test-endpaper

GEOMETRY FORMULAS

A = area, S = lateral surface area, V = volume, h = height, B = area of base, r = radius, l = slant height, C = circumference, s = arc length

Parallelogram	Triangle	Trapezoid	Circle	Sector
$\begin{vmatrix} h \\ A = bh \end{vmatrix}$	$A = \frac{1}{2}bh$	$\begin{vmatrix} \leftarrow a \rightarrow \\ h \\ A = \frac{1}{2}(a+b)h \end{vmatrix}$	$A = \pi r^2, C = 2\pi r$	$ \leftarrow r \rightarrow $ $A = \frac{1}{2}r^{2}\theta, s = r\theta$ $(\theta \text{ in radians})$
Right Circular Cylinder	Right Circular Cone	Any Cylinder or Prisr	n with Parallel Bases	Sphere
<u>↑</u> h				
$V = \pi r^2 h, S = 2 \pi r h$	$V = \frac{1}{3}\pi r^2 h, S = \pi r l$	V = Bh		$V = \frac{4}{3}\pi r^3, S = 4\pi r^2$

ALGEBRA FORMULAS

THE QUADRATIC FORMULA	THE BINOMIAL FORMULA			
The solutions of the quadratic equation $ax^2 + bx + c = 0$ are	$(x+y)^n = x^n + nx^{n-1}y + \frac{n(n-1)}{1 \cdot 2}x^{n-2}y^2 + \frac{n(n-1)(n-2)}{1 \cdot 2 \cdot 3}x^{n-3}y^3 + \dots + nxy^{n-1} + y^n$			
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	$(x-y)^n = x^n - nx^{n-1}y + \frac{n(n-1)}{1 \cdot 2}x^{n-2}y^2 - \frac{n(n-1)(n-2)}{1 \cdot 2 \cdot 3}x^{n-3}y^3 + \dots \pm nxy^{n-1} \mp y^n$			

TABLE OF INTEGRALS

BASIC FUNCTIONS

1.
$$\int u^n du = \frac{u^{n+1}}{n+1} + C$$

$$2. \int \frac{du}{u} = \ln|u| + C$$

$$3. \int e^u du = e^u + C$$

$$4. \int \sin u \, du = -\cos u + C$$

$$5. \int \cos u \, du = \sin u + C$$

$$\mathbf{6.} \quad \int \tan u \, du = \ln|\sec u| + C$$

7.
$$\int \sin^{-1} u \, du = u \sin^{-1} u + \sqrt{1 - u^2} + C$$

8.
$$\int \cos^{-1} u \, du = u \cos^{-1} u - \sqrt{1 - u^2} + C$$

9.
$$\int \tan^{-1} u \, du = u \tan^{-1} u - \ln \sqrt{1 + u^2} + C$$

$$10. \int a^u du = \frac{a^u}{\ln a} + C$$

$$11. \int \ln u \, du = u \ln u - u + C$$

$$12. \int \cot u \, du = \ln|\sin u| + C$$

13.
$$\int \sec u \, du = \ln|\sec u + \tan u| + C$$
$$= \ln\left|\tan\left(\frac{1}{4}\pi + \frac{1}{2}u\right)\right| + C$$

14.
$$\int \csc u \, du = \ln|\csc u - \cot u| + C$$
$$= \ln|\tan \frac{1}{2}u| + C$$

15.
$$\int \cot^{-1} u \, du = u \cot^{-1} u + \ln \sqrt{1 + u^2} + C$$

16.
$$\int \sec^{-1} u \, du = u \sec^{-1} u - \ln|u + \sqrt{u^2 - 1}| + C$$

17.
$$\int \csc^{-1} u \, du = u \csc^{-1} u + \ln|u + \sqrt{u^2 - 1}| + C$$

test-endpaper

RECIPROCALS OF BASIC FUNCTIONS

$$18. \int \frac{1}{1 \pm \sin u} du = \tan u \mp \sec u + C$$

$$19. \int \frac{1}{1 \pm \cos u} du = -\cot u \pm \csc u + C$$

20.
$$\int \frac{1}{1 \pm \tan u} du = \frac{1}{2} (u \pm \ln|\cos u \pm \sin u|) + C$$

21.
$$\int \frac{1}{\sin u \cos u} du = \ln |\tan u| + C$$

22.
$$\int \frac{1}{1 \pm \cot u} du = \frac{1}{2} (u \mp \ln|\sin u \pm \cos u|) + C$$

$$23. \int \frac{1}{1 \pm \sec u} du = u + \cot u \mp \csc u + C$$

24.
$$\int \frac{1}{1 \pm \csc u} du = u - \tan u \pm \sec u + C$$

25.
$$\int \frac{1}{1+e^u} du = u - \ln(1 \pm e^u) + C$$

POWERS OF TRIGONOMETRIC FUNCTIONS

26.
$$\int \sin^2 u \, du = \frac{1}{2}u - \frac{1}{4}\sin 2u + C$$

27.
$$\int \cos^2 u \, du = \frac{1}{2}u + \frac{1}{4}\sin 2u + C$$

$$28. \int \tan^2 u \, du = \tan u - u + C$$

29.
$$\int \sin^n u \, du = -\frac{1}{n} \sin^{n-1} u \cos u + \frac{n-1}{n} \int \sin^{n-2} u \, du$$

29.
$$\int \sin^n u \, du = -\frac{1}{n} \sin^{n-1} u \cos u + \frac{n-1}{n} \int \sin^{n-2} u \, du$$
30.
$$\int \cos^n u \, du = \frac{1}{n} \cos^{n-1} u \sin u + \frac{n-1}{n} \int \cos^{n-2} u \, du$$

31.
$$\int \tan^n u \, du = \frac{1}{n-1} \tan^{n-1} u - \int \tan^{n-2} u \, du$$

32.
$$\int \cot^2 u \, du = -\cot u - u + C$$

$$33. \int \sec^2 u \, du = \tan u + C$$

$$34. \int \csc^2 u \, du = -\cot u + C$$

35.
$$\int \cot^n u \, du = -\frac{1}{n-1} \cot^{n-1} u - \int \cot^{n-2} u \, du$$

35.
$$\int \cot^n u \, du = -\frac{1}{n-1} \cot^{n-1} u - \int \cot^{n-2} u \, du$$
36.
$$\int \sec^n u \, du = \frac{1}{n-1} \sec^{n-2} u \tan u + \frac{n-2}{n-1} \int \sec^{n-2} u \, du$$

37.
$$\int \csc^n u \, du = -\frac{1}{n-1} \csc^{n-2} u \cot u + \frac{n-2}{n-1} \int \csc^{n-2} u \, du$$

PRODUCTS OF TRIGONOMETRIC FUNCTIONS

38.
$$\int \sin mu \sin nu \, du = -\frac{\sin(m+n)u}{2(m+n)} + \frac{\sin(m-n)u}{2(m-n)} + C$$
39.
$$\int \cos mu \cos nu \, du = \frac{\sin(m+n)u}{2(m+n)} + \frac{\sin(m-n)u}{2(m-n)} + C$$

39.
$$\int \cos mu \cos nu \, du = \frac{\sin(m+n)u}{2(m+n)} + \frac{\sin(m-n)u}{2(m-n)} +$$

40.
$$\int \sin mu \cos nu \, du = -\frac{\cos(m+n)u}{2(m+n)} - \frac{\cos(m-n)u}{2(m-n)} + C$$

40.
$$\int \sin mu \cos nu \, du = -\frac{\cos(m+n)u}{2(m+n)} - \frac{\cos(m-n)u}{2(m-n)} + C$$
41.
$$\int \sin^m u \cos^n u \, du = -\frac{\sin^{m-1} u \cos^{n+1} u}{m+n} + \frac{m-1}{m+n} \int \sin^{m-2} u \cos^n u \, du$$

$$= \frac{\sin^{m+1} u \cos^{n-1} u}{m+n} + \frac{n-1}{m+n} \int \sin^m u \cos^{n-2} u \, du$$

PRODUCTS OF TRIGONOMETRIC AND EXPONENTIAL FUNCTIONS

42.
$$\int e^{au} \sin bu \, du = \frac{e^{au}}{a^2 + b^2} (a \sin bu - b \cos bu) + C$$

43.
$$\int e^{au} \cos bu \, du = \frac{e^{au}}{a^2 + b^2} (a \cos bu + b \sin bu) + C$$

POWERS OF *u* MULTIPLYING OR DIVIDING BASIC FUNCTIONS

$$44. \int u \sin u \, du = \sin u - u \cos u + C$$

$$45. \int u \cos u \, du = \cos u + u \sin u + C$$

46.
$$\int u^2 \sin u \, du = 2u \sin u + (2 - u^2) \cos u + C$$

47.
$$\int u^2 \cos u \, du = 2u \cos u + (u^2 - 2) \sin u + C$$

48.
$$\int u^n \sin u \, du = -u^n \cos u + n \int u^{n-1} \cos u \, du$$

49. $\int u^n \cos u \, du = u^n \sin u - n \int u^{n-1} \sin u \, du$

50.
$$\int u^n \ln u \, du = \frac{u^{n+1}}{(n+1)^2} [(n+1) \ln u - 1] + C$$

51.
$$\int ue^u \, du = e^u (u-1) + C$$

52.
$$\int u^n e^u \, du = u^n e^u - n \int u^{n-1} e^u \, du$$

53.
$$\int u^{n} a^{u} du = \frac{u^{n} a^{u}}{\ln a} - \frac{n}{\ln a} \int u^{n-1} a^{u} du + C$$
54.
$$\int \frac{e^{u} du}{u^{n}} = -\frac{e^{u}}{(n-1)u^{n-1}} + \frac{1}{n-1} \int \frac{e^{u} du}{u^{n-1}}$$
55.
$$\int \frac{a^{u} du}{u^{n}} = -\frac{a^{u}}{(n-1)u^{n-1}} + \frac{\ln a}{n-1} \int \frac{a^{u} du}{u^{n-1}}$$

55.
$$\int \frac{a^u du}{u^n} = -\frac{a^u}{(n-1)u^{n-1}} + \frac{\ln a}{n-1} \int \frac{a^u du}{u^{n-1}}$$

$$56. \int \frac{du}{u \ln u} = \ln |\ln u| + C$$

POLYNOMIALS MULTIPLYING BASIC FUNCTIONS

57.
$$\int p(u)e^{au} du = \frac{1}{a}p(u)e^{au} - \frac{1}{a^2}p'(u)e^{au} + \frac{1}{a^3}p''(u)e^{au} - \cdots$$
 [signs alternate: $+ - + - \cdots$]

58.
$$\int p(u)\sin au \, du = -\frac{1}{a}p(u)\cos au + \frac{1}{a^2}p'(u)\sin au + \frac{1}{a^3}p''(u)\cos au - \cdots \quad \text{[signs alternate in pairs after first term: } + + - - + + - - \cdots \text{]}$$

59.
$$\int p(u) \cos au \, du = \frac{1}{a} p(u) \sin au + \frac{1}{a^2} p'(u) \cos au - \frac{1}{a^3} p''(u) \sin au - \cdots$$
 [signs alternate in pairs: $+ + - - + + - - \cdots$]

60.
$$\int \frac{u \, du}{a + bu} = \frac{1}{b^2} [bu - a \ln|a + bu|] + C$$

64.
$$\int \frac{u \, du}{(a+bu)^3} = \frac{1}{b^2} \left[\frac{a}{2(a+bu)^2} - \frac{1}{a+bu} \right] + C$$

61.
$$\int \frac{u^2 du}{a + bu} = \frac{1}{b^3} \left[\frac{1}{2} (a + bu)^2 - 2a(a + bu) + a^2 \ln|a + bu| \right] + C$$
 65.
$$\int \frac{du}{u(a + bu)} = \frac{1}{a} \ln\left| \frac{u}{a + bu} \right| + C$$

65.
$$\int \frac{du}{u(a+bu)} = \frac{1}{a} \ln \left| \frac{u}{a+bu} \right| + C$$

62.
$$\int \frac{u \, du}{(a+bu)^2} = \frac{1}{b^2} \left[\frac{a}{a+bu} + \ln|a+bu| \right] + C$$

66.
$$\int \frac{du}{u^2(a+bu)} = -\frac{1}{au} + \frac{b}{a^2} \ln \left| \frac{a+bu}{u} \right| + C$$

63.
$$\int \frac{u^2 du}{(a+bu)^2} = \frac{1}{b^3} \left[bu - \frac{a^2}{a+bu} - 2a \ln|a+bu| \right] + C$$

67.
$$\int \frac{du}{u(a+bu)^2} = \frac{1}{a(a+bu)} + \frac{1}{a^2} \ln \left| \frac{u}{a+bu} \right| + C$$

RATIONAL FUNCTIONS CONTAINING $a^2\pm u^2$ IN THE DENOMINATOR (a>0)

68.
$$\int \frac{du}{a^2 + u^2} = \frac{1}{a} \tan^{-1} \frac{u}{a} + C$$

70.
$$\int \frac{du}{u^2 - a^2} = \frac{1}{2a} \ln \left| \frac{u - a}{u + a} \right| + C$$

69.
$$\int \frac{du}{a^2 - u^2} = \frac{1}{2a} \ln \left| \frac{u + a}{u - a} \right| + C$$

71.
$$\int \frac{bu+c}{a^2+u^2} du = \frac{b}{2} \ln(a^2+u^2) + \frac{c}{a} \tan^{-1} \frac{u}{a} + C$$

72.
$$\int \sqrt{u^2 + a^2} \, du = \frac{u}{2} \sqrt{u^2 + a^2} + \frac{a^2}{2} \ln (u + \sqrt{u^2 + a^2}) + C$$

75.
$$\int \frac{du}{\sqrt{u^2 + a^2}} = \ln(u + \sqrt{u^2 + a^2}) + C$$

INTEGRALS OF
$$\sqrt{a^2 + u^2}$$
, $\sqrt{a^2 - u^2}$, $\sqrt{u^2 - a^2}$ AND THEIR RECIPROCALS $(a > 0)$

72. $\int \sqrt{u^2 + a^2} du = \frac{u}{2} \sqrt{u^2 + a^2} + \frac{a^2}{2} \ln(u + \sqrt{u^2 + a^2}) + C$

75. $\int \frac{du}{\sqrt{u^2 + a^2}} = \ln(u + \sqrt{u^2 + a^2}) + C$

76. $\int \frac{du}{\sqrt{u^2 - a^2}} = \ln|u + \sqrt{u^2 - a^2}| + C$

76.
$$\int \frac{du}{\sqrt{u^2 - a^2}} = \ln|u + \sqrt{u^2 - a^2}| + C$$

74.
$$\int \sqrt{a^2 - u^2} \, du = \frac{u}{2} \sqrt{a^2 - u^2} + \frac{a^2}{2} \sin^{-1} \frac{u}{a} + C$$

77.
$$\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1} \frac{u}{a} + C$$

POWERS OF u MULTIPLYING OR DIVIDING $\sqrt{a^2 - u^2}$ OR ITS RECIPROCAL

$$78. \int u^2 \sqrt{a^2 - u^2} \, du = \frac{u}{8} (2u^2 - a^2) \sqrt{a^2 - u^2} + \frac{a^4}{8} \sin^{-1} \frac{u}{a} + C$$

$$81. \int \frac{u^2 \, du}{\sqrt{a^2 - u^2}} = -\frac{u}{2} \sqrt{a^2 - u^2} + \frac{a^2}{2} \sin^{-1} \frac{u}{a} + C$$

81.
$$\int \frac{u^2 du}{\sqrt{a^2 - u^2}} = -\frac{u}{2} \sqrt{a^2 - u^2} + \frac{a^2}{2} \sin^{-1} \frac{u}{a} + C$$

79.
$$\int \frac{\sqrt{a^2 - u^2} \, du}{u} = \sqrt{a^2 - u^2} - a \ln \left| \frac{a + \sqrt{a^2 - u^2}}{u} \right| + C$$
82.
$$\int \frac{du}{u\sqrt{a^2 - u^2}} = -\frac{1}{a} \ln \left| \frac{a + \sqrt{a^2 - u^2}}{u} \right| + C$$

82.
$$\int \frac{du}{u\sqrt{a^2 - u^2}} = -\frac{1}{a} \ln \left| \frac{a + \sqrt{a^2 - u^2}}{u} \right| + C$$

80.
$$\int \frac{\sqrt{a^2 - u^2} \, du}{u^2} = -\frac{\sqrt{a^2 - u^2}}{u} - \sin^{-1} \frac{u}{a} + C$$

83.
$$\int \frac{du}{u^2 \sqrt{a^2 - u^2}} = -\frac{\sqrt{a^2 - u^2}}{a^2 u} + C$$

POWERS OF u MULTIPLYING OR DIVIDING $\sqrt{u^2 \pm a^2}$ OR THEIR RECIPROCA

84.
$$\int u\sqrt{u^2 + a^2} \, du = \frac{1}{3}(u^2 + a^2)^{3/2} + C$$

90.
$$\int \frac{du}{u^2 \sqrt{u^2 \pm a^2}} = \mp \frac{\sqrt{u^2 \pm a^2}}{a^2 u} + C$$

85.
$$\int u\sqrt{u^2 - a^2} \, du = \frac{1}{3}(u^2 - a^2)^{3/2} + C$$

91.
$$\int u^2 \sqrt{u^2 + a^2} \, du = \frac{u}{8} (2u^2 + a^2) \sqrt{u^2 + a^2} - \frac{a^4}{8} \ln\left(u + \sqrt{u^2 + a^2}\right) + C$$

86.
$$\int \frac{du}{u\sqrt{u^2+a^2}} = -\frac{1}{a} \ln \left| \frac{a+\sqrt{u^2+a^2}}{u} \right| + C$$

92.
$$\int u^2 \sqrt{u^2 - a^2} \, du = \frac{u}{8} (2u^2 - a^2) \sqrt{u^2 - a^2} - \frac{a^4}{8} \ln|u + \sqrt{u^2 - a^2}| + C$$

87.
$$\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a} \sec^{-1} \left| \frac{u}{a} \right| + C$$

93.
$$\int \frac{\sqrt{u^2 + a^2}}{u^2} du = -\frac{\sqrt{u^2 + a^2}}{u} + \ln(u + \sqrt{u^2 + a^2}) + C$$
94.
$$\int \frac{\sqrt{u^2 - a^2}}{u^2} du = -\frac{\sqrt{u^2 - a^2}}{u} + \ln|u + \sqrt{u^2 - a^2}| + C$$

88.
$$\int \frac{\sqrt{u^2 - a^2} \, du}{u} = \sqrt{u^2 - a^2} - a \sec^{-1} \left| \frac{u}{a} \right| + C$$

88.
$$\int \frac{\sqrt{u^2 - a^2} \, du}{u} = \sqrt{u^2 - a^2} - a \sec^{-1} \left| \frac{u}{a} \right| + C$$
95.
$$\int \frac{u^2}{\sqrt{u^2 + a^2}} \, du = \frac{u}{2} \sqrt{u^2 + a^2} - \frac{a^2}{2} \ln \left(u + \sqrt{u^2 + a^2} \right) + C$$

89.
$$\int \frac{\sqrt{u^2 + a^2} \, du}{u} = \sqrt{u^2 + a^2} - a \ln \left| \frac{a + \sqrt{u^2 + a^2}}{u} \right| + C$$

89.
$$\int \frac{\sqrt{u^2 + a^2}}{u} du = \sqrt{u^2 + a^2} - a \ln \left| \frac{a + \sqrt{u^2 + a^2}}{u} \right| + C$$
96.
$$\int \frac{u^2}{\sqrt{u^2 - a^2}} du = \frac{u}{2} \sqrt{u^2 - a^2} + \frac{a^2}{2} \ln |u + \sqrt{u^2 - a^2}| + C$$

97.
$$\int \frac{du}{(a^2 - u^2)^{3/2}} = \frac{u}{a^2 \sqrt{a^2 - u^2}} + C$$

INTEGRALS CONTAINING
$$(a^2 + u^2)^{3/2}$$
, $(a^2 - u^2)^{3/2}$, $(u^2 - a^2)^{3/2}$ $(a > 0)$

97.
$$\int \frac{du}{(a^2 - u^2)^{3/2}} = \frac{u}{a^2 \sqrt{a^2 - u^2}} + C$$
100.
$$\int (u^2 + a^2)^{3/2} du = \frac{u}{8} (2u^2 + 5a^2) \sqrt{u^2 + a^2} + \frac{3a^4}{8} \ln(u + \sqrt{u^2 + a^2}) + C$$

98.
$$\int \frac{du}{(u^2 \pm a^2)^{3/2}} = \pm \frac{u}{a^2 \sqrt{u^2 + a^2}} + C$$

98.
$$\int \frac{du}{(u^2 \pm a^2)^{3/2}} = \pm \frac{u}{a^2 \sqrt{u^2 \pm a^2}} + C$$
101.
$$\int (u^2 - a^2)^{3/2} du = \frac{u}{8} (2u^2 - 5a^2) \sqrt{u^2 - a^2} + \frac{3a^4}{8} \ln|u + \sqrt{u^2 - a^2}| + C$$

99.
$$\int (a^2 - u^2)^{3/2} du = -\frac{u}{8} (2u^2 - 5a^2) \sqrt{a^2 - u^2} + \frac{3a^4}{8} \sin^{-1} \frac{u}{a} + C$$

POWERS OF u MULTIPLYING OR DIVIDING $\sqrt{a + bu}$ OR ITS RECIPROCAL

$$\frac{102.}{\int u\sqrt{a+bu}\,du} = \frac{2}{15b^2}(3bu - 2a)(a+bu)^{3/2} + C$$

$$\frac{103.}{\int u^2\sqrt{a+bu}\,du} = \frac{2}{105b^3}(15b^2u^2 - 12abu + 8a^2)(a+bu)^{3/2} + C$$

$$\frac{104.}{\int u^n\sqrt{a+bu}\,du} = \frac{2u^n(a+bu)^{3/2}}{b(2n+3)} - \frac{2an}{b(2n+3)}\int u^{n-1}\sqrt{a+bu}\,du$$

$$\frac{2}{\sqrt{-a}}\tan^{-1}\sqrt{\frac{a+bu}{-a}} + C \quad (a < 0)$$

$$\frac{2}{\sqrt{-a}}\tan^{-1}\sqrt{\frac{a+bu}{-a}} + C \quad (a < 0)$$

$$\frac{105.}{\int \frac{u\,du}{\sqrt{a+bu}}} = \frac{2}{3b^2}(bu - 2a)\sqrt{a+bu} + C$$

$$\frac{106.}{\int \frac{u^2\,du}{\sqrt{a+bu}}} = \frac{2}{15b^3}(3b^2u^2 - 4abu + 8a^2)\sqrt{a+bu} + C$$

$$\frac{106.}{\int \frac{u^n\,du}{\sqrt{a+bu}}} = \frac{2u^n\sqrt{a+bu}}{b(2n+1)} - \frac{2an}{b(2n+1)}\int \frac{u^{n-1}\,du}{\sqrt{a+bu}}$$

$$\frac{111.}{\int \frac{\sqrt{a+bu}\,du}{u^n}} = -\frac{(a+bu)^{3/2}}{a(n-1)u^{n-1}} - \frac{b(2n-5)}{2a(n-1)}\int \frac{\sqrt{a+bu}\,du}{u^{n-1}}$$

POWERS OF u MULTIPLYING OR DIVIDING $\sqrt{2au-u^2}$ OR ITS RECIPROCAL

112.
$$\int \sqrt{2au - u^2} \, du = \frac{u - a}{2} \sqrt{2au - u^2} + \frac{a^2}{2} \sin^{-1} \left(\frac{u - a}{a} \right) + C$$
113.
$$\int u \sqrt{2au - u^2} \, du = \frac{2u^2 - au - 3a^2}{6} \sqrt{2au - u^2} + \frac{a^3}{2} \sin^{-1} \left(\frac{u - a}{a} \right) + C$$
117.
$$\int \frac{du}{u \sqrt{2au - u^2}} = -\frac{\sqrt{2au - u^2}}{au} + C$$
118.
$$\int \frac{u \, du}{\sqrt{2au - u^2}} = -\sqrt{2au - u^2} + a \sin^{-1} \left(\frac{u - a}{a} \right) + C$$
119.
$$\int \frac{u^2 \, du}{\sqrt{2au - u^2}} = -\frac{(u + 3a)}{2} \sqrt{2au - u^2} + \frac{3a^2}{2} \sin^{-1} \left(\frac{u - a}{a} \right) + C$$

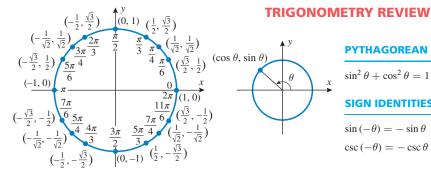
INTEGRALS CONTAINING $(2au - u^2)^{3/2}$

$$\frac{120. \int \frac{du}{(2au - u^2)^{3/2}} = \frac{u - a}{a^2 \sqrt{2au - u^2}} + C$$

$$121. \int \frac{u \, du}{(2au - u^2)^{3/2}} = \frac{u}{a\sqrt{2au - u^2}} + C$$

THE WALLIS FORMULA

122.
$$\int_0^{\pi/2} \sin^n u \, du = \int_0^{\pi/2} \cos^n u \, du = \frac{1 \cdot 3 \cdot 5 \cdot \dots \cdot (n-1)}{2 \cdot 4 \cdot 6 \cdot \dots \cdot n} \cdot \frac{\pi}{2} \begin{pmatrix} n \text{ an even} \\ \text{integer and} \\ n \ge 2 \end{pmatrix} \quad \text{or} \quad \frac{2 \cdot 4 \cdot 6 \cdot \dots \cdot (n-1)}{3 \cdot 5 \cdot 7 \cdot \dots \cdot n} \begin{pmatrix} n \text{ an odd} \\ \text{integer and} \\ n \ge 3 \end{pmatrix}$$



PYTHAGOREAN IDENTITIES

$$\sin^2 \theta + \cos^2 \theta = 1$$
 $\tan^2 \theta + 1 = \sec^2 \theta$ $1 + \cot^2 \theta = \csc^2 \theta$

SIGN IDENTITIES

$$\sin(-\theta) = -\sin\theta$$
 $\cos(-\theta) = \cos\theta$ $\tan(-\theta) = -\tan\theta$
 $\csc(-\theta) = -\csc\theta$ $\sec(-\theta) = \sec\theta$ $\cot(-\theta) = -\cot\theta$

COMPLEMENT IDENTITIES

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta \quad \cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta \quad \tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta \qquad \qquad \sin(\pi - \theta) = \sin\theta \qquad \cos(\pi - \theta) = -\cos\theta \quad \tan(\pi - \theta) = -\tan\theta$$

$$\csc(\pi - \theta) = \csc\theta \quad \sec(\pi - \theta) = -\cos\theta \quad \cot(\pi - \theta) = -\cot\theta$$

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$$\csc(\pi - \theta) = -\cos\theta \quad \cot(\pi - \theta) = -\cot\theta$$

$$\cot(\pi - \theta) = -\cot\theta$$

$$\cot(\pi$$

ADDITION FORMULAS

$$\sin{(\alpha + \beta)} = \sin{\alpha} \cos{\beta} + \cos{\alpha} \sin{\beta} \\ \sin{(\alpha - \beta)} = \sin{\alpha} \cos{\beta} - \cos{\alpha} \sin{\beta}$$

$$\tan{(\alpha + \beta)} = \frac{\tan{\alpha} + \tan{\beta}}{1 - \tan{\alpha} \tan{\beta}} \cos{(\alpha + \beta)} = \cos{\alpha} \cos{\beta} - \sin{\alpha} \sin{\beta}$$

$$\tan{(\alpha - \beta)} = \frac{\tan{\alpha} - \tan{\beta}}{1 + \tan{\alpha} \tan{\beta}} \cos{(\alpha - \beta)} = \cos{\alpha} \cos{\beta} + \sin{\alpha} \sin{\beta}$$

DOUBLE-ANGLE FORMULAS

HALF-ANGLE FORMULAS

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha \qquad \cos 2\alpha = 2 \cos^2 \alpha - 1 \\ \cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha \qquad \cos^2 \alpha = 1 - 2 \sin^2 \alpha \qquad \sin^2 \frac{\alpha}{2} = \frac{1 - \cos \alpha}{2} \qquad \cos^2 \frac{\alpha}{2} = \frac{1 + \cos \alpha}{2}$$