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State space analysis and control transfer function of a flyback converter

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Abstract

Flyback converters are used in automotive power supply applications for their advantage of galvanic isolation and the low number of components on a small PCB area. This paper is dedicated to state space analysis of flyback converter with parasitic elements. The Euler method also uses state-space matrices of the proposed flyback converter with nonideal flyback converter with a Bode plot of both open-loop and closed-loop systems. The advantage of this approach is to determine the stability of the open-loop system from a mathematical model and to estimate suitable PI tuning values for the closed-loop system. Transfer function and Bode plots have been obtained from state space matrices in MATLAB.

Keywords: transfer function; flyback converter; bode plot

1. Introduction

Nowadays, converters are used everywhere around us, and the flyback DC/DC converters are widely used. Since we are continuing to use more of these types of converters with the focus on minimization, the main advantage of these converters is their size mainly due to reduction of the transformer size. However, new components with better properties are being added, as well as new control methods are being used in order to improve the characteristics of these converters. To achieve the best results there are many options, starting with choosing the proper control method. It also depends on how we want the system to behave. Whether we want the fastest response possible but

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with a condition of higher overshoot or the opposite to have no overshoot with a slower response. This determines the stability of the controlled system, which can be detected by various methods, such as from Bode plot, Nyquist characteristic or pole-zero diagram.

2. State space analysis

To obtain the transfer function and estimate the stability of the system, it is necessary to know the dynamic behaviour of the converter.

A problem in power converter modelling is the nonlinear behaviour of the converter during switching. This problem can be overcome by using the state space averaging method.

The operation can be divided into two intervals.

The circuit to determinate the state space analysis for a flyback converter with parasitic components, as shown in Fig. 1, consists of a DC source $u_Z(t)$, a switch S, which in this case is a mosfet with internal resistance R_S, the transformer T and its magnetizing inductance L_M, the diode D and its voltage drop U_D, its forward resistance R_D, a capacitor C and its internal serial resistance R_C and load resistance R.

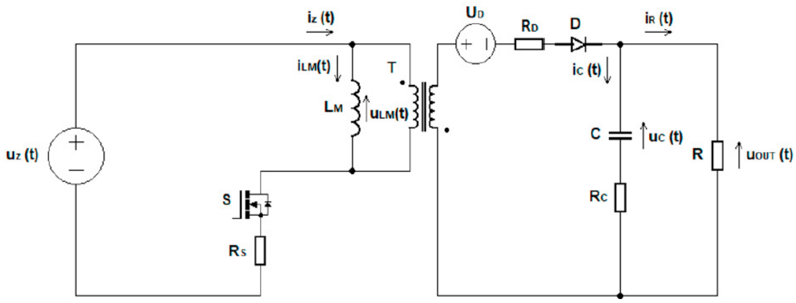

Fig. 1. Schematic of flyback converter with parasitic components

To use the state space analysis, we need to divide the circuit into the states in which the converter operates. In the case of a flyback converter, we can divide operation into three states. These states we can divide into two modes. The continuous conduction mode (CCM) and discontinuous conduction mode (DCM), while CCM mode has two states and DCM additional third state.

The first step is to determine the differential equations for the CCM and then obtain the state space equations describing the system.

We can start by solving the state space analysis for the circuit when the switch is open and then when the switch is closed or vice versa.

After applying this approach we obtain the following form for CCM:

$$\begin{bmatrix} \dfrac{di_{LM}}{dt} \\ \dfrac{du_C(t)}{dt} \end{bmatrix} = \begin{bmatrix} -\dfrac{R_S \cdot D_1}{L_M} + \dfrac{R_D \cdot nps \cdot D_2}{L_M} + \dfrac{R_C \cdot R \cdot nps^2 \cdot D_2}{R \cdot L_M - R_C \cdot L_M} & \dfrac{R \cdot nps \cdot D_2}{R \cdot L_M - R_C \cdot L_M} \\ -\dfrac{R \cdot nps \cdot D_2}{R \cdot C - R_C \cdot C} & -\dfrac{D_1}{R \cdot C + R_C \cdot C} - \dfrac{D_2}{R \cdot C - R_C \cdot C} \end{bmatrix} = \begin{bmatrix} i_{LM}(t) \\ u_C(t) \end{bmatrix} + \begin{bmatrix} -\dfrac{D_1}{L_M} & -\dfrac{nps \cdot D_2}{L_M} \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} u_Z(t) \\ U_D \end{bmatrix} \tag{1}$$

$$\begin{bmatrix} i_Z(t) \\ u_{OUT}(t) \end{bmatrix} = \begin{bmatrix} D_1 & 0 \\ \dfrac{nps \cdot R \cdot R_C \cdot D_2}{R - R_C} & \dfrac{R \cdot D_1}{R + R_C} + \dfrac{R \cdot D_2}{R - R_C} \end{bmatrix} \begin{bmatrix} i_{LM}(t) \\ u_C(t) \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} u_Z(t) \\ U_D \end{bmatrix} \tag{2}$$

To solve the state space analysis for the DCM, we need to solve the third state, which occurs during the second state when the current reaches zero due to low load. During this interval the switch and diode are not conductive, which means that the third state needs to be solved only for load and output capacitor.

The state space matrices for DCM mode look like this:

$$\begin{bmatrix} \frac{di_{LM}}{dt} \\ \frac{du_C(t)}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{R_S \cdot D_1}{L_M} + \frac{R_D \cdot nps \cdot D_2}{L_M} + \frac{R_C \cdot R \cdot nps^2 \cdot D_2}{R \cdot L_M - R_C \cdot L_M} & \frac{R \cdot nps \cdot D_2}{R \cdot L_M - R_C \cdot L_M} \\ -\frac{R \cdot nps \cdot D_2}{R \cdot C - R_C \cdot C} & -\frac{D_1 + D_3}{R \cdot C + R_C \cdot C} - \frac{D_2}{R \cdot C - R_C \cdot C} \end{bmatrix} = \begin{bmatrix} i_{LM}(t) \\ u_C(t) \end{bmatrix} + \begin{bmatrix} -\frac{D_1}{L_M} & -\frac{nps \cdot D_2}{L_M} \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} u_Z(t) \\ U_D \end{bmatrix} \tag{3}$$

$$\begin{bmatrix} i_Z(t) \\ u_{OUT}(t) \end{bmatrix} = \begin{bmatrix} D_1 & 0 \\ \frac{nps \cdot R \cdot R_C \cdot D_2}{R - R_C} & \frac{R \cdot \left(D_1 + D_3\right)}{R + R_C} + \frac{R \cdot D_2}{R - R_C} \end{bmatrix} \cdot \begin{bmatrix} i_{LM}(t) \\ u_C(t) \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u_Z(t) \\ U_D \end{bmatrix} \tag{4}$$

By averaging method and perturbation, we can also obtain a small-signal model.

3. Euler method

The Euler method is a first-order numerical procedure for solving differential equations with initial value. There are various Euler methods, but we can use the indirect Euler method, whose continuous equation with state matrices can be used to solve state space matrices.

We can use this method to verify the correctness of the state space matrices. As a suitable software for solving state space matrices, we can use MATLAB.

First we need to assign values for all variables in matrices, i.e. for resistance of all parasitic components, voltage drop of the diode, switching time, output filter capacitance, load, duty cycle, transformer turns ratio, input voltage, which is the DC output voltage after rectification of 230 VAC and the output voltage is chosen to be 12 V.

Table 1. Paramerers of flyback converter.

Parameters	Symbol	Value
Input voltage (VDC)	$u_Z(t)$	325 V
Output voltage	$u_{OUT}(t)$	12 V
Switching frequency	fsw	100 kHz
Turns ratio of transformer	nps	27
Transformer magnetizing inductance	L_M	0.210 H
Capacitor	C	200 μF
ESR of the Capacitor	R_C	0.090 Ω
Load resistance	R	5 Ω
Mosfet resistance	R_S	0.070 Ω
Diode resistance	R_D	0.200 Ω
Diode voltage drop	U_D	0.65 V
Duty cycle	D	0.5

Output voltage is then as follows:

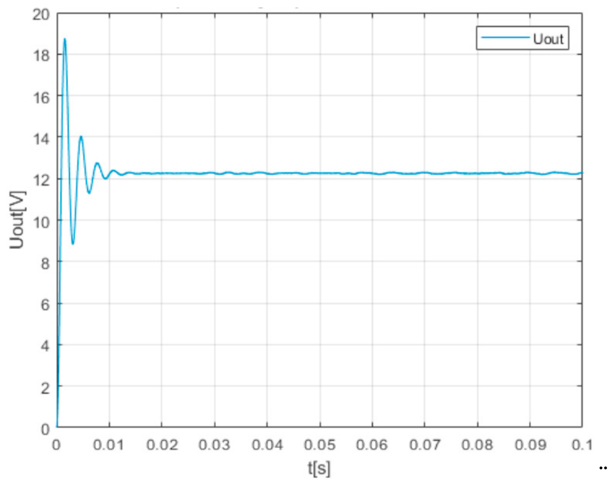

Fig. 2. Output voltage of flyback converter state space matrices by Euler method in MATLAB

4. Transfer function

From the state space analysis we can obtain the transfer function. First, we need to determine the transfer function equation with state space matrices of the converter model for both intervals, considering that our converter is operates in CCM mode.

The equation of the control transfer function with the state space matrices looks as follows:

$$\frac{Y}{D}=\left(C_1\cdot D_1+C_2\cdot D_2\right)\cdot\left[I-\left(A_1\cdot D_1+A_2\cdot D_2\right)\right]^{-1}\cdot\left(A_1-A_2\right)\cdot X+\left(B_1-B_2\right)\cdot U+\left(C_1-C_2\right)\cdot X+\left(E_1-E_2\right)\cdot U \tag{5}$$

Where Y represents the output, D is the duty cycle, A, B, C, E are the state space matrices, I is the unit matrix with size of A matrix, U contains the independent inputs, X contains the state variables.

We can use this equation in MATLAB to obtain the control transfer function of the flyback model with parasitic elements.

The control transfer function with distributed poles and zeros (zpk transformation) is as follows:

$$Gvd(s)=\frac{21047\cdot(s+9830)\cdot\left(s^2+828.6s+4.328e06\right)^3}{\left(s^2+828.6s+4.328e06\right)^4} \tag{6}$$

One of the methods to determine the stability of the system is by analysis of the logarithmic frequency characteristic (Bode plot).

By the command in the MATLAB, we obtain following characteristic:

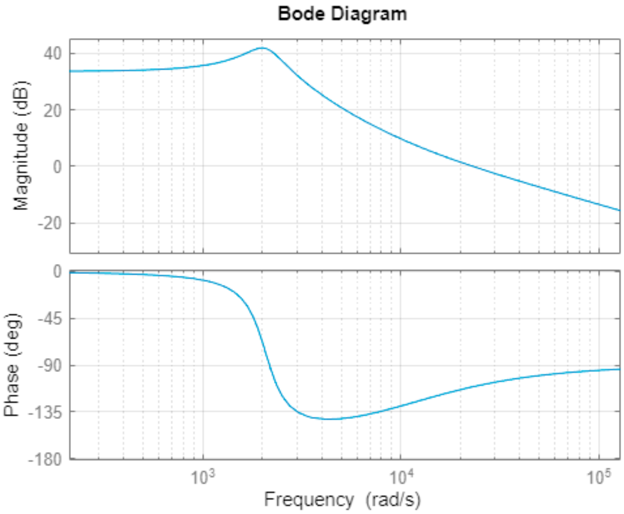

Fig. 3. Bode plot of open loop flyback converter transfer function

The system stabilized at higher frequencies at the -90° phase level, which is due to parasitic elements in the circuit. The transfer function contains one pole as the characteristic has a slope of -20dB/decade. It also contains complex roots due to overshoot, when the amplitude reaches 42dB.

The phase of the Bode plot must not go below -180° at a gain equal to 0dB, otherwise the system is unstable. The value of the phase margin at this gain crossover tells us about the stability of the system. The higher the phase margin φm, the more stable the system is. In our case, the phase at the crossover frequency is $\varphi = 112.5°$, and thus the phase margin has a value of $\varphi m = 67.5°$. At the same time, the quality factor is given as the overshoot of the amplitude and tells about the occurrence of oscillations and can lead to a decrease in stability because it shifts the phase crossover frequency. In our case, the quality factor is $Q = 18.6$, because the difference between the steady-state initial value and the overshoot is 8.5 dB.

We can determine the stability by Pole-Zero map and Nyquist diagram. The poles are located in the left part of the s-plane and therefore the system is stable, as can be seen in fig. 4. The poles contains complex roots due to the overshoot, which can be seen in the Bode diagram. The grid shows lines with a constant damping ratio as well as its frequency.

The Nyquist stability criterion can also be used, where we see that the frequency characteristic of an open-loop system is located to the right of the critical point -1, and the system is therefore stable.

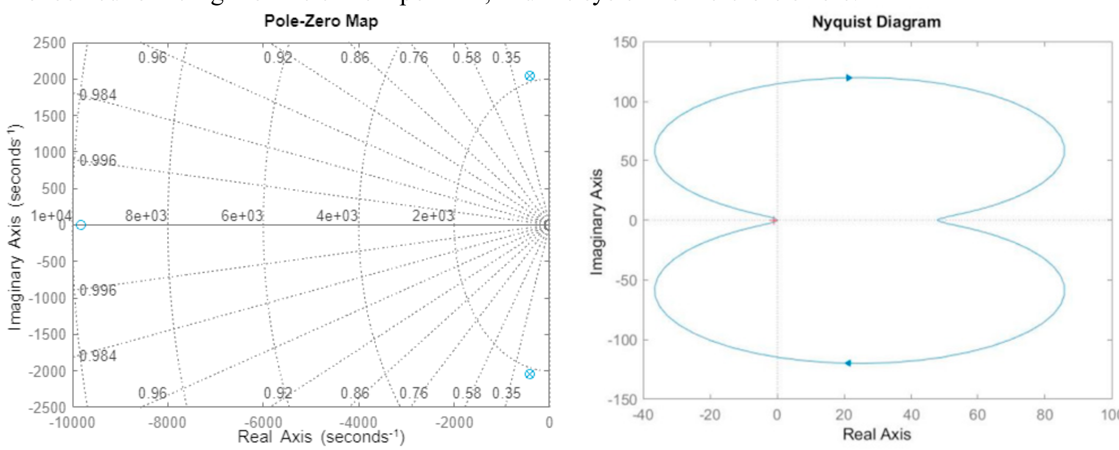

Fig. 4. Bode plot of open loop flyback converter transfer function

By adding a PI controller, we can control the given system. We choose the values using the empirical method in the pidtool library in MATLAB. The given controller is set to the following values of P=0.001 and I=2, and hence the compensator Gc is as follows:

$$Gc(s) = \frac{0.001s + 2}{s} \tag{7}$$

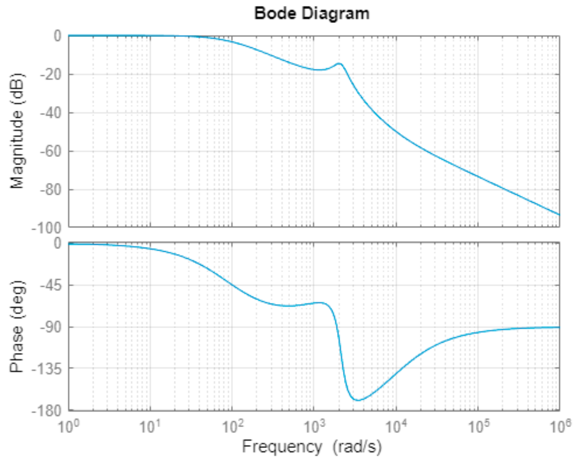

Fig. 5. Bode plot of the control transfer function of the closed loop system with PI compensator

5. Conclusion

In this paper we obtained state space matrices of the flyback converter with parasitic elements. At the end of the state space analysis of the nonideal flyback converter, we used the obtained matrices using Euler method in MATLAB to display the output voltage. The state space matrices were also used to obtain a transfer function to estimate the stability of the system. Based on Bode plot as well as the Pole-Zero map and the Nyquist diagram, we were able to determine the stability of the system. However, on the Bode Plot of the transfer function we see an overshoot and a relatively small phase margin, which can be corrected by adding a compensator. The addition of the PI compensator caused the phase margin to shift and reduced the overshoot. The voltage drop across the diode does not affect the Bode plot of the transfer function because it does not appear in equation. By increasing the parasitic elements of the components we can reduce the overshoot and increase the phase margin, as we can see from equations. The obtained state space matrices can also be used to calculate various quantities such as voltage or current of the proposed model.

Acknowledgements

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