+(-1)^{n+1}\frac{x^{2n+1}}{4n^2-1}$	0.1	0.6	$\frac{1+x^2}{2}\operatorname{arctg} x-\frac{x}{2}$
19	$1+\frac{x^2}{2}+\ldots+\frac{x^{2n}}{(2n)!}$	0.1	0.6	$\operatorname{ch} x$
20	$1+\frac{2x}{1!}+\ldots+\frac{(2x)^n}{n!}$	0.1	0.6	e^{2x}
21	$1+2\frac{x}{2}+\ldots+\frac{n^2+1}{n!}(\frac{x}{2})^n$	0.1	0.6	$(\frac{x^2}{4}+\frac{x}{2}+1)e^{\frac{x}{2}}$
22	$1-\frac{x^2}{2}+\frac{x^3}{3}+\ldots+(-1)^{n-1}\frac{n-1}{n!}x^n$	0.0	1.0	$(1+x)e^{-x}$
23	$x-\frac{x^3}{3}+\ldots+(-1)^n\frac{x^{2n+1}}{2n+1}$	0.0	0.5	$\operatorname{arctg} x$
24	$1+\frac{x^2}{1}+\frac{x^4}{2}+\ldots+\frac{x^{2n}}{n!}$	0.0	1.0	e^{x^2}
25	$\frac{1}{4}+\frac{x^4}{4^2}+\ldots+\frac{x^{4n}}{4^{n+1}}$	0.0	1.0	$\frac{1}{4-x^4}$
26*	$-\cos x+\frac{\cos 2x}{2^2}+\ldots+(-1)^n\frac{\cos nx}{n^2}$	$\frac{\pi}{5}$	π	$\frac{1}{4}(x^2-\frac{\pi^2}{3})$
27*	$1+\frac{\cos x}{1!}+\ldots+\frac{\cos nx}{n!}$	0.1	0.6	$e^{\cos x}\cdot\cos(\sin x)$
28*	$\cos x+\frac{\cos 2x}{2}+\ldots+\frac{\cos nx}{n}$	$\frac{\pi}{5}$	$\frac{6\pi}{5}$	$-\ln\lvert 2\sin\frac{x}{2}\rvert$

Table 2

Transcendental Equations and Recommended Method of Solution

#	Equation	Interval with Root	Recommended Method	Approx value of root
1	$e^x + \ln x - 10x = 0$	[3, 4]	Newthon	3.5265
2	$\cos x - e^{-\frac{x^2}{2}} + x - 1 = 0$	[1, 2]	Dichotomy	1.0804
3	$1 - x + \sin x - \ln(1+x) = 0$	[1, 1.5]	iterations	1.1474
4	$3x - 14 + e^x - e^{-x} = 0$	[1, 3]	Newthon	2.0692
5	$\sqrt{1-x} - \mathrm{tg}x = 0$	[0, 1]	Dichotomy	0.5768
6	$x + \cos(x^{0,52} + 2) = 0$	[0.5, 1]	iterations	0.9892
7	$3\ln^2 x + 6\ln x - 5 = 0$	[1, 3]	Newthon	1.8832
8	$0{,}6 \cdot 3^x - 2{,}3x - 3 = 0$	[2, 3]	Dichotomy	2.4200
9	$x^2 - \ln(1+x) - 3 = 0$	[2, 3]	iterations	2.0267
10	$2x \cdot \sin x - \cos x = 0$	[0.4, 1]	Newthon	0.6533
11	$e^x + \sqrt{1+e^{2x}} - 2 = 0$	[-1, 0]	Dichotomy	-0.2877
12	$\ln x - x + 1{,}8 = 0$	[2, 3]	iterations	2.8459
13	$x \cdot \mathrm{tg}\, x - \frac{1}{3} = 0$	[0.2, 1]	Newthon	0.5472
14	$\mathrm{tg}\frac{x}{2} - \mathrm{ctg}\frac{x}{2} + x = 0$	[1, 2]	Dichotomy	1.0769
15	$0{,}4 + \mathrm{arctg}\sqrt{x} - x = 0$	[1, 2]	iterations	1.2388
16	$3\sin\sqrt{x} + 0.35x - 3.8 = 0$	[2, 3]	iterations	2.2985
17	$0{,}25x^3 + x - 1{,}2502 = 0$	[0, 2]	Newthon	1.0001
18	$x + \sqrt{x} + \sqrt[3]{x} - 2{,}5 = 0$	[0.4, 1]	Dichotomy	0.7376
19	$x - \frac{1}{3 + \sin 3.6x} = 0$	[0, 0.85]	iterations	0.2624
20	$0{,}1x^2 - x\ln x = 0$	[1, 2]	Newthon	1.1183
21	$\mathrm{tg}\, x - \frac{1}{3}\mathrm{tg}^3 x + \frac{1}{5}\mathrm{tg}^5 x - \frac{1}{3} = 0$	[0, 0.8]	Dichotomy	0.3333
22	$\arccos x - \sqrt{1 - 0{,}3x^3} = 0$	[0, 1]	iteration	0.5629
23	$3x - 4\ln x - 5 = 0$	[2, 4]	Newthon	3.23
24	$\cos\frac{2}{x} - 2\sin\frac{1}{x} + \frac{1}{x} = 0$	[1, 2]	Dichotomy	1.8756
25	$\sqrt{1-0{,}4x^2} - \arcsin x = 0$	[0, 1]	iteration	0.7672
26	$e^x - e^{-x} - 2 = 0$	[0, 1]	Newthon	0.8814
27	$\sin(\ln x) - \cos(\ln x) + 2\ln x = 0$	[1, 3]	Dichotomy	1.3749
28	$x - 2 + \sin\frac{1}{x} = 0$	[1.2, 2]	iteration	1.3077