Types of Numbers

Outline:

- Real Numbers
- Rational Numbers
- Irrational Numbers
- Integers
- Natural Numbers
- Prime Numbers

Surds

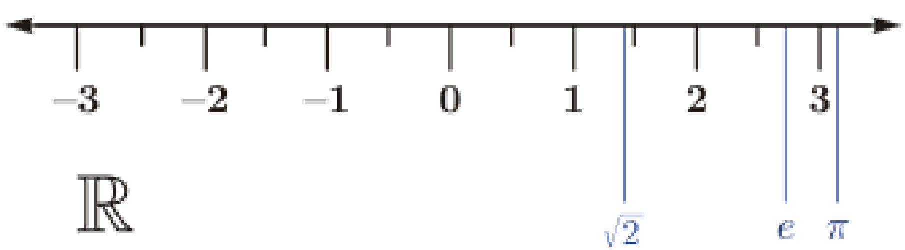
Indices / Powers – Rules of Indices

Real Numbers ($\mathbb{R}$)

In Mathematics a real number is a value that represents a quantity along a continuous line.

The Symbol of the set of real numbers is denoted by: $\mathbb{R}$

Positive or negative, large or small, whole numbers or decimal numbers are all Real Numbers.



Real Numbers

Real numbers can be represented geometrically by points on an infinitely long straight line called the Real Number line.

Each point on this line represents a unique real number and each real number can be represented by a unique point on this line.

There are two kinds of real numbers:

1. Rational Numbers
2. Irrational Numbers

Rational Numbers

Rational Numbers **($\mathbb{Q}$)**

A rational number is a real number that can be written as a ratio of two integers. A rational number written in decimal form is terminating or repeating.

Definition:

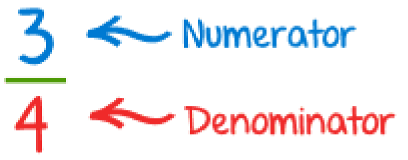
In mathematics, a rational number is any number that can be expressed as the quotient or fraction p/q of two integers, p and q, with the denominator q not equal to zero

E.g. $\frac{2}{3}, \frac{-1}{3}, \frac{1}{2}, 5$

5 is also a rational number since 5= 5/1

Rational Numbers

Note on fractions:



We call the top number the **Numerator.**

We call the bottom number the **Denominator.**

Irrational Numbers

Irrational Numbers

If a number cannot be expressed as the quotient or fraction p/q of two integers, p and q, with the denominator q not equal to zero, then, the number is called an irrational number.

E.g. $\sqrt{2}$, π , $\frac{\sqrt{3}}{3}$ are irrational numbers

Questions

Q1) Identify which of the following are rational / irrational numbers.

1. 2
2. 5
3. 0.333
4. 0.666
5. 0.125
6. $\sqrt{9}$
7. $\sqrt{3}$

9. $\sqrt{5}$
10. 5^2
11. $\sqrt{3}$ x $\sqrt{3}$
12. 2.5
13. 0.125
14. $\sqrt{1}$
15. $\sqrt{2}$
16. 0.01

Integers

Set of numbers that consists of positive and negative whole numbers including zero is called Integers. Integers are denoted by ($\mathbb{Z}$)

Includes:

- the counting numbers {1, 2, 3, ...},
- zero {0},
- and the negative of the counting numbers {-1, -2, -3, ...}

We can write them as $\mathbb{Z}$ **= {..., -3, -2, -1, 0, 1, 2, 3, ...}**

Examples of integers: -16, -3, 0, 1, 198

Natural Numbers

Positive integers including zero are called Natural Numbers. These integers are denoted by ($\mathbb{N}$)

Includes:

- the counting numbers **{0, 1, 2, 3, 4 ...}**,

Prime Numbers

If a natural number r excluding 1 is divisible by r alone, then r is known as a Prime Number.

- Example **{ 2, 3, 5, 7, 11, 13, 17 ...}**,

Q2) Find all the prime numbers less than 50.

Note: Apart from two are there any even-numbers that could be treated as Prime Numbers?

Question

Q3) Identify which of the following sets are made of real numbers, rational numbers, irrational numbers, integers, natural numbers by observing their elements.

1. {1, 3, 5, 6, 20}
2. {-1, -2, -5, -7, -9}
3. {-1, 0, 1}
4. $\{\sqrt{2}, \sqrt{3}, \sqrt{5}\}$
5. {0.5, 0.333, 0.25, 0.75}

Surds and Indices

Surds

Surds are irrational numbers where exact numerical values cannot be found.

E.g. $\sqrt{2}$ = 1.41213…

$\sqrt{5}$ = 2.23606…

$\sqrt{7}$ = 2.645751…

It is convenient to leave them as surds.

Question

Q4) Which of the following are surds?

1. $\sqrt{2}$
2. $\sqrt{3}$
3. $\sqrt{4}$
4. $\sqrt{8}$
5. 0.25
6. $\sqrt{9}$
7. $\sqrt{11}$
8. $3\sqrt{11}$

Simplification of a surd

E.g. Express $\sqrt{80}$ in its simplest form.

Answer: $\sqrt{80} = \sqrt{16 \times 5} = \sqrt{16} \times \sqrt{5} = 4\sqrt{5}$

Q5) Simplify the following

1. $\sqrt{20}$
2. $\sqrt{32}$
3. $\sqrt{40}$
4. $\sqrt{18}$
5. $\sqrt{84}$
6. $\sqrt{90}$

Rationalizing the denominator

If a simplification answer for a problem comes with surds in the denominator, it must be removed from the denominator. This is called rationalizing the denominator.

E.g. $\frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

Q6) Rationalize the following:

1. $\frac{3}{\sqrt{2}}$
2. $\frac{1}{\sqrt{7}}$
3. $\frac{7}{2\sqrt{2}}$
4. $\frac{\sqrt{2}}{\sqrt{80}}$

Indices / powers

Here, we will be looking at indices or powers. Either name can be used, and both names mean the same thing.

Basically, they are a shorthand way of writing multiplications of the same number.

So, suppose we have $4 \times 4 \times 4$

We write this as '4 to the power 3': 4^3

So $4 \times 4 \times 4 = 4^3$

The number 3 is called the power or index.

Note that the plural of index is indices.

Indices / powers

$9 = 3 \times 3$

Exponent/ Power

$9 = 3^2$

Number

Base

Indices / powers

Evaluate each of the following

1. 3^2
2. 2^4
3. $(2x)^2$
4. $(x/3)^2$
5. 4^3
6. $(2a)^5$
7. 3^5
8. 5^3

Laws of Indices

Rule-1

$a^m \times a^n = a^{(m+n)}$

1. $a^2 \times a^3$
2. $2a^4 \times 3a^3$
3. $a^{-2} \times a^3$
4. $a^2 \times a^{-3}$
5. $a^{-2} \times a^{-3}$

Laws of Indices

Rule-2

$(a^m)^n = a^{m \times n}$

Suppose we had a^4 and we want to raise it all to the power 3.

That is a^4 x a^4 x a^4

$(a^4)^3$

This means a^4 x a^4 x a^4

Now our first rule tells us that we should add the indices together.
So that is a^{12}

Laws of Indices

Rule-2

$(a^m)^n = a^{m \times n}$

1. $(a^2)^3$
2. $(2a^2)^5$
3. $(a^{-2}/2)^5$
4. $(a^{-2})^3$
5. $(a^2)^{-3}$
6. $(a^{-2})^{-3}$

Laws of Indices

Rule-3

$$a^m \div a^n = a^{(m-n)}$$

1. $a^3 \div a^2$
2. $2a^4 \div 3a^3$
3. $a^{-2} \div a^3$
4. $a^2 \div a^{-3}$
5. $a^{-2} \div a^{-3}$

Laws of Indices

Rule-4

$a^{m} = \frac{1}{a^{-m}}$ and $a^{-m}= \frac{1}{a^{m}}$

Evaluate each of the following leaving your answer as a proper fraction.

1. 2^{-9}
2. 3^{-5}
3. 4^{-4}
4. 5^{-3}
5. 7^{-3}

Laws of Indices

Rule-5

$a^0 = 1$

Evaluate each of the following.

1. 2^0
2. 23^0
3. $(-1)^0$
4. 0.5^0
5. $(-1.5)^0$

Laws of Indices

Rule-6

$$a^{\frac{1}{m}} = \sqrt[m]{a}$$

Evaluate each of the following.

1. $a^{\frac{1}{2}}$
2. $a^{\frac{1}{3}}$
3. $a^{\frac{2}{3}}$

Recap: Rules of Indices

Indices are a useful way of more simply expressing large numbers.

To manipulate expressions, we can consider using the Law of Indices.
These laws only apply to expressions with the **same base**

Rule1	$a^m \times a^n = a^{(m+n)}$
Rule2	$(a^m)^n = a^{m \times n}$
Rule3	$a^m \div a^n = a^{(m-n)}$
Rule4	$a^m = \frac{1}{a^{-m}}$ and $a^{-m} = \frac{1}{a^m}$
Rule5	$a^0 = 1$
Rule6	$a^{\frac{1}{m}} = \sqrt[m]{a}$

Basic Operations:

Addition	$+$
Subtraction	$-$
Multiplication	$\times$
Division	$\div$
Equal	$=$

Variables and Expressions

A mathematical quantity is anything that can be measured or counted. Some quantities remain constant. Others change or vary and are called *variable quantities.*

A **variable** is a symbol, usually a letter that represents the value(s) of a variable quantity.

An **algebraic expression** is a mathematical phrase that includes one or more variables.

A **numerical expression** is a mathematical phrase involving numbers and operation symbols, but no variables

Algebraic Expressions

Q1. Write algebraic expression for each word phrase.

1. 32 more than a number n
2. 58 less a number n
3. 58 less than a number n
4. 8 times a number n
5. The quotient of a number n and 5

Reasoning: Do the phrases 6 less a number y and 6 less than a number y mean the same thing?

Algebraic Expressions

Q2. Write algebraic expression for each word phrase.

1. 3 more than twice a number x
2. 9 less than the quotient of 6 and a number x
3. The product of 4 and the sum of a number x and 7
4. 8 less than the product of a number 4 and x
5. Twice the sum of number x and 8
6. The quotient of 5 and the sum of 12 and a number x

Algebraic Expressions

Q3. Write algebraic expression for each word phrase.

1. 12 fewer than n
2. The quotient of n and 8
3. 23 less than x
4. Two more than twice a number w.
5. 9 more than the difference of 17 and k.
6. 9.85 less than the product of 37 and t
7. 15 plus the quotient of 60 and w
8. Y minus 12

Algebraic Expressions

Q3. Write algebraic expression for each word phrase.

9. The product of 15 and c
10. The quotient of 17 and k
11. The sum of v and 3
12. A number t divided by 82
13. The sum of 13 and twice a number h
14. 6.7 more than the product of 5 and n
15. 7 minus the quotient of 3 and v
16. The product of 2.1 and the sum of 5 and k.

Order of operations

Use the order of operations to evaluate expressions.

Evaluate $7 + 4 \bullet 3$.

Is your answer 33 or 19?

You can get 2 different answers depending on which operation you did first. We want everyone to get the same answer so we must follow the **order of operations**.

Order of Operations

BODMAS

1. **B**rackets - ( ) or [ ]

   Note: Perform any operation(s) inside grouping symbols such as parentheses () and brackets [].

2. **O**rders: Powers / exponents (e.g. square roots)
3. **D**ivide and **M**ultiply (from left to right)
4. **A**dd and **S**ubtract (from left to right)

Order of Operations

"**P**lease **E**xcuse **M**y **D**ear **A**unt **S**ally" or **PEMDAS**.

ORDER OF OPERATIONS

1. **P**arentheses - ( ) or [ ]

   Note: Perform any operation(s) inside grouping symbols such as parentheses () and brackets [].

2. **E**xponents or Powers
3. **M**ultiply and **D**ivide (from left to right)
4. **A**dd and **S**ubtract (from left to right)

Evaluate the following

Example-1

$7 + 4 \bullet 3$

$= 7 + \underline{12}$ (Multiply.)

$= \underline{\mathbf{19}}$ (Add.)

Example-2

$14 \div 7 \bullet 2 - 3$

$= \underline{2} \bullet 2 - 3$ (Divide l/r.)

$= \underline{4} - 3$ (Multiply.)

$= \underline{\mathbf{1}}$ (Subtract.)

Evaluate the following

Example-3

$3(3 + 7)^2 \div 5$

$= 3(10)^2 \div 5$	(parentheses)
$= 3(100) \div 5$	(exponents)
$= 300 \div 5$	(brackets: multiplication)
= **60**	(division)

Evaluate the following

Example-4

$20 - 3 \bullet 6 + 10^2 + (6 + 1) \bullet 4$

$= 20 - 3 \bullet 6 + 10^2 + (7) \bullet 4$	(parentheses)
$= 20 - 3 \bullet 6 + 100 + (7) \bullet 4$	(exponents)
$= 20 - 18 + 100 + (7) \bullet 4$	(Multiply l/r.)
$= 20 - 18 + 100 + 28$	(Multiply l/r.)
$= 2 + 100 + 28$	(Subtract l/r.)
$= 102 + 28$	(Add l/r.)
= **130**	(Add.)

Questions

Q4. Evaluate the following:

1. $11^2 + 18 - 3^3 \cdot 5$
2. $16 - 2(10 - 3)$
3. $24 - 6 \cdot 4 \div 2$
4. $(6 - 2)^3 \div 3$
5. $\frac{2^4 - 1}{5}$
6. $5 \cdot 7 - 4^2 \div 2$
7. $12 - 25 \div 5$
8. $6 + 4 \div 2 + 3$
9. $5 \cdot 2^3 \div 2 + 8$
10. $\frac{4+3^4}{7-2}$

Evaluating a Variable Expression

To evaluate a variable expression:

1. substitute the given numbers for each variable.
2. use order of operations to solve.

Example-6

$n + (13 - n) \div 5$ for $n = 8$ (Substitute)

$= 8 + (13 - 8) \div 5$ (Parentheses)

$= 8 + 5 \div 5$ (Divide l/r.)

$= 8 + 1$ (Add l/r.)

$= \mathbf{9}$