## 2.2 Combinational and Arithmetic Circuits

Combinational circuits generate outputs based solely on current inputs without memory elements. Multiplexers, demultiplexers, decoders, and encoders are widely used in digital systems for data selection and encoding functions. A multiplexer, often called a data selector, takes multiple input signals and routes only one to the output based on control signals. A 4-to-1 multiplexer has four inputs, one output, and two control lines that determine which input is selected. Multiplexers are used in communication systems and data transmission. A demultiplexer performs the opposite function by taking a single input and directing it to one of multiple outputs based on control signals, commonly used in digital displays and signal routing.

A decoder converts binary input into a unique active output line. A 3-to-8 decoder takes a 3-bit input and activates one of eight output lines, essential for address decoding in memory circuits. An encoder, the inverse of a decoder, compresses multiple active inputs into a smaller number of outputs. A 4-to-2 encoder converts four input lines into a 2-bit binary representation, useful in data compression and priority encoding.

Binary arithmetic operations include addition and subtraction. Binary addition follows simple rules where 0 + 0 = 0, 0 + 1 = 1, 1 + 0 = 1, and 1 + 1 results in 10, with a carry of 1 to the next higher bit. A half-adder adds two single-bit binary numbers, producing a sum and a carry output, while a full-adder extends this by adding three bits, including a carry from the previous stage. Multi-bit adders such as ripple-carry adders combine multiple full-adders for complex calculations.

Binary subtraction can be performed directly or using the two's complement method. In two's complement, the negative of a number is obtained by inverting all bits and adding 1, allowing subtraction to be handled as addition. Unsigned binary numbers represent only positive values, while signed binary numbers use different methods to indicate positive and negative values. In sign-magnitude representation, the most significant bit (MSB) denotes the sign, with 0 indicating positive and 1 indicating negative. One's complement negates a number by flipping all bits, requiring an additional step to handle carries in arithmetic operations. Two's complement simplifies arithmetic by treating subtraction as addition, eliminating the need for separate subtraction circuits.