# National University of Computer & Emerging Sciences
# Karachi Campus

## Multivariable Calculus (MT2008) — Sessional-II Exam

Date: April 5th, 2024
Time: 8:30 am - 9:30 am
**Course Instructor(s)**
Dr. Fahad Riaz, Dr. Nazish Kanwal
Ms. Fareeha Sultan, MS. Alishba, & Ms. Uzma.

Total Time: 1 Hour
Total Marks: 30
Total Questions: 04

**Student Name** ______ **Roll No** ______ **Section** ______ **Student Signature** ______

**Attempt all questions. There are 4 questions and 1 page.**

**CLO #1**: Understand the basic concepts and know the basic techniques of differential and integral calculusof functions of several variables.

**Question 1** **[9 marks]**

(a) Let $f(x, y) = y\cos(2x) - \sin(2x)$.

i. $\boxed{\text{3 points}}$ Find the direction derivative of $f$ at $(0, 0)$ in the direction $\mathbf{i} - \mathbf{j}$.

ii. $\boxed{\text{2 points}}$ What is the value of the largest directional derivative of $f$ at $(0, 0)$.

(b) $\boxed{\text{4 points}}$ Find an equation for the tangent plane and parametric equations for the normal line to the surface $x^2y^3z^4 + xyz = 2$ at the point $(2, 1, -1)$.

**CLO #2**: Apply the theory to calculate the gradients, directional derivatives, arc length of curves, area of surfaces, and volume of solids. .

**Question 2** **[6 marks]**

(a) $\boxed{\text{4 points}}$ Evaluate the double integral over the rectangular region $R$.

$$\int\int_R \frac{xy}{x^2+1}\, dA; \quad R = \{(x, y) : 0 \le x \le 1,\ -3 \le y \le 3\}.$$

(b) $\boxed{\text{2 points}}$ Write a formula to find the volume of the solid enclosed between the surface $z = \frac{x}{y}$ and the rectangular region $R : 0 \le x \le 2,\ 1 \le y \le e^2$.

**CLO #3**: Solve problems involving maxima and minima, line integral and surface integral, and vector calculus.

**Question 3** **[ 5 marks]**

Compute the local minima of the given function by using **gradient descent algorithm** by taking **step size as 0.15** and initial point as **(2,2)**. Perform **three** iterations.

$$f(x, y) = 4x^2 + 2.5y^2 + 3xy - 5.5x - 4.1y$$

**CLO #3**: Solve problems involving maxima and minima, line integral and surface integral, and vector calculus.

**Question 4** **[10 marks]**

(a) $\boxed{\text{5 points}}$ Given the three points $P_1(1, 4)$, $P_2(5, 2)$, and $P_3(3, -2)$. Let

$$G(x, y) = (x-1)^2 + (y-4)^2 + (x-5)^2 + (y-2)^2 + (x-3)^2 + (y+2)^2$$

is the sum of the squares of the distances from point $P(x, y)$ to the three points $(P_1, P_2, \& P_3)$. Find the values of $x$ and $y$ so that this $G(x, y)$ is minimized.

(b) $\boxed{\text{5 points}}$ Use **Lagrange multipliers** to find the maximum and minimum values of the function **f** subject to the given constraint. Also find the points at which these values occurs.

$$f(x, y) = x^2 + y^2; \quad \text{subject to the constraint} \quad xy = 1.$$