Institute of Theoretical Physics
São Paulo State University

Aspects of Experimental High-Energy Physics

IV Journeys Into Theoretical Physics
Prof. Thiago Tomei
July 6-12, 2019
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Aspects of Experimental High-Energy Physics

Tomei

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Outline
- Introduction to the Standard Model
- Accelerators and Detectors
- Data Reconstruction
- Data Analysis

① The Standard Model
- → QFT based on gauge symmetry
- → model of electromagnetic, weak and strong interactions
- → reproduces extremely well the phenomenology of all observed particles
- → based on
  - → experimental discoveries
    - positron, muon, strange, charm, tau, ...
  - → QFT: particles are quanta of fundamental fields
    - QM + SR
  - → invariance under transformations that belong to symmetry groups
    - interaction comes as result of fundamental symmetries
- → successful predictions
  - → ∃ neutral currents that mediate weak inter.
  - → mass of W and Z bosons
  - → equal # of leptons and quarks in isospin doublets
  - → ∃ scalar neutral boson ⇝ Higgs

QFT: our best tool to describe the fundamental laws of nature

RQM doesn't work

- How does QFT improves from QM + Classical FT?
  - → dynamical degrees of freedom become operators that are functions of spacetime
    - ↳ QF obey appropriate commutation relations
  - → interaction of the fields are local ⇝ no "spooky" action at a distance

with symmetry, if combined (Lorentz, gauge), it becomes a powerful toll to describe interactions

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QFT for free fields

- → ∃ indistinguishable particles → all excitations of the same, e.g. electron, field
- → ∃ antiparticles
- → quantum statistics

QFT for interacting fields

- → appearance of processes creating and annihilating particles
- → association of interaction and exchange of particles

From particles to fields

particle of mass m and spin s → or helicity h

$$|\vec{p}, s_z, \sigma\rangle \qquad \begin{cases} \hat{\vec{p}}\,|\vec{p}, s_z, \sigma\rangle = \vec{p}\,|\vec{p}, s_z, \sigma\rangle \\ \hat{H}\,|\vec{p}, s_z, \sigma\rangle = E_{\vec{p}}\,|\vec{p}, s_z, \sigma\rangle \\ \hat{S}_z\,|\vec{p}, s_z, \sigma\rangle = s_z\,|\vec{p}, s_z, \sigma\rangle \end{cases} \qquad E_{\vec{p}} = \sqrt{p^2 + m^2}$$

σ: internal quantum numbers

etc

Lorentz invariance: $\mathcal{H}$ must contain all state vectors for all momenta on "the mass shell": $p^2 = p_\mu p^\mu = m^2$

Particle types labeled by s

Particles can also carry other internal charges
↳ e.g. electric charge

Single particle in the universe:

$$|\vec{p}, s_z, \sigma\rangle = a^\dagger_{\vec{p}, s_z, \sigma}|0\rangle$$

Multi-particles and statistics:

bosons
$$\left[a_{\vec{p}, s_z, \sigma}, a^\dagger_{\vec{p}', s_z', \sigma'}\right] = (2\pi)^3 2E_{\vec{p}}\,\delta^{(3)}(\vec{p} - \vec{p}')\,\delta_{s_z s_z'}\,\delta_{\sigma,\sigma'}$$

fermions
$$\left\{a_{\vec{p}, s_z, \sigma}, a^\dagger_{\vec{p}', s_z', \sigma'}\right\} = (2\pi)^3 2E_{\vec{p}}\,\delta^{(3)}(\vec{p} - \vec{p}')\,\delta_{s_z s_z'}\,\delta_{\sigma,\sigma'}$$

all other (anti-)commutator vanish

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Non-interacting particle states are built by repeated application of the creation operators

Indistinguishable particles: labeled only by occupation numbers

It is convenient to put all possible 1 particle "states" together by Fourier transforming. E.g., for $s=0$,

$$\Phi_+(\vec{x},t) \doteq \int \frac{d^3k}{(2\pi)^3} \frac{1}{2E_{\vec{k}}} a_{\vec{k}} \, e^{-ik_\mu x^\mu} \Big|_{k_0 = E_{\vec{k}}}$$

If the particle carries a charge, the anti-particle is distinct and we define

$$\Phi_-(\vec{x},t) \doteq \int \frac{d^3k}{(2\pi)^3} \frac{1}{2E_{\vec{k}}} b^\dagger_{\vec{k}} \, e^{ik_\mu x^\mu} \Big|_{k_0 = E_{\vec{k}}}$$

From the earlier commutation relations, we might show that the field

$$\Phi(\vec{x},t) = \Phi_+(\vec{x},t) + \Phi_-(\vec{x},t)$$

obeys

$$[\Phi(x), \Phi(y)^\dagger] = 0 \quad \text{for } (x-y)^2 < 0$$

$\rightarrow$ spacelike

$\rightarrow$ Causal field

Notice also that $\Phi(x)$ satisfies the Klein-Gordon equation

$$(\partial_\mu \partial^\mu + m^2)\,\Phi(x) = 0$$

$\rightarrow$ $E^2 = p^2 + m^2$, $\vec{p} \to -i\nabla$, $E \to i\frac{\partial}{\partial t}$

It can be derived from the Lagrangian density

$$\mathcal{L} = \partial_\mu \Phi^\dagger(x) \partial^\mu \Phi(x) - m^2 \Phi^\dagger(x)\, \Phi(x)$$

following the usual variational principle from class mech

$$\partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \Phi)} - \frac{\partial \mathcal{L}}{\partial \Phi} = 0$$

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Other Free Lagrangians

Dirac:

$$\mathcal{L} = i\bar{\psi}\gamma^\mu \partial_\mu \psi - m\bar{\psi}\psi$$

Leads to Dirac eq for $s=1/2$ particles

$$i\gamma^\mu\partial_\mu\psi - m\psi = 0$$

$A^\mu = (\phi, \vec{A})$

Free EM

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}, \quad \text{with } F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

Leads to Maxwell's Equations for the free EM field

$$\partial_\mu F^{\mu\nu} = 0$$

V in Schrödinger's is arbitrary

We could choose any form for the interactions, but let's take a closer look at EM

EM fields $\quad \vec{E} = -\nabla\phi - \frac{\partial \vec{A}}{\partial t}; \quad \vec{B} = \nabla \times \vec{A}$

invariant under gauge transformation

choosing the right gauge can simplify EM problems

$$\phi \to \phi' = \phi - \frac{\partial \chi}{\partial t}, \quad \vec{A} \to \vec{A}' = \vec{A} + \nabla\chi$$

Hamiltonian that gives rise to Lorentz force:

$$\mathcal{H} = \frac{1}{2m}(\vec{p} - q\vec{A})^2 + q\phi$$

$\vec{p} \to -i\nabla$

$E \to i\frac{\partial}{\partial t}$

Schrödinger's for particle in an EM field

$$i\frac{\partial\psi}{\partial t} = \left[\frac{1}{2m}(-i\nabla - q\vec{A})^2 + q\phi\right]\psi$$

$$\frac{1}{2m}(-i\vec{D})^2\psi = iD_0\psi, \quad \text{with } \begin{cases} \vec{D} = \nabla - iq\vec{A} \\ D_0 = \frac{\partial}{\partial t} + iq\phi \end{cases}$$

But if we take the free S.E. and make $\nabla \to \vec{D}$ and $\frac{\partial}{\partial t} \to D_0$, we get the same equation

If we make the gauge transformations $(\phi, \vec{A}) \xrightarrow{G} (\phi', \vec{A}')$, does the solution of

$$\frac{1}{2m}(-i\vec{D}')^2\psi' = iD_0'\psi'$$

describe the same Physics?

No! we need to make a phase transformation to the matter field Tomei

$$\psi' = \underbrace{\exp(iq\chi)\,\psi}_{U(1)\ \text{transformation}}$$

Derivatives transform as

$$\vec{D}'\psi' = \left[\nabla - iq(\vec{A} + \nabla\chi)\right]\exp(iq\chi)\,\psi$$
$$= \exp(iq\chi)\nabla\psi + iq\nabla\chi\exp(iq\chi)\psi - iq\vec{A}\exp(iq\chi)\psi - iq\nabla\chi\exp(iq\chi)\psi$$
$$= \exp(iq\chi)\vec{D}\psi$$
$$D_0'\psi' = \exp(iq\chi)D_0\psi$$

Schrödinger's maintain its form, since

$$\frac{1}{2m}\left(-i\vec{D}'\right)^2\psi' = \frac{1}{2m}\left(-i\vec{D}'\right)\left(-i\vec{D}'\psi'\right)$$
$$= \frac{1}{2m}\left(-i\vec{D}'\right)\left[-i\exp(iq\chi)\vec{D}\psi\right]$$
$$= \exp(iq\chi)\frac{1}{2m}\left(-i\vec{D}\right)^2\psi$$
$$= \exp(iq\chi)\left(iD_0\right)\psi = iD_0'\psi'$$

both fields describe the same Physics, since $|\psi|^2 = |\psi'|^2$

In order to make all variables invariant, we should substitute

$$\nabla \to \vec{D}, \quad \frac{\partial}{\partial t} \to D_0$$

and the current $\vec{J} \sim \psi^*(\nabla\psi) - (\nabla\psi)^*\psi$ also becomes gauge invariant

$$\psi^{*\prime}(\vec{D}'\psi') = \psi^*\exp(-iq\chi)\exp(iq\chi)\vec{D}\psi$$
$$= \psi^*(\vec{D}\psi)$$

Could we reverse the argument?

When we demand that a theory is invariant under a space-time dependent phase transformation, can this procedure impose the specific form of the interaction with the gauge field?

↳ Can symmetry imply dynamics?

QED - Our Best Theory

Free electron Lagrangian. $\mathcal{L}_e = \bar{\psi}(i\gamma_\mu\partial^\mu - m)\psi$

Tewei

Impose invariance under local phase transformation

$$\psi \to \psi' = \exp(i\alpha(x))\psi$$

Introduce photon field and coupling via covariant derivative

$$D_\mu = \partial_\mu + ieA_\mu \qquad A_\mu \to A'_\mu = A_\mu + \frac{1}{e}\partial_\mu \alpha(x)$$

This determines the interaction term with the electron

$$\mathcal{L}_{int} = -e\bar{\psi}\gamma_\mu \psi A^\mu$$

Introduce the free-photon Lagrangian

$$\mathcal{L}_A = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}, \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

And the QED Lagrangian comes out as

$$\mathcal{L}_{QED} = \bar{\psi}(i\gamma_\mu \partial^\mu - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - e\bar{\psi}\gamma_\mu \psi A^\mu$$

absence of photon mass terms $\frac{1}{2}m^2 A_\mu A^\mu$ for they break the gauge symmetry

Focus on the interaction we have proposed

$$-e\underbrace{\bar{\psi}\gamma^\mu\psi}_{J^\mu} A_\mu = -e\bar{\psi}^A(\gamma^\mu)_A{}^B\psi_B A_\mu$$

This is an interaction between a photon and two electrons, and is conveniently represented by

$$= -ie(\gamma^\mu)_A{}^B$$

of course just a simplification

Interpretation: $A_\mu \sim a + a^\dagger \quad \bar{\psi} \sim b^\dagger + c \quad \psi \sim b + c^\dagger$

Leads to transitions like

$$\langle\gamma|a^\dagger bc|e^+e^-\rangle \quad\text{or}\quad \langle\gamma e^-|a^\dagger b^\dagger b|e^-\rangle$$
$$e^+e^- \to \gamma \qquad\qquad e^- \to e^-\gamma$$

strictly, need a fourth particle to absorb momentum, but can occur as a "virtual" process

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Testing QED - Anomalous Magnetic Dipole Moment

Dirac's

$$i\hbar \frac{\partial \psi}{\partial t} = \left[c\vec{\alpha} \cdot \left(\vec{p} - \frac{e}{c}\vec{A}\right) + \beta mc^2 + e\phi \right] \psi$$

Pauli equation for the "large component" of the spinor:

$$i\hbar \frac{\partial \xi}{\partial t} = \left[\frac{\vec{p}^{\,2}}{2m} - \frac{e}{2mc}\left(\vec{L} + g_e \vec{S}\right) \cdot \vec{B} \right] \xi \qquad g_e \approx 2$$

$g_e \longrightarrow$ gyromagnetic factor

Anomalous magnetic dipole moment (a_e):

$$a_e = \frac{g_e - 2}{2}$$

Pauli: first-order prediction ("tree level"): $a_e = 0$

Dirac: higher-order contributions ("loops") $a_e \neq 0$

a_e receives contributions, in principle, from all interactions

$$a_e = a_{QED} + a_{EW} + a_{HAD} + a_{NEW} \quad \longrightarrow \frac{\alpha}{\pi} ?$$

QED's contribution can be written as a series in (α/π).

$$a_{QED} = \sum_{n \geq 1} A_n(\ell) \left(\frac{\alpha}{\pi}\right)^n + \sum_{n \geq 2} B_n(\ell, \ell') \left(\frac{\alpha}{\pi}\right)^n$$

A_n: dimensionless and universal (don't depend on the lepton flavour)

$A_1 = 0.5$

$A_2 = -0.328478965$ — 7 diagrams, 1950(w), 1958

$A_3 = +1.181241456$ — 72 diagrams, 1996

$A_4 = -1.91298(84)$ — 891 diagrams, 2003

$A_5 = +7.795(336)$ — 12672 diagrams, 2014 omei

Measurement of a_e

Penning trap: magnetic trap at low temperatures

The spin flip frequency for a given magnetic field is related to g_e

0.00119(5) 4.2% 1947

0.0011596521884(43) 4ppb 1987

$a_e^{theory} = 0.00115965218164\,3(763)$ } nine significant

$a_e^{exp} = 0.00115965218073(28)$ } digits

Basic Structure of SM

- 1+2 gauge interactions
  - $SU(3)_c$ strong (aka QCD)
  - $SU(2)_L \times U(1)_Y$ electroweak (EW)
  - electroweak symmetry breaking (EWSB)
    weak interactions and EM are observed as separated phenomena at low energies
- two kinds of matter particles
  - quarks subject to all three interactions
  - leptons subject to EW interaction only
- gauge mediators
  - photon (γ) for ~~the~~ EM
  - W^+, W^-, Z^0 for weak interactions
  - gluon (g) for strong interactions
- Scalar field (ϕ) / Higgs boson (H)

Basics of QCD

- Symmetry group is $SU(3)_c$
- three quark colors: R, G, B — transform under the fundamental representation of SU(3): a triplet
- quantum of the gauge field: gluon
  - eight bicolored varieties (color + anticolor)
  - transform under the adjoint representation of SU(3) - the eight generators of SU(3)

- since the gluons can carry color themselves, they can self-interact
  - there are qqg, ggg and $gggg$ vertices in the theory.
    Compare with the single $ee\gamma$ vertex in QED
- the theory is <u>renormalizable</u>
  - when making higher order calculations in QFT, we encounter divergences
  - renormalization is a collection of techniques to address those divergences
  - observables remain finite (renormalized); "bare" parameters in $\mathcal{L}_0$ are formally infinitesimal
    - in QFT we also learn how to do it with renormalized parameters from the start
  - a non-renormalizable theory is not amenable to standard perturbative calculations
- a price to pay: coupling constant α_s depends on interaction energy scale Q

QCD Running Coupling - Asymptotic Freedom and Confinement

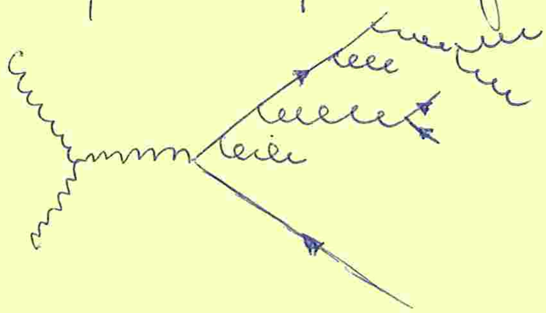
- the presence of gluon self interactions + qqg vertex leads to an expression for $\alpha_s(Q^2)$

$$\alpha_s(Q^2) = \frac{\alpha_s(\mu^2)}{1 + \frac{\alpha_s(\mu^2)}{12\pi}(33 - 2\underbrace{n_f}_{\text{number of flavours}})\log\left(\frac{Q^2}{\mu^2}\right)}$$

μ^2: renormalization scale

- asymptotic freedom: for high Q^2 (short distances), α_s becomes very small → quarks become quasi-free
- confinement: for low Q^2, α_s becomes very ~~high~~ large → no isolated quarks
- hadrons: colorless bound states
  → mesons: $q\bar{q}$
  or
  barions: qqq

- when you calculate a process via Feynman diagrams, you assume the initial and final states are free particles, but there is no such thing as a free quark (or gluon)
- quarks and gluons - partons - are bound inside hadrons, but in that state they are quasi free! The parton distribution function (pdf) $u^P(x)$ gives the prob of having a parton of type u inside the proton
- final state q's and g's radiate/branch and their energy gets diluted in a parton shower. The branchings are primarily soft and colinear - after a given point the process has to be treated non-perturbatively (high α_s)
- eventually, the whole system changes phase into a set of hadrons. Hadrons that come from a parton keep its original direction, forming a ~~parton~~ hadronic jet.



Basics of the Electroweak Model

- symmetry group is $SU(2)_L \times U(1)_Y$
- quarks come in six flavours:

  u, d, c, s, t, b
- leptons come in six flavours:

  $e, \nu_e, \mu, \nu_\mu, \tau, \nu_\tau$
- Left-handed particles ψ_L form a weak isospin doublet $(\uparrow, \downarrow)$. Right-handed particles ψ_R are weak isospin singlets. All particles have also a hypercharge Y
- the quantum for the $SU(2)_L$ gauge field are the weak bosons W_1, W_2, W_3; for the $U(1)_Y$ field it is the B boson
- again the theory is renormalizable

... and this has nothing to do with the real particles we talked about previously! Notice that

- the Lagrangian can't have fermion mass terms: $\bar{\psi}\psi = \bar{\psi}_R \psi_L + \bar{\psi}_L \psi_R$ has mixed symmetry
- The W_i, B bosons are massless, whilst the weak bosons are massive

Electroweak symmetry breaking

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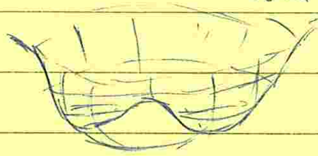
- add to the Lagrangian a complex scalar field ϕ



$$\mathcal{L}_{scalar} = |D_\mu \phi|^2 - \mu^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2, \text{ with } \phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

- ϕ is an $SU(2)_L$ doublet, with hypercharge suitably chosen
- choose μ, λ such that the vacuum expectation value of ϕ is not zero
- the ground state of ϕ is now assymmetric, but the system as whole still is. The $SU(2)$ symmetry is broken (hidden)
- rewrite ϕ as

$$\phi(x) = \exp\left(i \frac{\sigma_i}{2} \theta^i(x)\right) \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}$$

rewrite $\mathcal{L}$ and mass terms appear (after field rotation by angle θ_W) for W and Z bosons

→ Yukawa couplings of the form $\bar{\psi}\psi\phi$ give mass to fermions as well

The Higgs Boson

- one last field remains in the theory after EWSB; its quantum is the Higgs boson
- its mass is <u>not fixed</u> from low energy Physics
  - → fine structure α, Fermi's G_F, Weinberg angle θ_W fix all other terms in the Lagrangian
- Higgs properties are exquisitely dependent on its mass
- discovery on July 4th 2012 by the ATLAS and CMS collab
- all properties as expected by the SM, $m_H = 125.2$ GeV

High Energy Hadron Collisions

Full recipe for calculations

- calculate hard matrix elements from perturbative QFT

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- embed initial state partons in protons via structure functions
- add corrections for higher order + non-perturbative processes to the process
  - → initial and final state radiation
  - → underlying event → i.e. what happens to the rest of the hadron?
  - → hadronisations and decays of unstable particles

② Accelerators and Detectors

Why accelerate?

$E = h\nu$ → smaller distances

$E = mc^2$ → new massive particles (e.g. μ, π, etc)

Collider vs Fixed Target → HEP

$$E_T = \left[\cancel{m_1^2 c^4}^{\sim 0} + \cancel{m_2^2 c^4}^{\sim 0} + 2\left(E_1 E_2 - \vec{p}_1 \cdot \vec{p}_2 c^2\right)\right]^{1/2}$$

Fixed Target

$E_1 = E_B$ and $E_2 = m_A c^2$ Solid iron $8.5 \cdot 10^{28}$ atoms/m^3

$$E_T = \left[2 m_A c^2 E_B\right]^{1/2}$$

Collider

$E_1 = E_B$ $E_2 = E_B$ $\Rightarrow$ $|\vec{p}_1| = |-\vec{p}_2| \approx E_B/c$

$E_T = 2E_B$ beam: 1 proton/m^3

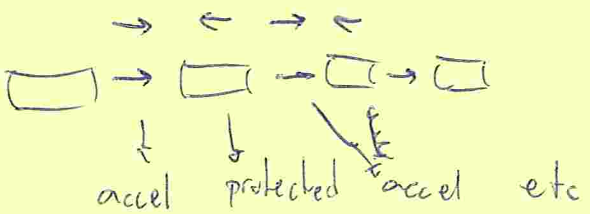
Energy or Luminosity or Simplicity or Cheap, etc

Linear Accelerators

→ energy limited by lenght

→ beam lost after collision

Accel: drift tubes inside RF caves



Circular Accel - Synchrotron
- → $\vec{B}$ bends and $\vec{E}$ accelerates ↗ synchronized with velocity
- → pp, ep - different sets of magnets
- → $p\bar{p}$, e^+e^- - same set, one tube
  - → need antiparticles
    - · positron: easy, pick from light on material $e^-\gamma \to e^- e^+ e^-$
    - · $\bar{p}$: hard, proton-nucleus collisions

Accelerator Components
- RF Cavities
  - → niobium
  - → Int Linear Collider plans for 35 MV/m
  - → Lenght for 500 GeV beams? → large
- Magnets
  - → dipoles (bending)
    - super conducting: 14.3m long, 8.35 T
    - proton energy: 7 TeV → min ring circ?
  - → quadrupoles (focusing)
    - impossible to focus on both directions!
      - → FODO cell
        - → alternate both and focus overall

Synchroton Radiation
- → accelerate ⇒ radiation
- → particles lose energy when circling $\Delta E = \frac{4\pi e^2 \beta^2}{3R}\left(\frac{E}{m}\right)^4$
- → energy limit for e^-e^+ machine to $\lesssim$ 100 GeV/beam ↳ light radiates much more
  - → higher energy machines (Tevatron, LHC) are hadron colliders
- → synchroton radiation is useful on its own, though
  - ↳ HE photons for material studies

Tomi

Cern Complex

One accel. feeds the next one $\rightsquigarrow$ more cost-effective

Particle Detection

Some $\exists$ long enough to fly and reach detector

- p, e, γ, ν are stable

  ν hardly interacts

- $n, \mu, K^0_L, \pi, K^\pm$ live for ≥ 10 ns

  $10\text{ ns} \cdot c = 3\text{ m}$

Two divisions needed

Charged $(p, e, \mu, \pi, K^\pm)$ or neutral (γ, ν, n, K^0_L)

Hadrons $(p, n, \pi, K^\pm, K^0_L)$ vs others (e, γ, μ, ν)

Other particles must be reconstructed

$Z \to e^+e^-$, $t \to Wb \to \mu\nu b$

To detect, we must make parts interact with bulk matter

After interaction, we have electric/light signals to reconstruct the particles from data is another problem, that we will deal with later

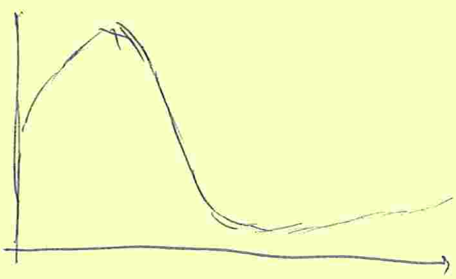
Energy Loss of Charged Heavy Particles

$\to$ dominant: collision with atoms (mainly excitation and ionization)

Bethe-Bloch formula $\to$ $\left(\frac{dE}{dx}\right)_{coll}$ $\rightsquigarrow$ how far can the particle go before it stops?

$\hookrightarrow$ good description for

$0{,}1 < \gamma\beta < 10^3$



En. Loss of Charged Light Particles (e^-, e^+)

$\hookrightarrow$ bremsstrahlung effect becomes relevant

$\hookrightarrow$ dominant process for high energy (> 1 GeV)

Bremsstrahlung Approx and Radiation Lenght

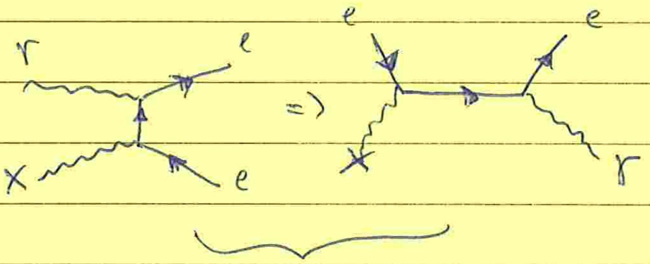
$$E(x) = E_0 \cdot \exp\left(-\frac{x}{X_0}\right)$$

↗ comes from approx @ HE for en. loss

X_0: radiation lenght

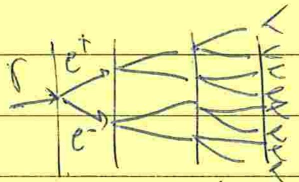
Interactions of Photon - LE
↳ photoelectric effect
↳ Compton scattering

HE
→ pair production



pair production has to be related to bremsstrahlung due to symmetries of QED

electromag. shower



PP BS PP BS ...

Interaction of Hadrons with Matter
: hadron int. with nucleus
→ elastic σ_{el}
→ inelastic σ_{in}
→ total $\sigma_{tot} = \sigma_{el} + \sigma_{in}$

dom. by σ_{in}
$\sigma_{el} \sim 10\,mb$
$\sigma_{in} \propto A^{2/3}$ ↗ atomic mass

$$\sigma_{tot} = \sigma_{tot}(pA) \approx \sigma_{tot}(pp) \cdot A^{2/3}$$

hadronic int lenght: $\lambda_{int} \sim A^{1/3}$ ↗ characterises longitudinal and transverse profile of hadronic shower

$$N = N_0 \exp\left(-\frac{x}{\lambda_{int}}\right)$$

$$\frac{\lambda_{int}}{X_0} \sim A^{4/3} \rightarrow \lambda_{int} \gg X_0$$

↳ hadronic calorimeter needs to be larger (# layers) than EM calorimeter

Scintillators

↳ converts energy from a particle passage into light that will be collected by a photosensor

↳ possible due to fluorescence ↗ e^- goes from high en. level to low

↳ has to be transparent to its own light, the conversion must be efficient and light must be detectable by photosensors

Types and applications

→ part. counter

→ image display

→ en. measurement at very high rate

→ ~~Magge~~ trackers

Gas Chambers

charged particles ionise the gas

electrons drift through the gas and generate electric signal

↳ external $\vec{E}$ applied

gain and relevant effects

→ gain improved by avalanche of ions and e^- from secondary ionisation

→ relevant effects

- recomb. and e^- attachment
- delta effects
- diffusion
- mobility of charges
- avalanche process

Semiconductor Detectors

↳ radiation damage resistant

↳ µm of precision, but high cost

↳ signal generated by passage of charged particle, produces e^--hole pairs

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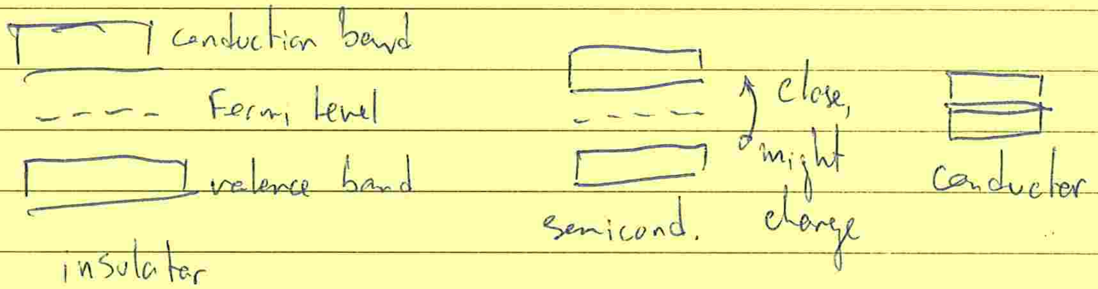
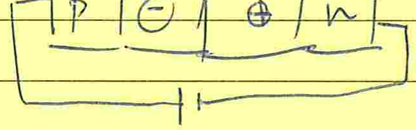
Energy Bands



Silicon lattice: add another element (Sb or B, eg) for doping

n-type add a e^-

p-type add a hole



μm precision in measurement

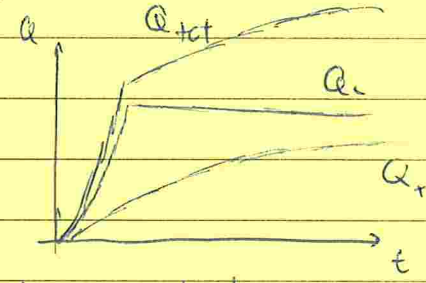
~~Charge Carrier Mobility + Drif Vel.~~ Charge Carrier Mobility and Drif Vel.

$v_D \sim 10^7$ cm/μs ↝ Saturation

For v_D typical in 100 μm, drift time ~ 10 ns

when charged particle passes through det, generates several electron-hole pairs

Signal Rise Time
- ~1 ns for Si det.
- → loss of charges
- → $\frac{\vec{v}}{\vec{E}} \neq$ cte
- → charge distributed on surface, might have mismeasurement



Tracker
- → charged particles, non-destructive
- → medium produces signals as particle passes
- → strong $\vec{B}$ to determine charge and momenta

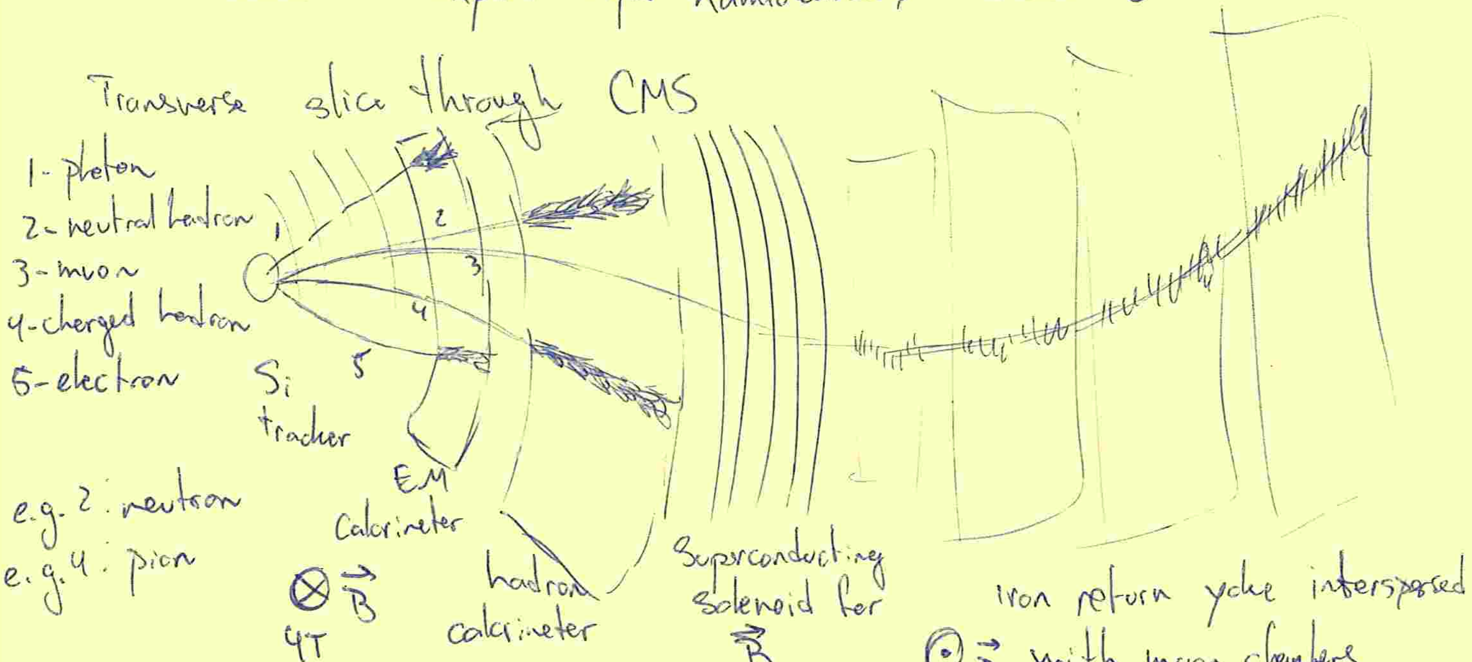
types: at least 2
- → Semiconductor → high rates, close to int point, expensive
- → gases → low rates, far from int point, cheap

Calorimeter
→ charged or neutral particles, destructive (totally absorbs part. after shower)
→ dense material to produce interaction
→ active material to produce measurable quantity

2 types:
sampling: alternates passive absorbers and active detectors
homogeneous: absorber and active detector @ same time

Integrating it all
→ we wish $(E, \vec{p})$, q, m of all in collision
→ non-destructive before destructive
→ observe passage without disturbing (small X_0 and λ_{int})
→ track part. and measure $\vec{p}$
→ shower in calorimeter and measure energy
→ special case 1: muons
→ very high range; 2m of Cu to stop 20 GeV muon
→ alternative: calorimeter before tracker (slowing down and leaving only muon)
→ special case 2: neutrinos
→ only weak interaction → small σ
→ dedicated exps: Super-Kamiokande, MiniBooNE

Transverse slice through CMS



e.g. 2: neutron
e.g. 4: pion

$\otimes \vec{B}$ 4T

2T $\odot \vec{B}$
↳ $\otimes$ comes back outside solenoid

transducer

③ Data Reconstruction

↳ we need to convert the energy deposited on det. to electric signals and, usually, digitalise the signals for further data processing

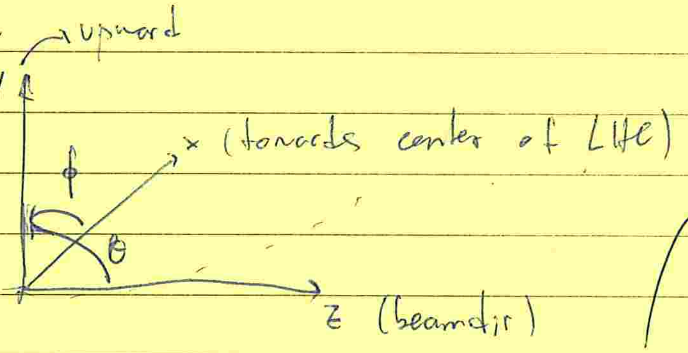
- local reconstruction
  - starts with det. readout and transform into intelligible data - a "hit"
  - low level info (tracker)
  - not much more than "charged particle passed through this sensor"
- global reconstruction
  - picks hits and tries to group @ higher level
  - reconstruct hits to obtain a trajectory or the deposit of a photon, as an example
- global event description
  - uses high-level objects to describe event as a whole
  - Connecting trackers, clusters, etc to reconstruct an electron, a muon, etc

Coordinate System

$$\eta = \log\left(\tan\left(\frac{\theta}{2}\right)\right)$$

$$\rho = \sqrt{x^2+y^2}$$

pseudorapidity



unknown $\vec{P}_0$

system: $\{\rho, \phi, \eta\}$ → for hadron colliders

Local Reconst. - Calorimeter as e.g.

↳ depends on implementation details

e.g. CMS's crystal EM cal. (ECAL)

→ around $8 \cdot 10^4$ $PbWO_4$ (lead glass) scintillating crystals

→ dense ($8.3\,g/cm^3$), short rad. lengths (0.89 cm), small Mol. radius (2.2 cm)

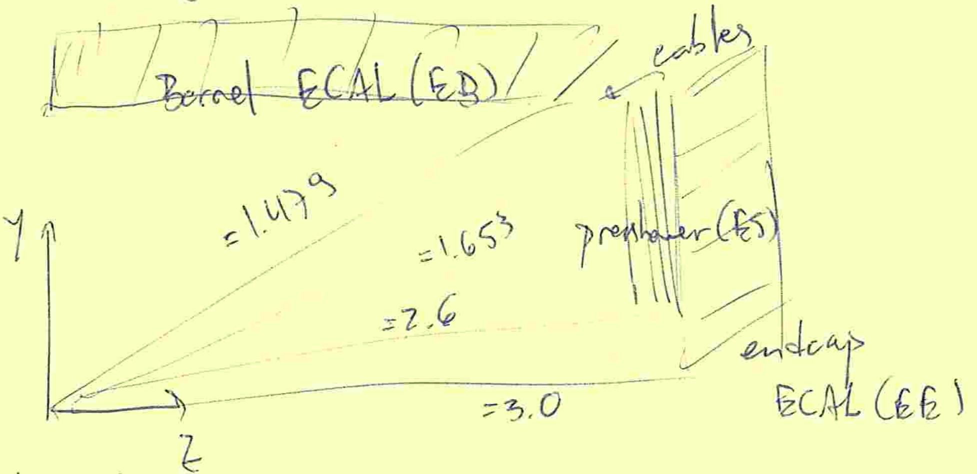
→ 80% of light emitted in 25 ns, ~420 nm, 4.5 photoel./MeV

Tomei

Readout: avalanche photodiodes (APDs) and vacuum phototriodes (VPTs) for barrel ($|\eta| < 1.479$) and endcap ($1.479 < |\eta| < 3$) regions, respec.

→ APD: exploit photoel. eff. in an avalanche mode diode to convert light → elect.

→ VPT = single-stage photon multipliers



ECAL Readout Chip

• ECAL readout & precision performance reqs.: front end signals digitized to better than 12 bits

. strategy: many gain ranges (×12, ×6, ×1) to span overall dynamic range

Digitise & transmit signals only for highest unsaturated range

12 bit Analog-to-digital converter (ADC) is now enough

parallel gain channels in multi-gain pre-amplifier (MGPA) coupled to multi-channel ADC

channel-in-range decision taken by digital logic following conversion stages

→ radiation damage ⇒ transparency loss! → need recallibration twice a week

ECAL Local Reconstruction

Hit reconstruction → estimation of $(E, t, \vec{x})$ of energy deposit in ECAL crystal

@ a given time sampling, 12 bit word for ADC counts + bits to encode gain (×12, ×6, ×1). For given event we want to save, we take 10 time samples, 3 readings before selected bunch-crossing + 7 after → this is called a "digi

Toni

From an ECAL "digi" we want to get: amplitude (peak of pulse shape), pedestal (baseline), jitter (time of pulse max) and χ^2 of the fit. After we want a global scale factor (ADC counts to GeV) and inter calibration constants

Reconstruct the pulse: multifit method allows for better energy information

Calibrate the Energy

→ use particles of very well known mass to calibrate detector

$\pi^0 \to \gamma\gamma$ mass at 135 MeV

$Z \to ee$ mass at 91.1876 GeV

→ exploit ϕ symmetry to intercallibrate crystals

→ exploit $E/p \simeq 1$ for HE electrons

Global Reconstruction: e.g. Tracker

Track reconstruction → using hits to obtain estimates for $\vec{p}$ and $\vec{x}$ of charged parts. responsible for the hits

↳ translate local position (coordinates) to global

↳ misalignment of detector elements ⇒ additional uncertainty

Traditionally two steps

→ track finding: decide within set of reconst. hits $\{x_i\}$ which subset makes a given track candidate &

→ track fitting: from subset of hits that give a candidate estimate parameters - production vertex position $\vec{x}_v$ and momentum @ vertex $\vec{p}_v$ - of originating particle

↳ connect the dots extreme pro edition

Motion in $\vec{B}$

$$\frac{d\vec{p}}{dt} = e\vec{v}\times\vec{B}$$

$\vec{B}$ does no work

$$m_0\gamma\frac{d\vec{v}}{dt} = e\vec{v}\times\vec{B}$$

$$m_0\gamma\frac{d^2\vec{r}}{dt^2} = e\frac{d\vec{r}}{dt}\times\vec{B}$$

use path lenght along track, s, instead of time t

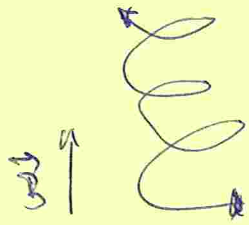
$ds = v\,dt$

Tonei

$$m_0 \gamma v \frac{d^2\vec{r}}{ds^2} = e \frac{d\vec{r}}{ds} \times \vec{B}$$

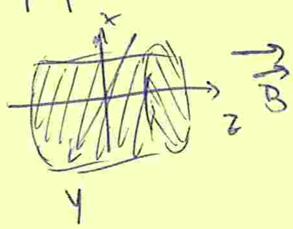
$$\frac{d^2\vec{r}}{ds^2} = \frac{e}{p} \frac{d\vec{r}}{ds} \times \vec{B}$$

Inhomogeneous $\vec{B}(s)$: solve diff. eq.
Homogeneous $\vec{B}(s)$: helix trajectory

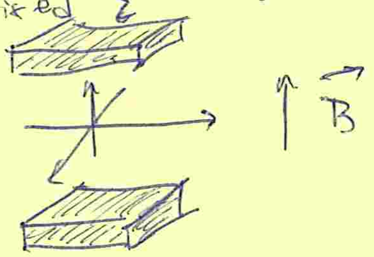


Magnetic Spectrometer

Solenoidal
- cylindrical
- deflection in x-y (ρ-ϕ) plane
- tracking dets. arranged in cyl. shells
- measurement of curved trajs. in ρ-ϕ planes at fixed ρ



Dipole
- rectangular
- deflection in y-z plane
- track. det in parallel planes
- meas. curved traj. in y-z planes @ fixed z



The Helix Equation

Parametric form

$$x(s) = x_0 + R\left[\cos\left(\Phi_0 + \frac{hs\cos\lambda}{R}\right) - \cos\Phi_0\right]$$
$$y(s) = y_0 + R\left[\sin\left(\Phi_0 + \frac{hs\cos\lambda}{R}\right) - \sin\Phi_0\right]$$
$$z(s) = z_0 + s\sin\lambda$$

λ: dip angle

$h = \pm 1$: sense of rotation

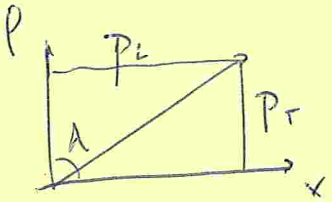
proj on x-y plane: circle

x_0, y_0, z_0: coord @ $s=0$

Φ_0: slope of tangent to circle @ $s=0$ → initial angle

In x-y, we can relate the curvature radius R to transverse momentum p_T as

$$R = \frac{p_T}{0{,}003\,qB}$$



Toni

Methods of Track finding

Conformal Mapping

Circle going through origin $(x-a)^2+(y-b)^2 = R^2 = a^2+b^2$

$$u \equiv \frac{x}{x^2+y^2}, \quad v \equiv \frac{y}{x^2+y^2} \Rightarrow v = \frac{1}{2b} - u\frac{a}{b}$$

How to obtain track candidates

- convert $(x,y) \to (u,v)$
- in (u,v) coord system, larger momentum $\Rightarrow$ line closer to origin

→ approx line as going through origin - hits of a track have constant ϕ_v

in practice: search for peaks in the histogram of ϕ_v variable of all hits

Hough Transform

$$R = \frac{p_T}{0.003\, qB}$$

we are interested in parts originating @ or close to int. point

Trajectory of such in transverse plane is given by

$$\frac{r}{2R} = \sin(\varphi - \phi) \approx \varphi - \phi$$

$$\phi = \Phi_0^{\text{helix}} - \frac{\pi}{2}$$

ϕ: angle of track @ transverse plane @ origin

small angle approx valid for $p_T \gtrsim 2\,\text{GeV}$ tracker

Combine eqs.

$$\phi = \varphi - \frac{0.0015\, qB}{p_T} \cdot r$$

One hit in (r,φ) maps to straight line in track parameter space $(q/p_T, \phi)$

If several hits produced by same part, lines intersect @ same point

→ locate intersection to identify track

→ intersect. coords. provide track parameters

→ caveat: grad of each line $\propto$ hit radius r - always positive

better use r_T, ϕ_T instead: $r_T = r_0 - T$, $\phi_T = \phi_{track}(r_T)$

↳ r_T can have both signs → larger range of line grads and + precision @ measurement of intersect point

Tonei

Track Road & Following → local, start from a subset of hits

→ Track Road

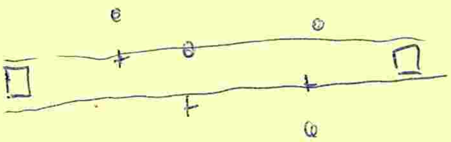
start from hits that could be from same part. Use expected traj. shape to interpolate amongst hits, creating a road around the trajectory. All hits fall in the road to constitute track candidate

→ Track following:

start from "track seed" → subset of hits that for some reason are designed to be + accurate. From seed, extrapolate until next det element containing a hit. The closest becomes part of track candidate. Iterate until

1) too many layers have missing hits

2) end of det system is reached



Elements of Track Fitting

· track parametrization: depends on geometry

↳ helix traj in solenoid: $\{x_0, y_0, \Phi_0, \lambda, R\}$

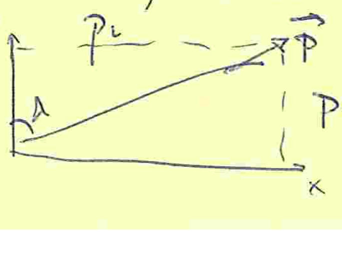
· track model: how parameter or state vector $\vec{q}$ @ given surface k depends on state vector at a diff. surface i: $\vec{q}_k = G_{k|i}(\vec{q}_i)$, where $G_{k|i}$ is the track propagator from surface i → j

· error propagation: during track param. estimation procedure, propagation of the track parameter cov. matrix along with parameters themselves is often requested

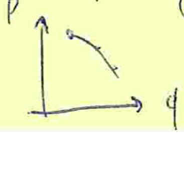
· material effects: ionization en. loss, multiple Coulomb scattering, bremsstrahlung, all affect the part. prop.

· measurement model: $\vec{h}_k$ describes functional dependence of measured quant in layer k, $\vec{m}_k$ on the state vector @ same layer: $\vec{m}_k = \vec{h}_k(\vec{q}_k)$

Momentum Measurement

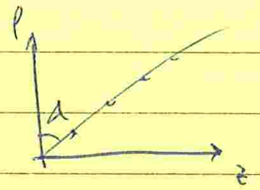


ρ-φ plane: transverse momentum p_T measured through curvature radius R



$p_T = p \cos\lambda = 0.003\, B R$

In ρ-z plane, we measure dip angle λ

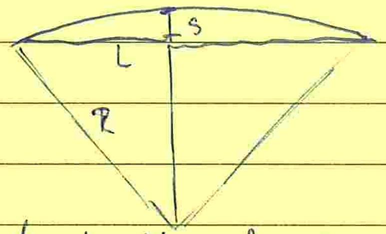


Orders of magnitude

$p_T = 1\ \mathrm{GeV},\quad B = 2T,\quad R = 1.67\ m$

$p_T = 10\ \mathrm{GeV},\quad B = 2T,\quad R = 16.7\ m$

The sagitta s



$$R^2 = L^2 + (R-s)^2$$

$$R = \frac{s}{2} + \frac{L^2}{2s}$$

$$s \ll R \Rightarrow R \approx \frac{L^2}{2s}$$

Assuming track length of 1 m: $p_T = 1\ \mathrm{GeV} \Rightarrow s = 7.4\ cm$

$p_T = 10\ \mathrm{GeV} \Rightarrow s = 0.74\ cm$

Having p_T and λ, the total momentum is

$$p = \frac{p_T}{\cos\lambda} = \frac{0.003\ BR}{\cos\lambda}$$

The error in the momentum is easily calculated

$$\frac{\partial p}{\partial R} = \frac{p_T}{R}$$

$$\frac{\partial p}{\partial \lambda} = -p_T \tan\lambda$$

$$\left(\frac{\Delta p}{p}\right)^2 = \left(\frac{\Delta R}{R}\right)^2 + (\tan\lambda\, \Delta\lambda)^2$$

Need to study

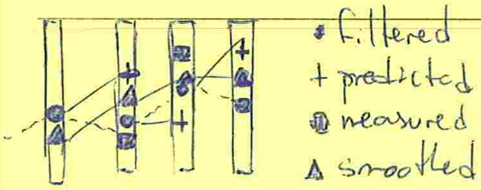
→ error in radius measured in bending plane ρ-ϕ

→ error on dip angle

→ contrib. of multiple scattering to $\vec{p}$ resolution

Hadronic collider: main emphasis on transverse momentum p_T. Since elementary processes happen amongst partons not @ rest @ lab frame, we employ $\vec{p}$ cons. only in transverse plane

Kalman Filter



- to start the KF we need a "seed"
- the position on next plane is predicted
- the measurement is considered
- pred and meas. are merged (filtered)
- iterate: pred. - meas. - filt. - ...

Tone:

Filtering: weighted $\bar{x}$ of new measurement x_n and new pred x_p
Smoothing: refine each position estimate with info from all estimates

Kalman Filter - The Equations

Starting point
- initial set of parameters (seeds): first measurements

Dimensions
- P parameters for track model
- D meas. $\vec{m}_k$ @ each point (usual. D < P)
- K meas. points ($N = K \times D$)
- state of system @ index k is $\vec{q}_k$

Iterative method
- Using system eq. $\vec{q}_k = \vec{G} \cdot \vec{q}_{k-1} + \vec{w}_k$
  - $\vec{G} = P \times P$ matrix, $\vec{w}$ perturb. assoc. with cov. matrix $V_{\vec{w}}$ $(P \times P)$
- Add new point, update param & cov using meas. eq. $\vec{m}_k = \vec{H} \cdot \vec{q}_k + \vec{\epsilon}_k$
  - $\vec{H} = D \times P$ matrix, $\vec{\epsilon}$ meas. error assoc. with diag cov $D \times D$ matrix V_m
  - weighted $\bar{x}$ of pred + meas. using variance $\Rightarrow \chi^2$ fit

$$\vec{q}_k = \left(V^{-1}_{k|k-1} \vec{q}_{k|k-1} + H^T V^{-1}_{m_k} \vec{m}_k\right)\left(V^{-1}_{k|k-1} + H^T V^{-1}_{m_k} H\right)^{-1}$$

Global Event Description - aka Particle Flow
- all subsystems provide basic elements
- basic idea: correlate basic elements to identify <u>all final-state particles</u>. Combine corresponding meas. for ultimate accuracy in part properties
  - → initially developed and used by ALEPH (LEP)
  - → not used successfully in hadron colliders - detec granularity was not fine enough
- complicated algorithm that implements a set of links
  - e^- leave both tracks + EM cal deposits, γ only the latter
  - μ leave tracks in both Si tracker + μ spectrometers
  - charged hadrons leave tracks and h. calor. deposits, neutral only the latter
  - e^- emit γ during bremsstrahlung, all E must be reclustered
  - a hadronic τ decay produces just a few hadrons + 1 ν
- the output of the PF algorithm is a list of final-state reconstructed particles

Hadronic Jets & Missing p_T

↳ quarks & gluons fragment & hadronize => colimated high-momentum spray of hadrons flying roughly in the same direction

↳ hadronic jet

Jet algorithm: iterative tuneable process that clusters close particles & creates jet objects

Can also cluster all observed particles

↳ important cross check: total transverse momentum in hadron collision should be zero

↳ if after the overall clustering a sizeable p_T remains, that would be evidence for an undetected part being produced

Missing transverse momentum $\vec{p}_T^{\,miss}$:

$$\vec{p}_T^{\,miss} = -\sum_i \vec{p}_T^{\,i}$$

Online Reconstruction

→ collision rate close to 40MHz

↳ impossible to record & store all collisions @ LHC

→ Trigger system, only more interesting collisions are kept

↳ quasi-real time reconstruction

→ Level 1 (L1) trigger in CMS

→ customizable hardware (ASICs and FPGAs)

→ output rate: 100 kHz (det. readout constraint)

→ timing: 4μs (available buffer)

→ coarse granularity

→ High Level Trigger (HLT) in CMS

→ PC farm with light version of the offline software

→ output rate: 1kHz average over the LHC fill

→ timing: $\lesssim 380$ μs

→ full detector granularity

④ Data Analysis

Broad terms: 2 camps of LHC data analysis

- Measurements
  trying to measure some known S.M. quantity
  e.g. Higgs boson mass M_H, fully inclusive top quark production @ LHC @ 14 TeV, $\sigma(pp \to t\bar{t} + \text{anything})$
- Searches
  tries to uncover discrepancies between SM predictions & observed data
  e.g. searches for resonances, SUSY, dm...

This distinction is a bit artificial

- $H \to \mu\mu$ @ 13 TeV LHC not observed yet, but well defined prediction → we call it a search
- when searching for a new resonance, $pp \to Z' \to ee$, we usually make many assumptions of the values of mass, spin, etc
  Then try to measure production cross-section - usually comes up statistically compat. w/ zero

What do we actually measure?
↳ count # of events in a given config.

$$N_{obs} = \sigma \cdot \mathcal{L} \cdot A \cdot \epsilon$$

observed # of events for a given process

σ: production cross-section → purely theoretical-driven

$\mathcal{L}$: ↳ accel. luminosity ↳ how many parts we may hit through given space in given time

A: acceptance: ratio of detectable parts that actually go into det. volume ↳ technically, also theoretically-driven

ϵ: selection efficiency → $\epsilon = \epsilon_0 \cdot \epsilon_a$ subdivided

- ϵ_0: eff. of reconstructing a given set of objects @ detector
- ϵ_a: eff. of any further requirements done in the analysis

Given config.: determined by acceptance · efficiency product
Total number of events we observe in config is

$$N_{obs} = \sum_{i} N^{i}_{obs} + N_{Bonus}$$

$\sum_i$: all physics process that produce events in configuration

N_{Bonus}: ↳ non-collision effects

Some origin of N_{fakes}
- cosmic rays going through detector (Cosmic muon)
- longitudinal particles from beam interactions with accel. elements (beam halo)
- hardware failures. "hot cells", dead channels, high voltage spikes
- software failures. e.g. unusual config of hits leading to high number of fake tracks
- phase of moon (?) → Nucl. Instrum. Methods Phys. Res., A 357 (1995), 249-252
- seasonal variation of rain fall (???) → Proceedings of the 1999 Particle Accelerator Conference, New York, 1999
- schedule of French high-speed trains (?????) → Nucl. Instrum. Methods Phys Res., A 417 (1998) 9-15

Signal & Background

Since $\sum N^i_{obs}$ goes over all relevant physics process, we can't readily separate the process we want (signal) from the others (background)
- irreducible backgrounds
  share exact same final state as signal
  eg. $pp \to \gamma\gamma$ is irreducible background to $pp \to H \to \gamma\gamma$
  only option: model as well as possible
- reducible backgrounds
  final state of background ≠ signal, but due to various reasons end up being selected for analysis
  e.g. $Z \to ll$ is back. for $ZZ \to llqq$ search
  ↳ could appear as "dilepton + jet" final st. and jet could be mistaken to be $Z \to qq$ of latter

Separating Signal & Background

→ first task: search for observables that are differently distributed for signal & background

→ Some searches are easy: for $Z \to ll$: other searches are harder
- 2 high-p_T leptons
- same flavour, opposite charge
- invariant mass M_{ll} in 70-110 GeV range

Razor variables used for SUSY searches

$$M_R \equiv \sqrt{(|\vec{p}^{j1}| + |\vec{p}^{j2}|)^2 - (p_z^{j1} + p_z^{j2})^2} \quad \& \quad R^2 = \left(\frac{M_T^R}{M_R}\right)^2$$

$$M_T^R = \frac{1}{\sqrt{2}}\sqrt{p_T^{miss}(p_T^{j1} + p_T^{j2}) - \vec{p}_T^{\,miss} \cdot (\vec{p}_T^{\,j1} + \vec{p}_T^{\,j2})}$$

Tomi:

Machine Learning in Analysis

- Classification problems
  - output var. takes class labels
    - eg classify as signal or background
    - usually unpack classification & use output var. directly
- Regression problems
  - output var takes continuous values
    - e.g. energy of b-jet based on its kinematics + flavour content

Cut Based Analysis

- design set of cuts by as unbiased a procedure as possible
  - blind study → dont look at data until you have frozen the analysis
    ↳ avoid observer + confirmation bias
- choose cuts that optimise final overall accuracy of result
  - difficult tradeoff between stat. & syst. uncertainties
- always study marginal effect of each cut with tables & plots
  - cuts with no marginal effect (remove no events after all other cuts) are quite useless

Background Estimation

Some background always remains (or, if dont, there is uncertainty). How to estimate it?

- Fully trust simulation: not recommended, except maybe for backs. so small that even a large error would make no difference
- Trust "data corrected" sims. usually done by def. control regions ("side-bands") in which you expect similar behaviour of your background but near-absence of your signal
- Model back. "in situ": same, but your sidebands act simultaneously as meas. regions for some of the modality of search

Basic Concepts in Probability

- given all possible exclusive, elementary events X_i, prob of occurence of X_i is $P(X_i)$ and follows Kolmogorov axioms.
  - $P(X_i) \geq 0, \ \forall i$
  - $P(X_i \text{ or } X_j) = P(X_i) + P(X_j)$
  - $\sum_{\Omega} P(X_i) = 1$

· Frequentist prob

observe N events & n are of type ~~X~~, the prob that any single event will be type X is the "empirical" limit of frequency ratio

$$P(X) = \lim_{N\to\infty} \frac{n}{N}$$

· approx prob by making N large

· exps have to be repeatable - means all <u>relevant</u> conditions ~~should~~ are same. Good science should produce reproducible results

· Bayesian prob

→ degree of belief in X. Operational def based on <u>coherent bet</u>

→ "The idea is to determine how strongly a person believes that X will occur by determining how much he would be willing to bet on it, assuming that he would be willing to bet on it, assuming that he wins a fixed amount if X does later occur, and nothing if it fails to occur. Then $P(X)$ is defined as the largest amount he would be willing to bet, divided by the amount he stands to win" [James 2006]

↳ follows Kolmogorov

→ property of observer + observed system - ~~will~~ may change if observer obtains more knowledge - <u>subjective</u> prob

→ helps addressing some questions we want to answer

"What is the prob. the Universe is cosmologically flat?"

"What is the prob that Higgs vacuum is stable?"

Bayes Theorem

Discrete: $P(A|B) = \dfrac{P(B|A)\,P(A)}{P(B)} \overset{\text{Bayesian}}{\Longrightarrow} P(\theta_i|X^0) = \dfrac{P(X^0|\theta_i)\,P(\theta_i)}{P(X^0)}$

Continuous random var.: $g(Y|X) = \dfrac{p(X|Y)\,h(Y)}{g(X)} \overset{\text{Bayesian}}{\Longrightarrow} p(\theta|X^0) = \dfrac{p(X^0|\theta)\,p(\theta)}{\int p(X^0|\theta)\,p(\theta)\,d\theta}$

~~pdf~~ prior prob density for θ ← major problem is here

posterior prob density for θ ← pdf

likelihood function $L(\theta)$, not a pdf

Example
- two subspecies of beetle: common "C" and rare "R"
- an entomologist spots what might be a rare subspecies of beetle due to pattern X on its back
- rare subspecies: 98% has patter, common subspecies 5% has pattern
- rare subspecies, 0.1% of population
- how likely it is for the beetle to be rare?

$P(X|C) = 5\%$, $P(X|R) = 98\%$, $P(X|I) = 0.98 \cdot 0.001 + 0.05 \cdot 0.999$

$P(R|I) = 0.001$, $P(R|X) = ?$

$$P(R|X) = \frac{P(X|R)\,P(R|I)}{P(X|I)} \simeq 1.9\%$$

The Prior Problems - M. Pierini (CERN)
- the need of priors in bayesian statistics is a ~~th~~ problem for some physicist
  - ↳ priors depend on choices made on unscientific basis
    - ↳ J. Charles et al. hep-ph/0607246
- the lack of priors in non-bayesian statistics is a problem for some statistician
  - ↳ can't deal with hypothesis because they are not observable and thus are not random variables for the frequentist
    - ↳ J. Press, Subjective and Objective Bayesian Statistics

Did the Sun Just Explode? (It's night, so we're not sure) - XKCD

Frequentist Statistician (FS): this neutrino detector measures whether the Sun has gone nova

Bayesian Statistician (BS): then, it rolls two dice. If they both come up six, it lies to us. Otherwise, it tells the truth

FS: Let's try. Detector! Has the Sun gone nova?

Detector: *Rolls* Yes!

FS: the probability of this result happening by chance is $\frac{1}{36} = 0.027$. Since $p < 0.05$, I conclude that the Sun has exploded.

BS: bet you \$50 it hasn't.

Practical Answers to Statistical Questions

· Point estimation: find a single value $\hat{\theta}$ that is "as close as possible" to the true parameter θ we want to measure ⇒ usually: maximum likelihood estimator

$$\frac{\partial \log L}{\partial \theta_i} = 0$$

↳ optimal in asymptotic limit of large N

→ in general, better report $L(\theta)$ itself, at least near maxima

· Interval estimation: find range $\theta_a \leq \theta \leq \theta_b$ that contains true value θ_0 with prob β

→ 1D: trivial ⇒ Neyman construction + Feldman-Cousins "unified approach"

→ ND: profile likelihood (MINOS) → bonus: allow removal of nuisance parameters μ by maximising full likelihood @ each value of param. of interest θ

Hypothesis Testing - from Bob Cousins

Two hypotheses

H_0: null hypothesis. "The SM is a true description of nature @ scales probed by the LHC"

H_1: alternative hypothesis. "[INSERT YOUR MODEL HERE] is a true description of nature @ scales probed by the LHC"

$L(X, \theta)$ diff. for each H_i. How do we test H_0 vs H_1?

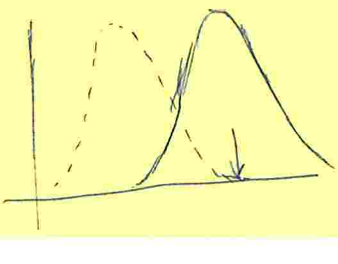
• For H_0, order possible observ. x from least extreme to most extreme using an order. principle (which can depend on H_1 as well). Choose a cutoff α (smallish number)

"Reject" H_0 if observed x_0 is in the most extreme fraction α of observ. x (generated under H_0) By construction

→ α: prob (with x gen. according to H_0) of rejecting H_0 when it's true

→ β: prob (with x gen. according to H_1) of not rejecting H_0 when it's false

Expected distribution of $2\log(L(0^-)/L(0^+))$ under pure pseudoscalar (--) and pure scalar hypotheses (histograms) for Higgs boson. Arrow indicates measured value @ CMS



A decision on whether or not to declare discovery (falsifying H_0) requires 2 more inputs, both of which can affect the choice of α

· Prior belief in H_0 vs H_1

· Cost of Type I error (false discovery claim) vs cost of Type II error (missed discovery)

A one-size-fits-all criterion of α corresponds to 5σ is without foundation!

Example: Ratio of Hadronic and Leptonic Cross-Sections in e^+e^- Reactions

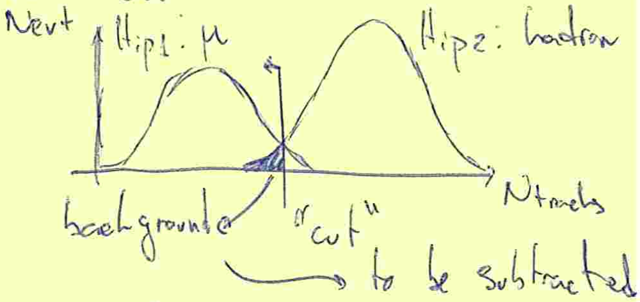
- Energy range where 5 quark flavours contribute and below Z resonance (for lowest order in perturbation theory)

$$R_\gamma = \frac{\sigma(e^+e^- \to \text{hadron})}{\sigma(e^+e^- \to \mu^+\mu^-)} = \frac{\sigma_{had}}{\sigma_{lep}}$$
$$= N_c \sum_q e_q^2 = N_c \frac{11}{9}$$

- Goal: determine or constrain number of colour states (N_c)
- Count # of events with hadronic (N_{had}) and leptonic (N_{lep}) final states

$$R_\gamma = \frac{\sigma_{had}}{\sigma_{lep}} = \frac{N_{had}/\mathcal{L}}{N_{lep}/\mathcal{L}} = \frac{N_{had}}{N_{lep}}$$

- Define event selection and estimate backgrounds, possibly with input from simu. to define discrimination var.
  - # of charged particles (N_{tracks}) in each event helps separate hadronic & leptonic events. Usually leptonic have few tracks, whilst hadronic has many more
  - There is some overlap: the selection has an eff. $\epsilon \leq 1$ to select given type of event



$$N^{meas}_{had/lep} = \epsilon^{meas}_{had/lep} \cdot N^{true}_{had/lep}$$

- $N^{meas}_{had/lep}$ should be corrected
  - ↳ in practice, we estimate the ϵs
    - ↳ with either simu. or data

Analysis

- ϵ is not only due to cut: $\epsilon = A \cdot \epsilon_{tri} \cdot \epsilon_{rec} \cdot \epsilon_{cut}$ (trigger, reconstruction, cut; ↳ det acceptance)
- background subtraction:

  $N^{true}_{lep} = (N^{meas}_{lep} - N_{bckg}) / \epsilon_{lep}$
- stat. uncertainties
  - can be estimated using all stat distrib. followed in each measurement
  - for counting exp. Poisson can be proper choice
  - for efficiency measurement (pass or fail), uncertainty following binomial would be preferable

Tomi

· · syst. uncertainties

· in general, depends on each analysis (no standard procedure)

- e.g. in the present case we could assign an uncertainty associated to the mismodelling of det. response by the simu., whic was used for efficiency correction

Measurements of R from different lepton colliding

- the relation for R_r in previous slides applies to region $\sqrt{s} > 10$ GeV (where quarks u,d,s,c,b contribute) and for from Z boson peak

· in this case, we have

$$R_r = N_c \frac{11}{9} \approx \frac{11}{3} \Rightarrow N_c \approx 3$$

Conclusions

HEP: field that studies the smallest building blocks of matter.

↳ equally powered by contributions from theorists, experimentalists, computer scientists, engineers... the harmonious coop. of these diff. groups is fund. to the success of the field

↳ theoretical side

↳ field has continued

↳ resounding success with SM of particles & fields

↳ extensions to SM continue to be proposed

↳ exploring new ideas and addressing additional data from astrophysics and cosmology

↳ experimental side

↳ field has moved to global colabs that design, build and operate extremely large and complex detectors

↳ data taken with detectors dwarfs all other scientific datasets to date and allow to measure properties of particles and fields to unprecedent precision

↳ long-term endeavour: exp time scales measured in decades

field is preparing for the challenges ahead, with new exps being proposed all around the world ionei

Finally, the LHC is scheduled to run at least until 2035.