

Graph-theoretic Models, Lecture 3, Segment 1

John Guttag

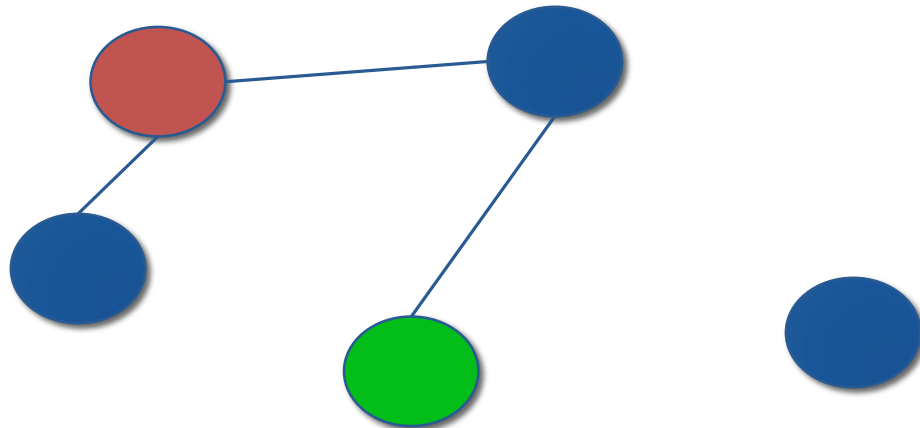
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Computational Models

- Programs that help us understand the world and solve practical problems
- Saw how we could map the informal problem of choosing what to eat into an optimization problem, and how we could design a program to solve it
- Now want to look at class of models called graphs

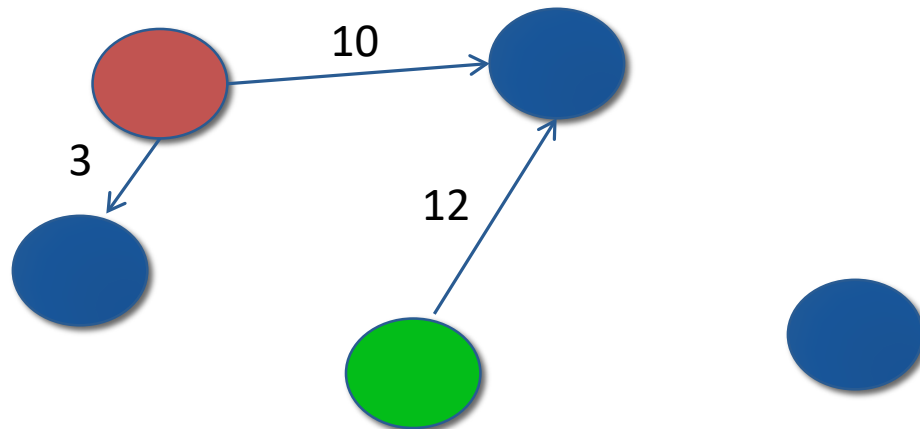
What's a Graph?

- Set of nodes (vertices)
 - Might have properties associated with them
- Set of edges (arcs) each consisting of a pair of nodes
 - Undirected (graph)
 - Directed (digraph)
 - Source (parent) and destination (child) nodes
 - Unweighted or weighted



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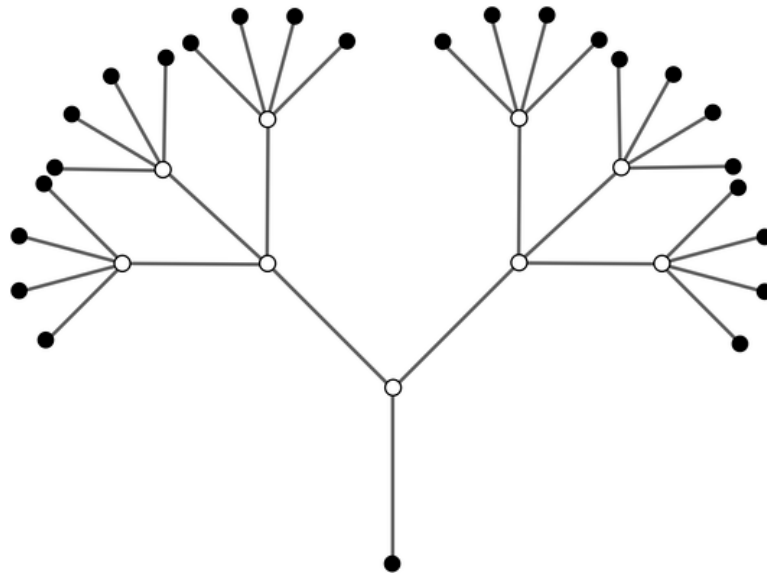


Why Graphs?

- To capture useful relationships among entities
 - Rail links between Paris and London
 - How the atoms in a molecule related to one another
 - Ancestral relationships

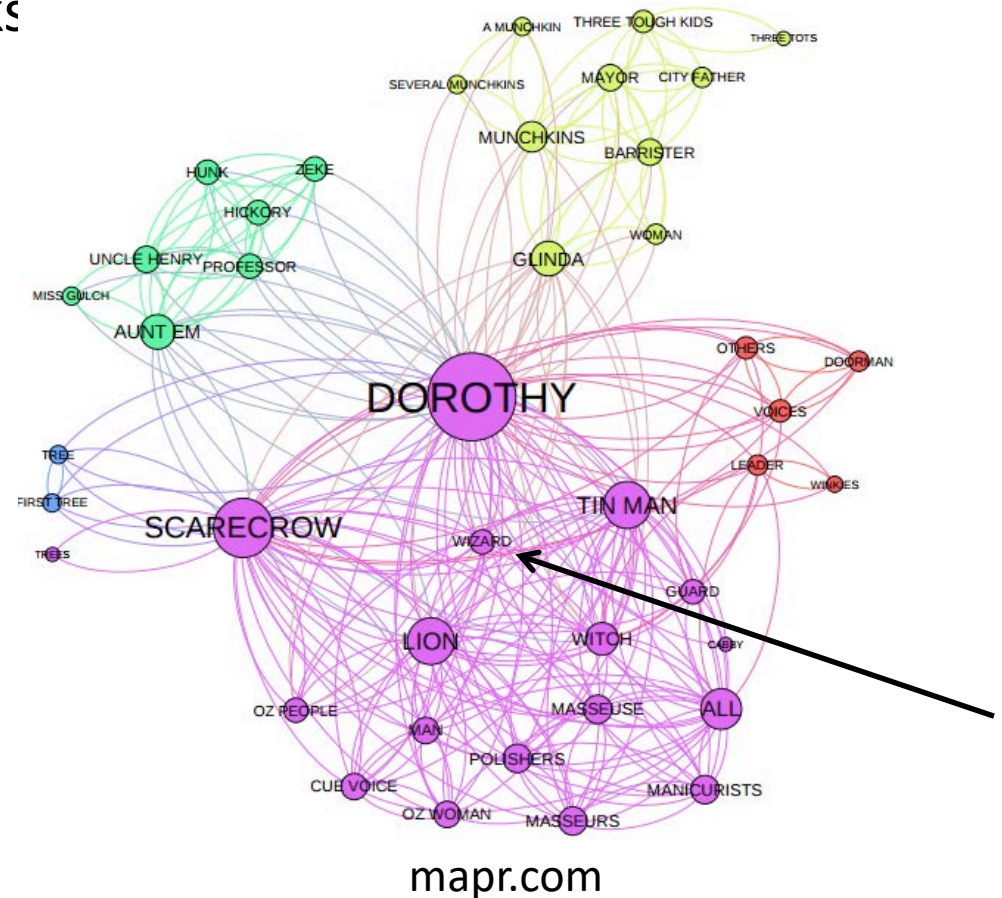
Trees: An Important Special Case

- A directed graph in which each pair of nodes is connected by a single path
 - Recall the search trees we used to solve knapsack problem

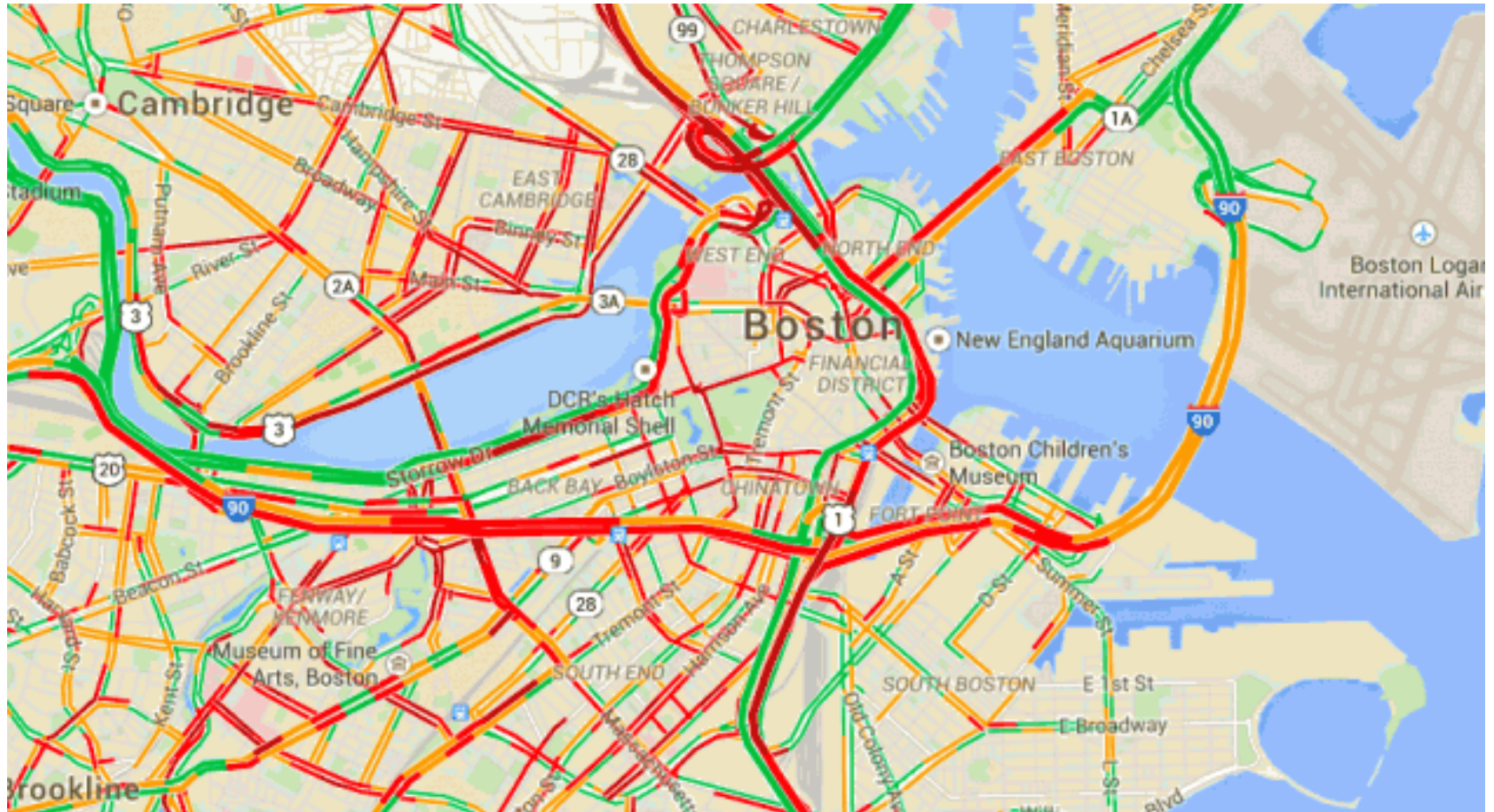


Why Graphs Are So Useful

- World is full of networks based on relationships
 - Computer networks
 - Transportation networks
 - Financial networks
 - Sewer networks
 - Political networks
 - Criminal networks
 - Social networks
 - Etc.



Graph Theory Saves Me Time Every Day



www.google.com

Getting John to the Office

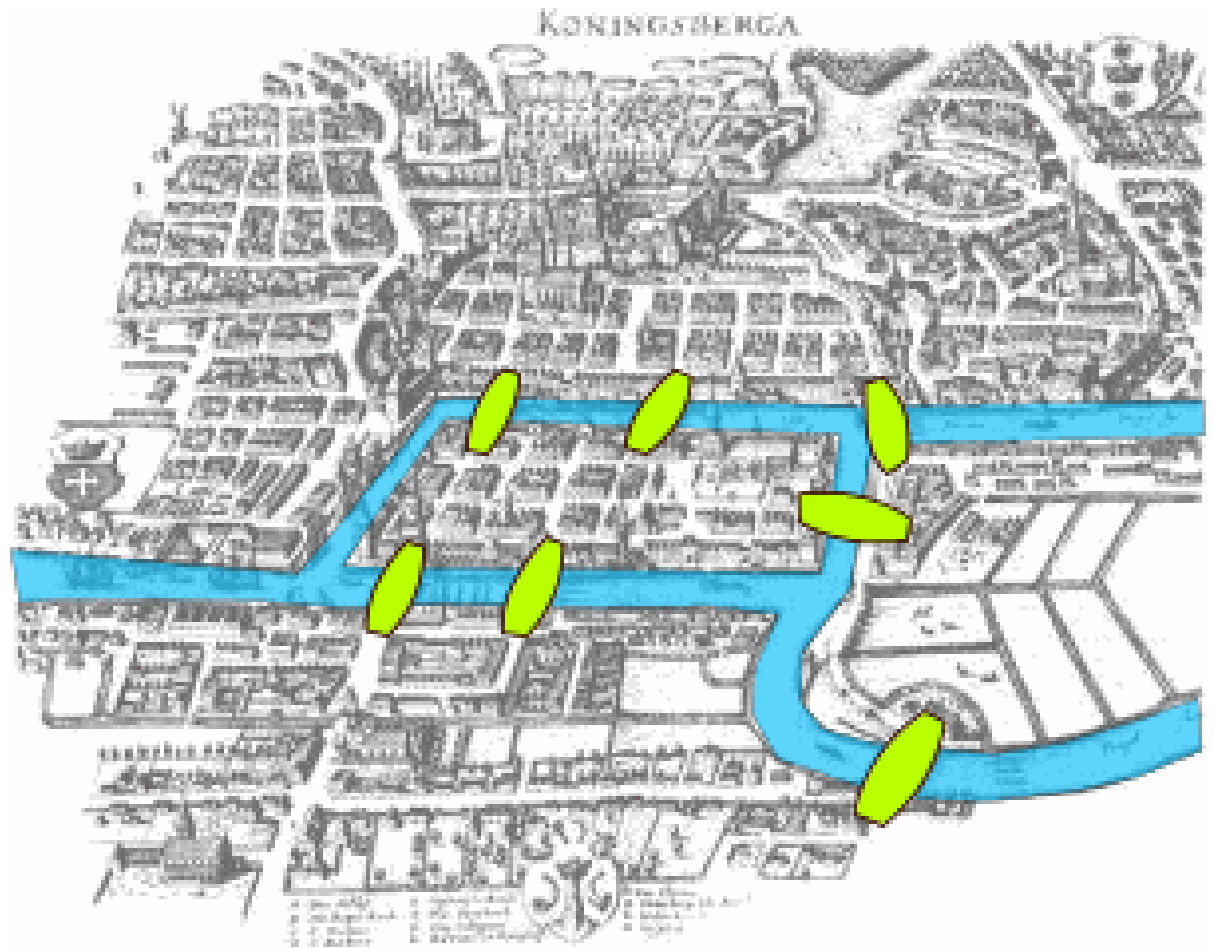
- Model road system using a digraph
 - Nodes: points where roads end or meet
 - Edges: connections between points
 - Each edge has a weight indicating time it will take to get from source node to destination node for that edge
- Solve a graph optimization problem
 - Shortest weighted path between my house and my office



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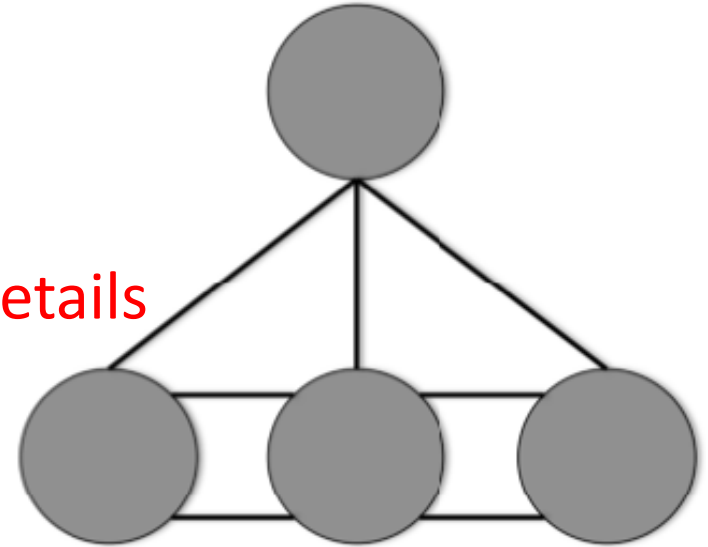
First Reported Use of Graph Theory

- Bridges of Königsberg (1735)
- Possible to take a walk that traverses each of the 7 bridges exactly once?



Leonhard Euler's Model

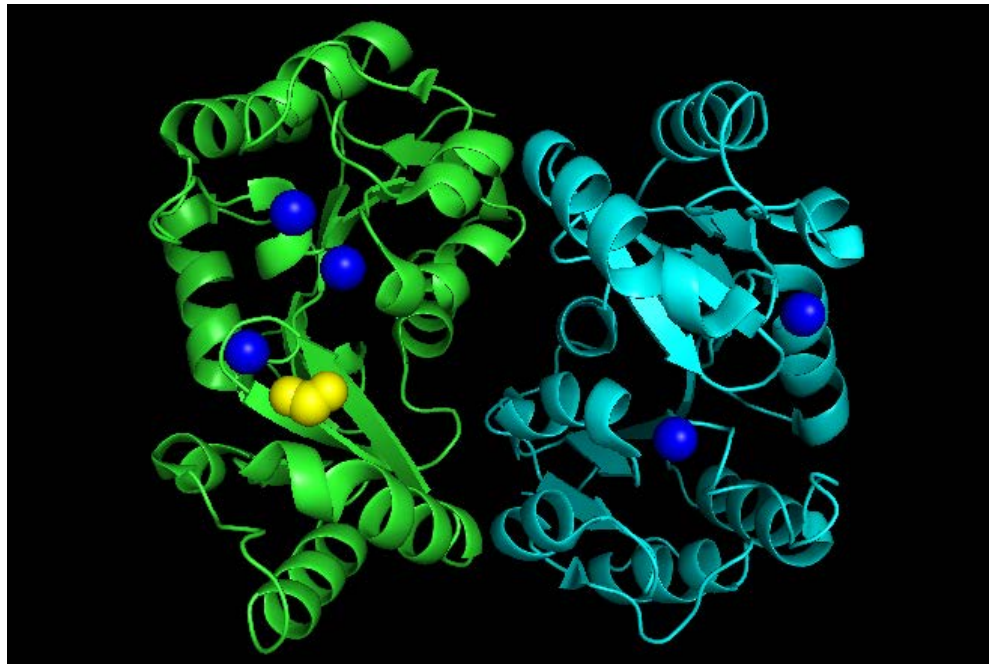
- Each island a node
- Each bridge an undirected edge
- **Model abstracts away irrelevant details**
 - Size of islands
 - Length of bridges



- Is there a path that contains each edge exactly once?

Next Segment

- Implementing graphs
- Some classic graph optimization problems



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