Problem Set -1

Please read all of the guidelines carefully before submitting the problem set. Each question is ***20 points*** and there are ***100 points*** in total.

Due date: Wednesday, January 26, 11:59 PM. Late submissions will be accepted with a penalty! (10% reduction per day – no submissions accepted two days after the deadline.)

Guidelines – Before You Start

1) **You should complete the problem set on your own.** Discussing ideas is fine; but, sharing answers and sharing code will be considered as plagiarism.
2) You will be using the **`Python`** programming language. You need to write your codes in an empty **.ipynb** file.
3) Make sure that you provide many comments to describe your code and the variables that you created.
4) Please use **LaTeX** or **MS Word** to submit your written responses (hand-written responses will not be graded).
5) For some of the coding exercises, you may need to do a little bit of "**Googling**" or review the documentation.

Deliverables:

1) The code of the problem set in **.ipynb** format (one file)
2) Short answers written with ***LaTeX*** or **MS Word** and exported in **.pdf** format (one file)

Questions

1) Write a program using **`Python`** that does the following:
   - Takes two matrices of any size as the input
   - Returns their **dot product** as the output

   Note: You cannot use pre-packaged algorithms for matrix operations for this question. You can use `numpy` or `pandas` **to store your data** (not for calculations).

   Please do the following:

   a. Please test the following matrix multiplications using your hand-written code and report the result:

$$\begin{bmatrix} -4 & -3 & -2 \\ 6 & 0 & -1 \\ 2 & 1 & 3 \end{bmatrix}\begin{bmatrix} 5 & 4 \\ 6 & 7 \\ -4 & -3 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \end{bmatrix}$$

   b. Compare the result to the packaged dot product `numpy.dot`. Are they same?

   c. Please add your `code` to your **.pdf** file and also save it as an **.ipynb** file

2) Assume that we have two (2) *d-dimensional* real vectors **x** and **y**. And denote by x_i (or y_i) the value in the *i-th* coordinate of **x** (or **y**). Prove or disprove the following statements by checking *non-negativity*, *definiteness*, *homogeneity*, and *triangle inequality*.

    a. The following distance function is a metric. (5 points)

$$L_1(x,y) = \sum_{i=1}^{d} |x_i - y_i|$$

    b. The following distance function is a metric. (5 points)

$$L_2(x,y) = \sqrt{\sum_{i=1}^{d} (x_i - y_i)^2}$$

    c. The following distance function is a metric. (10 points)

$$L_2^2(x,y) = \sum_{i=1}^{d} (x_i - y_i)^2$$

3) <u>Calculating by hand</u>, find the ***characteristic polynomial***, ***eigenvalues*** and the ***eigenvectors*** of the following matrix:

$$\begin{pmatrix} 4 & 4 & 4 \\ -2 & -3 & -6 \\ 1 & 3 & 6 \end{pmatrix}$$

4) Provide a <u>proof</u> for the following: Let ***A***, ***B***, and ***C*** be any n x n matrices:
    a. Show that **trace(ABC) = trace(CAB) = trace(BCA)** (10 points)
    b. **trace(ABC) = trace(BAC).** Provide a proof or a counterexample (10 points)

5) Let ***A*** and ***B*** be *n x n* matrices with ***AB = 0***.

    Each question below is 5 points. Provide a <u>proof</u> or <u>counterexample</u> for each of the following:

    a) ***BA = 0***
    b) Either ***A = 0*** or ***B = 0*** (or both)
    c) If ***det(A) = -3***, then ***B = 0***
    d) There is a vector ***v ≠ 0*** such that ***BAv = 0***