Exploration of Molecular Cloud and Star Formation

Uzair Tahamid Siam
University of Rochester, Physics and Astronomy Dept.
ASTR232W - The Milky Way Galaxy

10 January 2022

Abstract
Galaxies are aggregates of dusts, gases, clouds, cosmic rays, and stars. While they all play an important role in driving galactic astrophysics and processes, stars often play just as much an important role as hosts for solar systems. Understanding the formation of stars is crucial to understanding much of what interests us - planets, life, and the like. In this paper I will explore some of the processes that govern the formation of molecular clouds such as *converging flows* and *gravitational instabilities*, their collapse defined by *Jean's Mass* as well as *Bonnor-Ebert Mass*, and the upper and lower bounds of stellar mass in the formation process.

1 Formation of Molecular Clouds

Giant molecular clouds (GMCs) are the primary reservoirs of cold, star-forming molecular gas in the Milky Way and other galaxies we know of. The understanding of GMC clouds - their formation, evolution, and destruction - is essential to understanding star-formation and the formation of many other astrophysical entities. Our understanding of cloud structures and their evolution is an ongoing ordeal with fascinating discoveries happening regularly and my goal is to review what the current research says throughout this section.

1.1 Molecule Formation and Dust Grains

Before we understand the formation of *Giant* molecular clouds, let us first understand can molecules truly even form in the ISM? As mentioned in chapter-4 of '*The physics of the interstellar medium*' [3], from laboratory experiences, we already know that molecules form from atoms on surfaces containing atomic gas. In the ISM this occurs on dust surfaces. However, is the process of molecule formation on dusts efficient enough to outweigh H_2 losing mechanisms such as heating by starlight[1]. If we make the assumption that any H that arrives at a dust grain only leaves the grain as a H_2 then the rate at which H_2 is formed is given by

$$\frac{d}{dt}n(H_2) = \frac{1}{2}n(H)\pi a^2 n_g v_H \tag{1}$$

for grains of radius a, number density n_g and most probable velocity v_H. Now, we need to take into account the dusts that we are considering. Our obvious choice is gas that is known to cause visual extinction of light. The quantity $\pi a^2 Q_{ext} n_g$ is related to extinction, and if we assume $Q_{ext} \approx 1$, then the grains causing visual extinction give a value for $3 \cdot 10^{-26} n$ m^{-1}, so that H_2 forms in a gas at 100 K at the rate

$$\frac{d}{dt}n(H_2) \approx 3 \cdot 10^{-23} n\ n(H)\ m^{-3} s^{-1}$$

where $n = n(H) + 2n(H_2)$. This actually gives us a lower estimate of the formation rate as we expect other small grains to exist that do not have as significant contribution to visual extinction as larger grains. Now that we have our formation rate, we want to compare it to the rate of photodissociation. This rate depends very strongly on column density, N(H_2) m^{-2}. For clouds of moderate density - $n \approx 10^8\ m^{-3}$ - it appears that our formation rate at the center of the cloud is adequate to convert the bulk of hydrogen to molecular hydrogen. And given that this occurs at the center, self-shielding also protects the molecules from being photodissciated. So we can safely conclude that given our assumptions, molecule formation does occur on dust grains. The actual process of how these molecules form is rooted in chemistry and while that is fascinating, I will not cover that discussion in this paper.

1.2 Giant Molecular Cloud Formation

[2] The main mechanisms that we know lead to formation of GMC are converging flows driven by stellar winds, clustering of small clouds, gravitational instability, and magneto-gravitational instability. These vast physical phenomenon, however, involve one thing in common - converging flows, i.e. $\nabla \cdot \mathbf{u} < 0$ where $\mathbf{u}$ is the velocity. Each mechanism however acts over different sizes and timescales, leading to different densities in the clouds. As a consequence, these different mechanisms may dominate in different environments, and may lead to different cloud properties. For this paper, I will only engage with two of the phenomenon - localized converging flows and gravitational instabilities.

1.2.1 Converging flow

Stellar feedback processes such as the expansion of HII regions can drive converging jets and streams of gas that accumulate to become molecular clouds, either in the Galactic plane or above it. Morphological evidence for this process can be found in large-scale extinction maps of the Galaxy as shown in the Fig. 1. Locally – on scales up to ~ 100 pc – it is likely that these processes play a dominant role in molecular cloud formation, since on these scales the pressure due to local energy sources is typically $\frac{P}{k_B} \sim 10^4$ K cm^{-3} which is higher than the average ISM pressure nearby. A converging flow, however, is not by itself sufficient to form a MC; the detailed initial velocity, density must combine with thermal instability to pro-

[1] H_2 is photodissociated. The mechanism involves excitation from the ground state s_0 in the lowest vibrational level to an excited state s_j, followed by a cascade into the vibrational continuum of the ground state s_0, in which the molecule dissociates. A significant fraction ($\sim 20\%$) of such excitations lead to dissociation of the molecule into atomic H

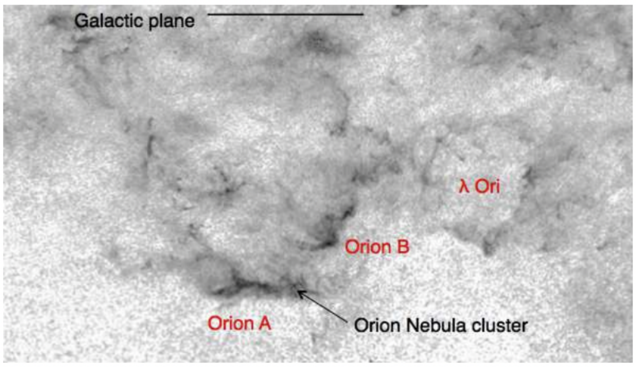

Figure 1. Extinction map towards the Orion-Monoceros molecular complex. Different features are located at different distances, but at the mean distance of Orion, the approximate size of the A and B clouds, as well as the Ori ring, is of the order of 50 pc.

duce fast cooling. This allows rapid accumulation of cold, dense atomic gas, and thus promotes molecule formation. For Solar neighborhood conditions, where there are low densities and relatively short timescales for coherent flows, Molecular clouds of a maximum of order $10^4\ M_\odot$ are created. As mentioned before, converging flows driven by other processes such as large-scale instabilities can produce higher-mass clouds which will eventually lead to the birth of higher mass stars.

1.2.2 Gravitational Instability

One of the alternative theories for massive clouds is that they form in a top-down manner [2] and one such process happens to be gravitational instability. Small axisymmetric changes in a single-phase thin gas disk with effective sound speed c_{eff}, surface density σ, and epicyclic frequency, the frequency at which a radially displaced fluid parcel oscillates, χ can occur whenever the Toomre parameter,

$$Q \equiv \frac{\kappa c_{\text{eff}}}{\pi G \sigma} \tag{2}$$

a criterion for the stability of *differentially* rotating disks, implies instability if $Q < 1$. In the absence of a pre-existing well defined stellar spiral patterns, gravitational instability in the combined gas-star disk can lead to flocculent or multi-armed spirals, as our current models show. In cases where a grand design - well defined spiral features - is present such as when driven by a tidal encounter, gas flowing through the spiral pattern supersonically experience shocks, which raise the density and as a result may cause gravitational collapse.

Clouds formed by gravitational instabilities in single-phase gas disks typically have masses about 10 times the two-dimensional Jeans mass [4] for a disk,

$$M_{J,2D} = \frac{c_{\text{eff}}^4}{G^2\sigma} \approx 10^8 M_\odot \left(\frac{c_{\text{eff}}}{7\ kms^{-1}}\right)^4 \left(\frac{\sigma}{M_\odot pc^{-2}}\right)^{-1} \tag{3}$$

At moderate gas surface densities, $\sigma < 100 M_\odot\ pc^{-2}$, away from spiral-arm regions, the masses of the molecular clouds are larger than those of observed GMCs when $c_{\text{eff}} \approx 7\ kms^{-1}$. The absence of such massive clouds is consistent with observations indicating that Q values are generally above the critical threshold (except possibly in high redshift systems), such that spiral-arm regions at high σ form observed GMCs via self-gravitating instability. Note however that all our arguments so far have been involving single-phase disks, the region of the ISM from which GMCs form are actually multi-phase where cold clouds are surrounded by warmer intercloud medium. Simulations that include a multi-phase medium find a broader spectrum of cloud masses, extending up to several $10^6 M_\odot$.

[2] Top-down manner implies that the largest imperfections are on the largest scales; they begin gravitating first, and as they do, these large imperfections fragment into smaller ones.

2 Collapse of Molecular Clouds - Birth of Stars

2.1 Star formation

[1] Formation of stars begins with the collapse of dense regions in molecular clouds, sometimes referred to as *stellar nurseries*. The collapse is governed by the gravitational instability of the clouds.

2.1.1 Jean's Mass and Length

While the dynamics of clouds is far from simple, I will make most of my arguments assuming the presence of only outward thermal pressure and inward gravitational pressure. This assumption allows us to use virial theorem [cite here] that describes the condition of equilibrium for a stable, gravitationally bound systems.

$$2K + U = 0 \tag{4}$$

If $2K > |U|$ then the cloud expands and if $|U| > 2K$ then the cloud collapses in on itself as the outward gas pressure is not enough to hold structure against the inward gravitational force. The boundary between these two cases describes the critical condition for stability when rotation, turbulence, and magnetic fields are neglected. Assuming a spherical cloud of uniform density, we get

$$U = -\frac{3GM_c^2}{5R_c} \tag{5}$$

where M_c and R_c are the mass and radius of the clouds. We can also estimate the kinetic energy of cloud using,

$$K = \frac{3Nk_BT}{2} \tag{6}$$

where N is the total number of particles with $N = M_c/\mu m_H$ where μ is the mean molecular weight. Using Equation 4, 5, and 6 and

$$\rho_0 = \frac{3M_c}{4\pi R_c^3} \to R = \left(\frac{3M_c}{4\pi\rho_0}\right)^{1/3}$$

we find that Jean's mass is given by,

$$M_J = \left(\frac{5kT}{G\mu m_H}\right)^{3/2} \left(\frac{3}{4\pi\rho_0}\right)^{1/2} \tag{7}$$

and

$$R_J = \left(\frac{15kT}{4\pi G\mu m_H \rho_0}\right)^{1/2} \tag{8}$$

If $M_C > M_J$ or $R_C > R_J$ then the cloud will collapse on itself.

2.1.2 Bonnor-Ebert Mass

The Jeans mass derivation assumes the absence of an important fact that there must exist an external pressure on the cloud due to the surrounding interstellar medium. The critical mass required for gravitational collapse in the presence of an external gas pressure of P_0 is given by the Bonnor-Ebert mass. In this section we will derive the dimensionless Bonnor-Ebert Mass using the *isothermal Lane-Emden* equation.

[5] We start with the assumption that we have a spherically symmetric cloud and that the process is *isothermal*, i.e. $\gamma = 1$. This gives us three equations,

$$\nabla P = \frac{dP}{dr}$$

$$\nabla^2 \phi = \frac{1}{r^2}\frac{d}{dr}\left(\frac{r^2 d\phi}{dr}\right)$$

$$\frac{d\phi}{dr} = -G\frac{M(r)}{r^2}$$

where ϕ is the gravitational potential, r is the radial distance from the center, and P represents pressure.

These three equations now give us the condition for hydrostatic equilibrium of a thermally supported spherical cloud,

$$\frac{1}{\rho(r)}\frac{dP}{dr} = -\frac{d\phi}{dr}$$

Using the ideal gas law, $P = \rho(r)a^2$, where a is the sound speed, and Poisson's equation for gravitational potential, $\nabla^2\phi = 4\pi G\rho(r)$, we get the equation,

$$\frac{a^2}{\rho(r)}\frac{d\rho}{dr} = -\frac{d\phi}{dr}$$

$$\therefore \frac{dln\rho(r)}{dr} = -\frac{d}{dr}\frac{\phi(r)}{a^2}$$

Now, let us assume that $\phi(0) = 0$ and $\rho(r) = \rho_c e^{-\phi(r)/a^2}$. This results in the Poisson equation being,

$$\nabla^2\phi = 4\pi G\rho_c e^{-\phi(r)/a^2}$$

Next we define $u \equiv \phi/a^2$ and $\xi \equiv (4\pi G\rho_c/a^2)^{1/2}r$, thus resulting in the Lane-Emden differential equation,

$$\frac{1}{\xi^2}\frac{d}{d\xi}\left(\xi^2\frac{du}{d\xi}\right) = e^{-u} \tag{9}$$

We then define the boundary conditions, $u(0) = 0$, $du/d\xi_{|\xi=0} = 0$

Next, we non-dimensionalize the mass. We start by consider the mass of the sphere:

$$M = \int_0^{r_0} 4\pi r^2\rho(r)dr$$

Using our definitions of ρ, u and ξ and substitution, we get,

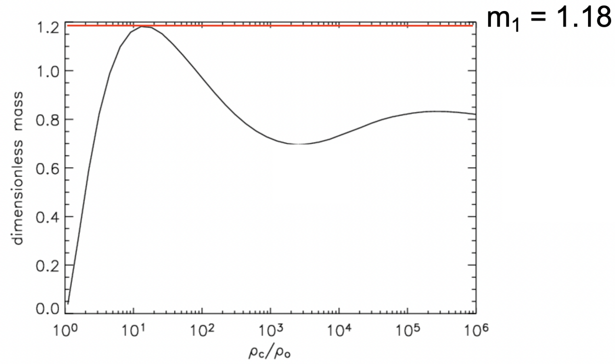

Figure 2. Dimensionless mass, m, vs Dimensionless density, ρ_c/ρ_0 used to find the values of m for unstable equilibrium positions. The first m, m_1 is shown on the plot with the red horizontal line with $m_1 = 1.18$

$$M = 4\pi\rho_c\left(\frac{a^2}{4\pi G\rho_c}\right)^{3/2}\int_0^{\xi_0} e^{-u}\xi^2 d\xi$$

Finally with the Lane-Emden equation we get the mass,

$$M = 4\pi\rho_c\left(\frac{a^2}{4\pi G\rho_c}\right)^{3/2}\xi^2\frac{du}{d\xi}$$

since,

$$\int_0^{\xi_0} e^{-u}\xi^2 d\xi = \xi^2\frac{du}{d\xi}$$

we can now define the dimensionless mass,

$$m \equiv \frac{{P_0}^{1/2}G^{3/2}M}{a^4}$$

$$\therefore M_{BE} = \frac{ma^4}{G^{3/2}{P_0}^{1/2}}$$

In addition, we can also define m as,

$$m = \left(\frac{4\pi\rho_c}{\rho_0}\right)^{-1/2}\left(\xi^2\frac{du}{d\xi}\right)_{\xi_0}$$

at the boundary. So, ρ_0 represents ρ at the boundary and similarly ξ_0 is ξ at the boundary. From Fig.2 we get $m = m_1 = 1.18$ which gives us the final result for the Bonner-Ebert Mass,

$$\therefore M_{BE} = \frac{1.18a^4}{G^{3/2}{P_0}^{1/2}} \tag{10}$$

Just like Jean's Mass and Length, we can define the conditions for collapse as,

- Increase external pressure
- Have a cloud with mass, $M_c > M_{BE}$

2.1.3 Free-fall time

[1] Considering the case of uniform density, ρ_0 and the absence of rotation, turbulence, or magnetic fields, we can calculate the

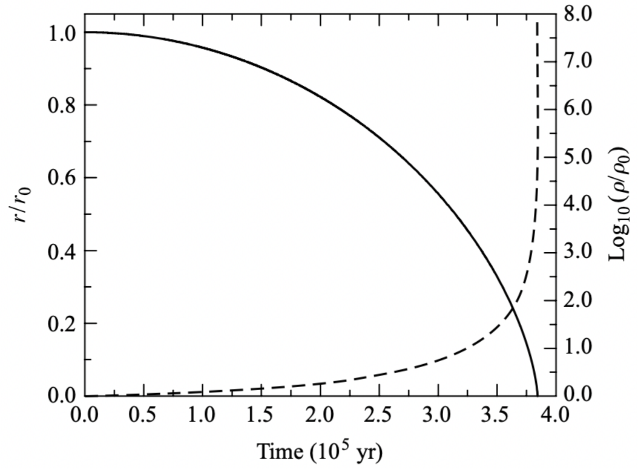

Figure 3. Homologous collapse of clouds with different densities and radii vs free-fall time. The higher the density, the longer the free-fall time and the larger the radius the shorter the free-fall time as described by Equation 12.

free-fall time required for the cloud to collapse. In addition, if we also make (the quite unrealistic) assumption that the pressure gradient is small, then the cloud is essentially in free-fall during the first part of its evolution. Furthermore, throughout the free-fall phase, the temperature of the gas remains nearly constant. This simplifies the problem into the differential equation,

$$\begin{aligned} \frac{d^2r}{dt^2} &= -G\frac{M_r}{r^2} \\ \Rightarrow \frac{d^2r}{dt^2} &= -\frac{4\pi G r_0^3 \rho_0}{3r^2} \\ \Rightarrow \frac{dr}{dt}\frac{d^2r}{dt^2} &= -\frac{4\pi G r_0^3 \rho_0}{3r^2}\frac{dr}{dt} \end{aligned} \tag{11}$$

Solving Equation 11 gives us a free-fall time,

$$t_{\mathrm{ff}} = \left(\frac{3\pi}{32G\rho_0}\right)^{1/2} \tag{12}$$

As you can see the time for free-fall is independent of the initial radius in the case of a homologous collapse. Fig. 3 shows the free fall time for varius densities.

2.2 Stellar mass distribution

[1] The mass of a star is one of its most fundamental properties. In this section we will look at the intriguing phenomenon of mass distribution of stars as well look at why the distribution exists.

2.2.1 Initial Mass Function (IMF)

The initial mass function illustrates the distribution of the number of stars that exist per unit area of the Milky Way per unit interval of logarithmic mass. Fig. **??** above shows a theoretical estimate of the IMF. However, IMF depends on several factors, such as the environment around the star clusters. You will notice that most stars form with relatively low mass and that there seems to be very massive stars and also a notable maximum mass to the stars.

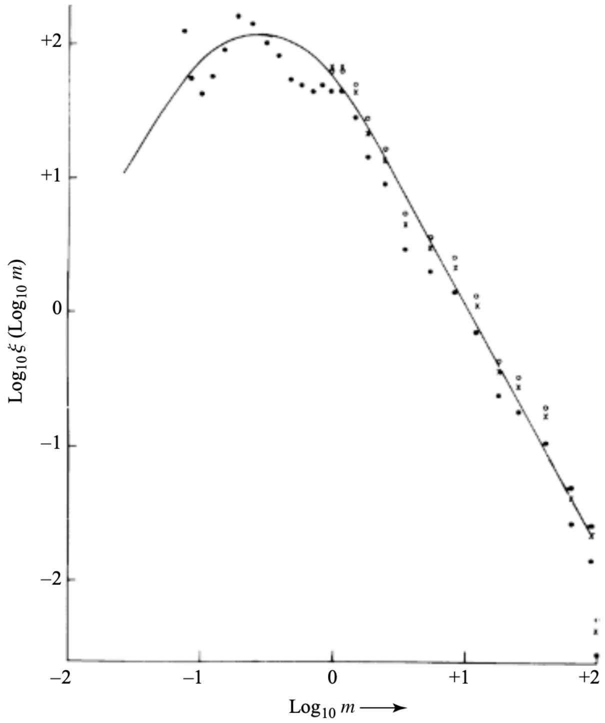

Figure 4. The initial mass function illustrating the distribution of the number of stars that exist per unit area of the Milky Way per unit interval of logarithmic mass. The solid line describes a theoretical functional form for the mass distribution.

2.2.2 Fragmentation

As clouds begin to collapse, the density, ρ increases. Assuming that the temperature, T, remains constant, i.e. the process is isothermal, looking at Equation 7 we can see that M_J decreases. This means that the cloud is more susceptible to collapse and that is exactly what happens throughout multiple regions in the collapsing cloud. If there exists any homogeneity throughout the cloud, then the regions that exceed M_J first will start to collapse followed by others thus creating multiple pockets of more dense regions. This is why stars frequently tend to form in groups ranging from binary star systems to clusters that contain hundreds of thousands of members. But when does this process stop? We made the assumption that the process is isothermal throughout the free-fall. But obviously that cannot be true. Once stars begin to form, the temperature changes. As the temperature changes we have to take into account the relation between that, the density of the cloud as well as the Jean's Mass. Using ideal gas law,

$$T \propto \rho^{\gamma-1}$$

This relation alongside Equation 7 gives us that,

$$M \propto \rho^{(3\gamma-4)/2}$$

For an adiabetic process where, $\gamma = 5/3$ we get $M \propto \rho^{1/2}$. Thus, as expected, M_J increases meaning that the cloud becomes less susceptible to collapse. Using a crude estimate where the outward pressure and inward pressure are balanced,

$$\begin{aligned} L_{\text{ff}} &= L_{\text{rad}} \\ \Rightarrow \frac{\Delta E_g}{t_{\text{ff}}} &= 4\pi\sigma R^2 T^4 e \\ \Rightarrow G^{3/2}\left(\frac{M_J}{R_J}\right)^{5/2} &= 4\pi\sigma R^2 T^4 e \\ \Rightarrow M_J &= \frac{4\pi R_J^{9/2} e\sigma T^4}{G^{3/2}} \end{aligned} \tag{13}$$

Using representative values into Equation 13 has shown that it predicts a mass, $M_{J_{min}} \approx 0.5\ M\odot$ at which the fragmentation process stops.

2.2.3 Eddington Luminosity

Now that we have an estimate of the lower bound of stellar masses, let's look at the upper bound. Stars have their own gravitational pressure acting on them. The pressure that resists that and keeps it stable, is the pressure from all the photons. At stable equilibrium,

$$\begin{aligned} \frac{dP_\gamma}{dr} &= \frac{dP_g}{dr} \\ \Rightarrow -\frac{\kappa\rho\bar{F}_{\text{rad}}}{c} &= -\frac{GM\rho}{r^2} \\ \Rightarrow -\frac{\bar{\kappa}L}{4\pi r^2 c} &= -\frac{GM}{r^2} \\ \therefore L_{\text{ED}} &= \frac{4\pi GMc}{\bar{\kappa}} \end{aligned} \tag{14}$$

L_{ED} represents the maximum luminosity that a star can have before the outward pressure exceeds the inward gravitational pressure thus causing the star to explode. Given that $L_{\text{ED}} \propto M$ we can now see that this Eddington Luminosity puts a higher bound on how massive stars can get before they become unstable.

3 Conclusion

In this paper I discussed the hierarchical processes of molecule formation on dust grains in the ISM, followed by molecular cloud formation from the molecules and finally the formation of stars from the collapsing molecular clouds. Understanding of the phenomenon that drive these processes such as converging flows, gravitational instabilities, and pressure gradients allows us to better understand physics throughout the ISM and not just star formation and many of these phenomenon are widely common in many other formation processes such as planet formation. That said, it is important to remember that many of these phenomena are still under studying and as our understanding of the universe grows so does our understanding of star and cloud formation.

4 Bibliography

References

[1] Bradley W. Carroll and Dale A. Ostlie. *An Introduction to Modern Astrophysics*. Ed. by San Francisco: Pearson Addison-Wesley. 2nd (International). 2007.

[2] C. L. Dobbs et al. "Formation of Molecular Clouds and Global Conditions for Star Formation". In: *Protostars and Planets VI* (2014). DOI: 10.2458/azu_uapress_9780816531240-ch001. URL: http://dx.doi.org/10.2458/azu_uapress_9780816531240-ch001.

[3] J. E. Dyson and D. A. Williams. *The physics of the interstellar medium*. 1997. DOI: 10.1201/9780585368115.

[4] Duncan Forgan and Ken Rice. "The Jeans mass as a fundamental measure of self-gravitating disc fragmentation and initial fragment mass". In: *Monthly Notices of the Royal Astronomical Society* 417.3 (Aug. 2011), pp. 1928–1937. ISSN: 0035-8711. DOI: 10.1111/j.1365-2966.2011.19380.x. URL: http://dx.doi.org/10.1111/j.1365-2966.2011.19380.x.

[5] Tom Megeath. *Lecture 6: Modeling Molecular Cores as Bonnor-Ebert Spheres and Isothermal Spheres*. URL: http://astro1.physics.utoledo.edu/~megeath/ph6820/lecture6_ph6820.pdf.