DSCC 265
Assignment 1
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1. Let, the hypothesis be that the car is behind door 1. And we want to find the probability that the car is behind door 1 given that the door opened has a goat behind it. We will use ! to represent not. So,

$$C = \text{The car is behind door 1}$$

$$G = \text{The goat is behind the door opened}$$

$$P(C|G) = \frac{P(G|C)P(C)}{P(G)}$$

We can write the marginal probability of the host showing a goat as,

$$P(G) = P(G|C)P(C) + P(G|!C)P(!C)$$

This gives us all the possibilities of the host showing the goat regardless of whether door 1 has a car or not.

$$P(G|C) = P(G \cap C)/P(C) = (1/3)/(1/3) = 1$$

The probability of having the goat behind the door that was opened given that the car is behind door 1 is just 1. Because the door with the car will obviously not be opened.

$$P(G|!C) = P(G\cap!C)/P(!C) = (2/3)/(2/3) = 1$$

The probability that the host shows the goat, given that there is a goat behind door 1, is again always 1. The host always shows a door with a goat.

From the problem itself we know that, $P(C) = 1/3$ ($P(!C) = 1 - P(C) = 2/3$)and that is our prior.

$$\therefore P(G) = (1/3) + (2/3) = 1$$

Which again makes sense because the host will always open a door that has a goat. He will never open a door with a car.

So, now we can find,

$$\begin{aligned} P(C|G) &= \frac{P(G|C)P(C)}{P(G)} \\ &= \frac{(1)\cdot(1/3)}{1} = 1/3 \end{aligned}$$

We also find that,

$$P(!C|G) = 1 - P(C|G) = 2/3$$

This shows us that it is more probable that the car is not behind door 1. So, we can conclude that it is more advantageous to switch.

2. For the two teams to reach the 7^{th} game team A has to win 3 games and has to lose 3 games (i.e. team B wins the 3 games). This is just a binomial distribution. Let, $P(A) = 0.55$ be the probability that A wins and $P(B) = 0.45$ be the probability that B wins.

$$\begin{aligned} P_6(3) &= \binom{6}{3}(P(A))^3(P(B))^3 \\ &= \binom{6}{3}(0.55)^3(0.45)^3 \\ &= 0.3032 \end{aligned}$$

3. Attached in .ipynb file
4. Attached in .ipynb file
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